SEMINARIO DI GEOMETRIA E ALGEBRA

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Bence Csajbok
(Politecnico di Bari)

Small, complete subsets of groups without a 3-term arithmetic progression

Abstract. Let G denote an Abelian group of odd order, written additively. A 3-term arithmetic progression of G, 3-AP for short, is a set of three distinct elements of G of the form g, g+d, g+2d, for some elements g, d of G.

Let S denote a subset of G. Then S is called 3-AP-free if it does not contain a 3-AP. Moreover, S is called complete 3-AP-free if S is 3-AP-free and it is not contained in a larger 3-AP-free set. We will say that S is 3-AP saturating if for each element x of G S there is a 3-AP of G consisting of x and two elements of S. Clearly, S is complete 3-AP-free if and only if it is 3-AP-free and 3-AP saturating.

A classical problem in additive combinatorics is to obtain good upper and lower bounds for the maximal size of a 3-AP-free set. In the special case when G=GF(3)n (GF(q) denotes the finite field of order q) then the 3-AP-free sets are exactly the affine point sets without a collinear triple. In general, point sets of V=GF(q)n without a collinear triple are called (affine) caps. In finite geometry it is a classical problem to find small complete caps (a cap is complete if it is not contained in a larger cap) and small saturating sets (a point set S is saturating if for each point x of V S there is a collinear triple ofV consisting of x and two elements of S).

In this talk I will present constructions of complete 3-AP-free sets and 3-AP saturating sets whose size is close to the trivial lower bound. We will be mostly concerning the cases when G is a finite vector space or a cyclic group.

This is a joint work and work in progress with Zoltán Lóránt Nagy.

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