

# ACIPDif22

## School-Workshop on Analysis, Control & Inverse Problems for Diffusive Systems with Application to Natural and Social Sciences

Bari, July 18-22, 2022



The poster features a light blue background with a large teal shape on the left. At the top left, there are logos for GUPBESG, STEP, and the European Union. A circular image of a building is on the top right. The text is arranged in columns, listing organizers, mini courses, and speakers. At the bottom, there are logos for the University of Bari Aldo Moro, the Department of Mathematics, INdAM, and MIUR-PRIN.

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**SCHOOL-WORKSHOP ON  
ANALYSIS, CONTROL & INVERSE  
PROBLEMS FOR DIFFUSIVE  
SYSTEMS WITH APPLICATION TO  
NATURAL AND SOCIAL SCIENCES**  
ACIPDif22

DIPARTIMENTO DI MATEMATICA  
UNIVERSITA' DI BARI ALDO MORO  
BARI, JULY 18-22, 2022

Also via zoom

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#### Speakers

- F. ALABAU-BOUSSOUIRA, *Université de Lorraine (Francia)*  
U. BICCARI, *Universidad de Deusto (Spagna)*  
L. BOCIU, *NC State University (USA)*  
F. BUCCI, *Università degli Studi di Firenze (Italia)*  
R. CAPUANI, *Università degli Studi della Tuscia (Italia)*  
M. CONTI, *Politecnico di Milano (Italia)*  
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Addolorata Salvatore, *Università degli Studi di Bari Aldo Moro*

## ACIPDif22

### School-Workshop on Analysis, Control & Inverse Problems for Diffusive Systems with Application to Natural and Social Sciences

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#### ABSTRACTS - TALKS

#### The Alabau-Boussouira's optimal-weight convexity method with applications to nonlinear stabilization of PDE's, ODE's, and their discretization schemes

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The purpose of this talk is to present the optimal-weight convexity method in the form introduced by the speaker in 2004 (see [1] for the announcement and [2] for the corresponding general presentation for abstract PDE's (with some first optimal energy estimates) and its further extensions in [6] for further sharp developments and general optimality results for the ODE cases, or for discretized PDE's. The general goal of the method is to derive constructive, sharp, and optimal (or quasi-optimal) energy estimates for nonlinearly damped PDE's or ODE's with a flexibility and robustness which allow extensions to various frames and related questions.

A general synthesis is presented in [7] with comparisons with previous existing results in the literature, their interests and their limitations (see the references [19-23]). The optimal-weight convexity method also allows to handle various applications to examples of concrete PDE's besides the wave equation with localized or boundary nonlinear dampings, such as as the plate equations in [3] and [9], combined with indirect closed loop control for linear and nonlinear stabilization of Timoshenko beams (see [4] and [9]), and for the nonlinear stabilization of coupled systems in velocities (see [14]), as well as for the nonlinear boundary stabilization of degenerate wave equation in [11] (see [10] for the first presentation of the results). The optimal-weight convexity method is also a key for sharp energy estimates for memory damped PDE's (see [5]).

The optimal-weight convexity method has also been extended to nonlinear stabilization of continuous and discretized PDE's in [13] (see also [12] for a first presentation) on first order abstract PDE's with various applications and for general uniform discretization schemes in space, in time, and in fully in space-time.

In the first part, we first show how to derive quasi-optimal energy decay rates for general nonlinear dissipations. This can be performed by a direct method (see [1],[2], [6], [7]) or an indirect method (see [8]).

We also give an overview on the optimality of energy decay rates, in particular for the semi-discretized PDE's is also presented as proved in [2] and [6] and compared to the ones obtained in [19] by Martinez and Vancostenoble. Note that the constructive energy estimates derived in [19] are not always optimal (see [2] and [6] for a proof). The proof of the optimality of the energy estimates resulting from the optimal-weight convexity method are based on an energy comparison principle and comparison between pointwise and energy estimates. Note that in the case of discretized PDE's, the resulting estimates given in [6] are nonuniform with respect to the discretization parameter.

The second part of the talk will be devoted to the discretized problems and uniformity with respect to the discretization parameters. This aspect is considered in [13] (see [12] also for a first presentation) with the optimal-weight convexity method combined with ideas coming from the pioneering works on the numerical aspects for control theory of Glowinski and Li (see [17], [18]), and Ervedoza and Zuazua [15] (see also [16]). More precisely we show how uniform optimal energy decay rates with respect to the discretization parameters can be derived by adding a numeric viscosity, in the nonlinear damping cases.

