SEMINARIO DI GEOMETRIA E ALGEBRA

UNIBA - POLIBA

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Gröbnerian and Gröbner Free Techniques on Non-assocative Algebras

Abstract. In this talk we are investigating to what extent Gröbnerian technologies and the Gröbner-free approach can allow to describe the structure of the non-associative magmas.

We consider a $\mathbb{F}$–vector space $V, \dim_{\mathbb{F}}(V) = n$, label and enumerate as $\mathbf{S} = \{x_1, \dots, x_n\}$ an independent basis of it, and consider on V a non-associative binary operation $\circ : V \times V \mapsto V$ and the magma $\overline{\mathbf{S}}$ generated by it. In this setting, Gröbner bases have been studied in several papers which defined as Buchberger reduction the natural reduction and proved that any interreduced set is a reduced Gröbner basis G of the ideal $\mathbb{I}(G)$ it generates, so that neither S-pair completions nor criteria are needed. In order to describe such ideal $\mathbb{I}(G)$ we make use of the alternative description proposed by Buchberger and Möller in terms of functionals; this requires to define, for each $P := (a_1, \dots, a_n) \in \mathbb{F}^n$, the functional $L_P : \mathbb{F}\{\overline{\mathbf{S}}'\} \to \mathbb{F}$, to give an adaptation of Möller Algorithm and state a proper (and trivial) Cerlienco–Mureddu Correspondence, which allows us to prove that

$$\mathbb{I}(G) = \{f \in \mathbb{F}\{\overline{\mathbf{S}}'\} : L_P(f) = 0, P \in \mathbb{F}^n\}.$$

This allows us to answer the challenging query of Pistone, Riccomagno and Rogantin: they consider the design ideal

$$\mathbb{I}(\mathcal{F}) := \{f \in \mathbf{k}[x_1, x_2] : f(a, b) = 0, (a, b) \in \mathcal{F}\},$$

$\mathcal{F} := \{(0,0), (1,-1), (-1,1), (0,1), (1,0)\}$ and the model $1, x_1, x_1^2, x_2, x_2^2$ which is the most symmetric of the models in the statistical fan. But they correctly remark that such model is not a monomial basis related to a Gröbner basis because *to distroy symmetry is a feature of Gröbner basis computation*. It is sufficient to remove the irrelevant requirement that the monomial basis must be related to some termordering to produce a symmetric model for $\mathbb{I}(\mathcal{F})$.

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