SEMINARIO DI GEOMETRIA E ALGEBRA

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Non-compactness results for homogeneous almost contact and contact Riemannian manifolds

Abstract.

It is a classical result that every homogeneous Riemannian manifold with negative definite Ricci tensor must be noncompact. In particular, every homogeneous Einstein Riemannian manifold (M, g) with $Ric = cg$, $c < 0$ cannot be compact.

We shall discuss some related results concerning homogeneous η-*Einstein* almost contact and contact metric manifolds. These are odd-dimensional Riemannian manifolds (M, g) endowed with a mixed real-complex structure; indeed, they carry an almost CR structure (D, J), compatible with the metric, where $D \subset TM$ is a smooth distribution of codimension 1 and $J : D \to D$ is a partial complex structure on it. The distribution D is the kernel of a fixed globally defined 1-form η and the η-Einstein condition is a meaningful extension of the Einstein condition in this context, namely

$$Ric = ag + b\eta \otimes \eta, \quad a, b \in \mathbb{R}.$$

A systematic introduction to the theory of almost contact metric manifolds is devepoled in the monograph [1]. We shall concentrate on some relevant subclasses of these, like almost cosymplectic and pseudo-Hermitian manifolds.

We shall also present a non-compactness result valid for locally homogeneous, regular contact metric manifolds, under more general assumptions, including the η-Einstein case as a particular one. An example of application of this result will be illustrated, concerning contact metric manifolds which are symmetric CR manifolds according to [4] and their classification developed in [2] and [3].

References:

[1] D.E. Blair: *Riemannian geometry of contact and symplectic manifolds*, 2nd edn. Progress in Mathematics, vol. 203. Birkh§user, Boston (2010).

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[3] E. Boeckx: A full classification of contact metric (κ, μ)-spaces, *Illinois J. Math.* **44** (2000), 212–219.

[4] W. Kaup, D. Zaitsev: On symmetric Cauchy-Riemann manifolds, Adv. Math. **149** (2000), 145–181.

[5] A. Lotta, V. Martín-Molina: Some non-compactness results for locally homogeneous contact metric manifolds, *Results Math.* **77** (2022), no. 4, Paper No. 150, 18 pp.

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