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## Multilevel control

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We discuss the multilevel control problem for dynamical systems, consisting in designing a piece-wise constant control function taking values in a finite-dimensional set. We start by providing a characterization of multilevel controls through an optimal control approach, based on the minimization of a suitable cost functional. We do this by focusing on the Selective Harmonic Modulation problem in power electronic engineering, a practical application of multilevel control, which motivates our study [1, 2]. In a second moment, we address the problem of multilevel control for linear ODE systems fulfilling the Kalman rank condition for controllability [3].

## References

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## Analysis and Control in Fluid Flows through Deformable Porous Media

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Fluid flows through deformable porous media are relevant for many applications in biology, medicine and bio-engineering, including tissue perfusion and fluid flow inside cartilages and bones. We are interested in perfusion inside the eye and its connection to the development of glaucoma. Mathematically, the problem translates into the study of a quasi-static nonlinear poroelastic system, which is a system of PDEs of mixed parabolic-elliptic type. We answer questions related to ocular tissue biomechanics via well-posedness, sensitivity analysis, and optimal control for the PDE coupled system applied to the eye.

## Linear quadratic optimal control of evolution equations with finite memory

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The well-posedness of the Riccati equations is a fundamental question within the optimal control problem with quadratic functionals for linear partial differential equations (PDEs). Indeed, solving the corresponding Riccati equation yields the optimal cost operator (and the gain operator) which occur in the feedback law, thus allowing the synthesis of the optimal control. Forty years of research on this subject have brought about distinct functional-analytic frameworks that mirror parabolic PDEs, hyperbolic PDEs, and also certain coupled systems of hyperbolic/parabolic PDEs, along with the respective Riccati theories. When it comes to evolution equations with memory, a most relevant reference is the work by P. Cannarsa, H. Frankowska and E.M. Marchini (2013), which pertains to the Bolza problem for semilinear PDEs with memory; the obtained results are consistent with the more general context of a nonlinear dynamics and non-quadratic functionals to

be minimized. The aim of this talk is to outline how the approach utilized in the study of the linear-quadratic problem for memoryless control systems proves successful to deal with certain evolution equations with finite memory, thereby providing a first extension (to the realm of PDEs) of the Riccati-based theory recently devised by L. Pandolfi in a finite dimensional context.

(The talk is based on ongoing joint work with P. Acquistapace (Pisa).)

### **Random lift of set-valued maps and applications**

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We discuss some preliminary results about the lifting of set-valued maps defined between Polish spaces to set-valued maps defined between the corresponding spaces of probability measures. In particular, we are interested to establish conditions granting that some relevant properties (for instance semicontinuity, compactness of the images, Lipschitz continuity,...) of the original set-valued maps are conserved also in the lifted map.

The main motivation of the study is the dynamics and the control of multi-agent systems, where the macroscopical trajectory can be seen as the lift of the solution set-valued map of a differential inclusion expressing the microscopical dynamics of the agent, however our results can be extended to more general set-valued maps of agent trajectories provided that they enjoys some properties.

These results have been obtained in collaboration with Antonio Marigonda (University of Verona) and Michele Ricciardi (University of Verona).

### **Some unexplored questions arising in linear viscoelasticity**

*Monica Conti*

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We consider an abstract integrodifferential equation modeling the dynamics of linearly viscoelastic solids. The equation is known to generate a semigroup on a certain phase space, whose asymptotic properties have been the object of extensive studies in the last decades. Nevertheless, some relevant questions still remain open, with particular reference to the decay rate of the semigroup compared to the decay of the memory kernel, and to the structure of the spectrum of the infinitesimal generator of the semigroup. In this talk, we will briefly review the state of the art, and we provide some answers. Joint work with Filippo Dell'Oro and Vittorino Pata.

### **On the simultaneous controllability of parabolic systems**

*Luz De Teresa*

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In this talk we present some recent results concerning the null controllability of two uncoupled parabolic systems when we act with the same distributed control. In specific, we give some sufficient conditions in the case of two heat equations (work in collaboration with F. Araruna and F. Chaves-Silva) and in the case of two Stokes systems (work in collaboration with T. Takahashi and Y. Wu-Zhang). The key point in both results is to obtain an observability inequality of the adjoint systems with observation of the norm of the sum of the solutions of the adjoint equations on the control set. To this end we work with an extended "adjoint system" and prove a Carleman inequality in this new setting.

