Recovering the reaction and the diffusion coefficients in a linear parabolic equation

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Abstract. Let H be a real separable Hilbert space and $A : \mathcal{D}(A) \to H$ be a positive and self-adjoint (unbounded) operator. We consider the identification problem consisting in searching for a H-valued function u and a couple of real numbers λ and μ, the first one being positive, that fulfill the initial-value problem

$$u'(t) + \lambda\, Au(t) = \mu\, u(t), \quad t \in (0,T), \quad u(0) = u_0,$$

and the additional constraints

$$\|A^{r/2}u(T)\|^2 = \varphi \quad \text{and} \quad \|A^{s/2}u(T)\|^2 = \psi,$$

for some time-instant $T > 0$, where we denote by A^s and A^r the powers of A with exponents $r < s$. Provided that the given data u_0 and φ, $\psi > 0$ satisfy proper *a priori* limitations, using a Faedo-Galerkin approximation scheme we construct a unique solution (u, λ, μ) on the whole interval $[0, T]$, and exhibit an explicit continuous dependence estimate - of Lipschitz-type - with respect to the data. Also, we provide specific applications to second and fourth-order parabolic initial-boundary value problems.

We finally recall that the problem of recovering one single constant in the same equation from a final overdetermination of the above type have been successfully studied in [1] (in collaboration with A. Lorenzi) and in [2].

References

[1] A. Lorenzi and G. Mola, *Identification of a real constant in linear evolution equations in Hilbert spaces*, to appear on Inverse Problems and Imaging, **18** (2010), 321–355

[2] G. Mola, *Identification of the diffusion coefficient in linear evolution equations in Hilbert spaces*, Journal of Abstract Differential Equations and Applications **2** (2011), 18–28