## Deep Learning Approaches to Earth Observation Change Detection

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The interest in change detection in the field of remote sensing has increased in the last few years. Searching for changes in satellite images has many useful applications, ranging from land cover and land use analysis to anomaly detection. In particular, urban change detection provides an efficient tool to study urban spread and growth through several years of observation.

At the same time, change detection is often a computationally challenging and time-consuming task; therefore, a standard approach with manual detection of the elements of interest by experts in the domain of Earth Observation needs to be replaced by innovative methods that can guarantee optimal results with unquestionable value and within reasonable time.

We will present two different approaches to change detection (semantic segmentation and classification) that both exploit convolutional neural networks to address these particular needs, which can be further refined and used in post-processing workflows for a large variety of applications.

## Approximate control for the nonlinear Schrödinger equation on a torus via bilinear controls

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We consider the nonlinear Schrödinger equation (NLS) on a torus of arbitrary dimension. We assume the presence of an external potential field whose time-dependent amplitude plays the role of control. We ensure the approximate controllability between any pair of eigenstates with respect to the  $L^2$ -norm in arbitrarily small time. The result is proved by developing a multiplicative version of a geometric control approach introduced by Agrachev and Sarychev for additive controls. Our theory exploits specific saturation properties to construct dynamics for the (NLS) steering any eigenmode close to any other in any positive time.

Joint work in collaboration with Vahagn Nersesyan.

## Reachable spaces for heat equations with lower order terms

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he goal of this talk is to explain how perturbative arguments can be applied to derive a sharp description of the reachable space for heat equations having lower order terms.

The main result I will present is the following one. Let us consider an abstract system  $y' = Ay + Bu$ , where  $A$  is an operator generating a  $C^0$  semigroup  $(\exp(tA))_{t \geq 0}$  on a Hilbert space  $X$ , and  $B$  is a control operator, for instance a linear operator from an Hilbert space  $U$  to  $X$ , and let us assume that this system is null-controllable in  $X$  in any positive time. Then, setting  $R$  the reachable set of the system (that is all the states that can be achieved by y solution of  $y' = Ay + Bu, y(0) = 0$ ), the restriction of  $(\exp(tA))_{t \geq 0}$  to  $R$  forms a  $C^0$  semigroup on  $R$ .

Accordingly, the system  $y' = Ay + Bu$  is exactly controllable on  $R$ , and one can then perform classical perturbative arguments to handle lower order terms, as I will explain on a few examples.

This talk is based on a joint work with Kévin Le Balc'h (INRIA Paris) and Marius Tucsnak (Bordeaux).

## Controllability of Evolution Equations arising in Energy Balance Climate Models

*Giuseppe Floridia*

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In this talk we present some Energy Balance Climate Models (EBCM) and we prove some results concerning the approximate controllability of degenerate parabolic equations.

## Stabilizing controllers with improved performance for nonlinear PDEs: a NMPC approach

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We present a flexible and general method to ensure the stability of a Model Predictive Control (MPC) scheme applied to several nonlinear partial differential equations (PDEs) with the shortest possible horizon  $N = 2$  (known as instantaneous control). A suitable exponential controllability condition ensures the stability of the MPC scheme, deduced from exponential stability estimates given by an explicit feedback controller. In this way, we design an optimal stabilizing control with respect to given cost functionals. We apply the method to several examples, such as the linear heat equation, the Burgers' equation with Neumann boundary conditions, and the Schlögl system. Numerical simulations show the effectiveness of the method and the improved performance in the optimization process.

## Stabilization from the boundary in a third order in time nonlinear dynamics with applications to nonlinear acoustics

*Irena Lasiecka*

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A third-order (in time) Partial Differential Equation (PDE) systems arise naturally in a variety of second order PDE models where time relaxation parameter accounts for an extra derivative, which then leads to a singularly perturbed dynamics. It has been known since the sixties that such models, even in linear case, may be ill-posed in the sense of semigroups. This has motivated an extensive studies of third order dynamics from the point of view of semigroup theory and related stabilization. A class of third order models arising in nonlinear acoustics will be discussed. Such nonlinear (quasi-linear) Partial Differential Equation (PDE) describes nonlinear propagations of high frequency acoustic waves and it is motivated by an array of applications in engineering and medical sciences-including high intensity focused ultrasound [HIFU] technologies. The important feature is that the model resolves an infinite speed of propagation paradox associated with a classical second order in time equation. Replacing a classical heat transfer by heat waves gives rise to the third order in time derivative scaled by a small parameter  $\tau > 0$ , the latter represents the thermal relaxation time parameter and is intrinsic to the properties of the medium where the dynamics occurs. Stability/unstability of **heat-waves** in this hyperbolic like dynamics depend on the parameter which is a function of several variables such as :diffusivity, speed of sound and time relaxation. For the "critical" value of this parameter the dynamics exhibits an abrupt change from uniform stability to chaotic behavior. It is thus of interest to study "critical" case and to devise stabilization strategies which allow to obtain a controlled long time behavior. Of particular interest are feedback controls which are implemented on a part of the boundary only.

The aim of the lecture is to provide a brief overview of recent results in the area which is pertinent to boundary stabilization of both linear and non-linear third order dynamics in the "critical" case. Peculiar features associated with the third order dynamics lead to novel phenomenological behaviors. Applications to on-line feedback control will be also presented.

The work presented is in collaboration with Marcelo Bongarti and Jose Rodriguez.

## **Observability of infinitely many beams**

*Paola Loreti*

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We investigate the simultaneous observability of infinite systems of vibrating beams. We give an introduction discussing the observability of one beam, then we consider the problem of observability of infinitely many beams. The talk focuses on a Fourier approach and the main tool is a generalization of a theorem of Ingham. The talk is based on a joint work with V. Komornik and A. C. Lai.

## **References**

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## **The Maximum Principle for Lumped-Distributed Control Systems**

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In this work we derive necessary conditions of optimality for optimal control problems for lumped-distributed control systems.

Lumped-distributed control systems are assemblages of interconnected subsystems, some of which have a finite dimensional state spaces while other infinite dimensional state spaces.

Our analysis allows for endpoint state constraints, pathwise state constraints and terminal costs, that depend on state variable components associated with the lumped components of the overall system. It also allows for non-smooth data.

The distinctive feature of this system description is that the inhomogeneous term in the state equation, cost and constraints functions, are affected only by the projection of the state onto a finite dimensional subspace of the state space.

Descriptions of this nature arise in mechanical energy transmission systems, such as wind generators, automobile and aeronautical engineering. But they are also encountered in communication systems where, for example, a transmission line has an active load, and in thermal systems where a distributed thermal channel interacts with heat sinks and sources. This description also arises in the optimal control of hereditary systems, when the dynamic constraint is reformulated as a ‘delay-free’ evolution equation with an infinite dimensional state space.

A number of difficulties are encountered when we attempt to model the derivation of necessary conditions for problems with infinite dimensional state spaces on the standard, finite dimensional theory.

The most notable of these is that, for problems with constraints and problems with non-smooth data, perturbational methods, based on approximation and passage to the limit, may fail.

These difficulties are all, broadly speaking, connected with the fact that general infinite dimensional systems lack the compactness properties that are a key element in finite dimensional analysis.

This work identifies a class of infinite dimensional systems, encountered across a range of engineering disciplines, where the difficulties discussed above can be overcome.

Joint work with Richard Vinter

## **Asymptotic behavior of solutions to Hamilton-Jacobi-Bellmann equations**

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The analysis of the ergodic behavior of solutions to Hamilton- Jacobi-Bellmann equations has a long history going back to the seminal paper by [Lions, P.-L., Papanicolaou, G. and Varadhan, S.R.S]. Since this work, the subject has grown very fast and when the Hamiltonian is of Tonelli type a large number of results have been proved. A full characterization of the ergodic behavior of solutions to Tonelli Hamilton-Jacobi equations can be found in the celebrated weak KAM theory and Aubry-Mather theory. However, few results are available if the Hamiltonian fails to be Tonelli, i.e., the Hamiltonian is neither strictly convex nor coercive with respect to the momentum variable. In particular, such results cover only some specific structure and so, the general problem is still open. In this talk, I will present some recent results obtained in collaboration with Piermarco Cannarsa and Pierre Cardaliaguet concerning the long time-average behavior of solutions to Hamilton-Jacobi-Bellman equations. We will look, first, to the case of control of acceleration and, then, to sub-Riemannian control systems. Finally, we conclude this talk showing how the previous analysis applies to mean field game systems.

## **Modelling the olive quick decline syndrome epidemic: a short review**

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The olive quick decline syndrome (OQDS) is a devastating vascular disease caused by *Xylella fastidiosa* subsp. *pauca* (Xfp), a rod-shaped, Gram-negative, and quarantine bacteria for the European Union. The disease appeared few years ago, in a grove located near the city of Gallipoli, on the Ionian coast of the Salento peninsula (Apulia region, south-east Italy), and spread fast towards the North, in the provinces of Brindisi, Taranto, reaching the southern part of the Bri provive. To date, the pathogen spread on almost 50% of the Apulian territory, covering an area with more than 20 million of olive trees. The pathogen is transmitted by xylem sap-feeding insects, belonging to the order Hemiptera, among which *Philaenus spumarius*, *P. italosignus* and *Neophilaenus campestris* are the most important. Being a quarantine pathogen for the EU territory, control measures are under the coordination and responsibility of the Phytosanitary Authorities, and to date they rely on containment and mitigation measures, such as inspections, sampling, infected tree removal and vector control. Since the disease appearance, and due to the complexity of the variables driving the epidemics, detailed and rigorous mathematical and statistical analysis have been pursued. The OQDS spatio-temporal epidemics has three main pillars, i.e., the bacterial aggressiveness, the wide host range, and the vectors abundance and ubiquity. However, many other factors such as susceptible host density, human infrastructures, and interventions, as well as climatic conditions greatly contribute to shaping the spread and the control of the disease. Investigations on these aspects are crucial to understand drivers of entry, establishment, and spatiotemporal spread, as well as the type, timing, and effectiveness of disease management strategies. A short review of the recent papers dealing with the development of mathematical and statistical model of OQDS epidemic will be presented.

Joint work with M. Carlucci and G. Fragnelli

## **Recent results on the Hegselmann-Krause opinion formation models with time-dependent time delays**

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The Hegselmann-Krause model has been introduced in order to describe the evolution of  $N$  agents' opinions. In this talk, we discuss some recent results on the Hegselmann-Krause model with time-dependent time delay, seen here as a reaction time or a time for each agent to receive information from other agents. We will show that consensus is achieved even with more general time delays. Since the parameters appearing in the model are independent of the number of agents, we are able to study the transport equation associated to the particle system, obtained as its mean-field limit. Furthermore, we present a delayed control problem, namely a Hegselmann-Krause-type model with the presence of leadership. We will show that there exists a control which steers all agents to a prefixed state.

Joint work with Cristina Pignotti (University of L'Aquila)

### **On the metric regularity of affine optimal control problems and applications**

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In this talk we discuss some properties of the type of metric regularity of the set-valued map associated with the system of necessary optimality conditions for optimal control problems that are affine with respect to the control (shortly, affine problems). For such problems it is reasonable to extend the standard notions of metric regularity by involving two metrics in the image space of the map. The results describe the stability properties of the problem under perturbations.

### **Weak solutions for time-fractional evolution equations**

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In this talk we examine a definition for weak solutions of abstract fractional differential equations, introduced in [1]. The time-fractional derivative occurring in the equations is in the sense of the Caputo derivative. The abstract theory developed can be applied, for example, to two concrete equations of great interest in the theory of time-fractional partial differential equations: fractional diffusion-wave equations and fractional Petrovsky systems. Formalising the adequate definition for weak solutions is also useful to establish a result of hidden regularity for fractional Petrovsky systems, as we achieve in [2].

### **References**

- [1] P. Loreti, D. Sforza, Weak Solutions for Time-Fractional Evolution Equations in Hilbert Spaces. *Fractal Fract.* 2021, 5, 138. <https://doi.org/10.3390/fractalfract5040138>
- [2] P. Loreti, D. Sforza, Trace regularity for biharmonic evolution equations with Caputo derivatives. arXiv:2108.03417, submitted.

### **Uniform stabilization of the Boussinesq system in Besov spaces of low regularity by a localized, finite dimensional feedback controller pair boundary, interior**

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The presentation will be based on paper [1], which in turn is stimulated by paper [2] and requires, in particular, the contributions of [3] and [4]. The Boussinesq system on a  $3d$ -bounded domain couples the Navier-Stokes equations with a suitable diffusion equation. One assumes at the start that the linearized version has an unstable equilibrium solution. One then constructs explicitly a static, finite dimensional pair  $v, u$  of feedback controllers that exponentially stabilizes the original non-linear system in the vicinity of the unstable equilibrium solution, in the uniform norm of a suitable Besov-space of low regularity, with tight indices. Here,  $v$  is a scalar Dirichlet boundary control for the thermal equation, acting on an arbitrarily small connected component of the boundary. Instead,  $u$  is  $(d-1)$ -internal control [ $(d-1)$  is a novelty due to the B's model] for the fluid equation acting on an arbitrarily small interior collar supported by the boundary support of  $v$ . The case of selecting a boundary-localized control this time for the Navier-Stokes equation, as in [2], will be studied next.

## References

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### Bilinear Control of degenerate wave equations

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In this talk I will present a result of bilinear local exact controllability in finite time  $T > 0$  along the ground state for a degenerate wave equation with Neumann boundary conditions (see [2]). Due to such boundary conditions, the ground state is a constant function. We prove that there exists a threshold value of time  $T_0$  such that for  $T > T_0$  a classical result of local controllability along the ground state can be achieved. Thus, in this case the reachable set is an entire neighbourhood of the reference trajectory. For  $T < T_0$ , we show that the reachable set from the ground state is contained in a  $C^1$ -submanifold of infinite codimension. For  $T = T_0$  and strong degeneracy, a classical result of local controllability can be proved (as in the case  $T > T_0$ ), except for a countable set of values of the degeneracy parameter. Finally, for  $T = T_0$  and weak degeneracy, the reachable set is a  $C^1$ -submanifold of codimension 1.

The strategy of the proof follows the one proposed by K. Beauchard in [1] for the controllability of a non-degenerate wave equation, and it consists in the resolution of a moment problem coupled with an inverse mapping theorem. However, because of the degeneracy of our operator, new difficulties arise. Indeed, while for  $T > T_0$  it suffices applying Ingham theory to solve the moment problem, for  $T = T_0$  we need to extend the Kadec  $\frac{1}{4}$  Theorem and finally, for  $T < T_0$ , we combine general results on non-minimal families of exponentials with density properties of the eigenvalues of our degenerate operator.

This is a joint work with P. Cannarsa and P. Martinez.

## References

- [1] K. BEAUCHARD, Local controllability and non-controllability for a  $1D$  wave equation with bilinear control, *Journal of Differential Equations*, **250**, 4: 2064–2098 (2011)
- [2] P. CANNARSA, P. MARTINEZ AND C. URBANI, Bilinear control of a degenerate hyperbolic equation, arXiv:2112.00636,

### **Inverse problem for a nonlinear Fisher-KPP equation**

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In this joint work with P. Martinez, we consider a reaction-diffusion model of biological invasion in which the evolution of the population is governed by several parameters among them the intrinsic growth rate.

The knowledge of this growth rate is essential to predict the evolution of the population, but it is a priori unknown for exotic invasive species. We prove uniqueness and unconditional Lipschitz stability for the corresponding inverse problem, taking advantage of the positivity of the solution inside the spatial domain and studying its behaviour near the boundary with maximum principles. Our results complement previous works by Cristofol and Roques.

### **Optimal controller design via Brunovsky's normal form**

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By using the Brunovsky normal form, we provide a reformulation of the problem consisting in finding the controller which minimizes the controllability cost for finite dimensional linear systems with scalar controls.

This reformulation provides a novel perspective to the optimal design problem in the sense that it does not require the matrix generating the dynamics to be diagonalizable, and does not entail a randomization procedure (as done in past literature in diffusion equations and waves).

The resulting optimization problem reduces to a minimization of the norm of the inverse of a change of basis matrix, and allows us to stipulate the existence of minimizers, as well as non-uniqueness due to an invariance of the cost with respect to orthogonal transformations.

We provide illustrations generated by numerical experiments to both visualize these artifacts and also stipulate further directions and open problems.

This is a joint work with Borjan Geshkovski, MIT.