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TITOLO: “C*-algebras and von Neumann algebras in Quantum Probability” (CANVA)

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PI: Dott.ssa Maria Elena Griseta

ABSTRACT PROGETTO:

The project is related to Quantum Probability and Operator Algebras research area. In particular, the following topics have been studied:

First, we studied the C*-algebra generated by finitely many annihilation operators acting on the weakly monotone Fock space, called weakly monotone C*-algebra. More in detail, we gave an abstract representation for the weakly monotone C*-algebra, showing that it is isomorphic to a suitable quotient of a Cuntz-Krieger C*-algebra \mathcal{O}_A corresponding to a matrix A . Moreover, we studied a maximal abelian subalgebra (MASA) for this algebra and we determined its Gelfand spectrum. [1]

Successively, the notion of independence for \mathbb{Z}_2 -graded C*-subalgebras has been examined. Given two subalgebras of a given \mathbb{Z}_2 -graded C*-algebra, we established the conditions under which these subalgebras are C*-independent, exploiting the Schlieder property. In addition, we provided other notions of statistical independence for \mathbb{Z}_2 -graded von Neumann algebras like W*-independence or logical independence and we proved some relationships between them. Finally, we provided a characterization for the graded nuclearity property. [2]

The last part of the project is devoted to the study of C*-independence for \mathbb{Z}_2 -graded C*-subalgebras that do not commute with grading and extend the cross property on two \mathbb{Z}_2 -graded C*-algebras to their corresponding enveloping \mathbb{Z}_2 -graded von Neumann algebras. [3]

Finally, starting from a uniquely ergodic action of a locally compact group G on a compact space X_0 , we considered non-commutative skew-product extensions of the dynamics, on the crossed product $C(X_0) \rtimes_{\alpha} \mathbb{Z}$, through a 1-cocycle of G in T , with α commuting with the given dynamics. We first proved that any such two skew-product extensions are conjugate if and only if the corresponding cocycles are cohomologous. We then studied unique ergodicity and unique ergodicity w.r.t. the fixed-point subalgebra by characterizing both in terms of the cocycle assigning the dynamics. [4]

References:

- [1] M.E. Griseta, Janusz Wysoczański, “*Finitely generated weakly monotone C*-algebra*”, Infinite Dimensional Analysis, Quantum Probability and Related Topics, DOI: <https://doi.org/10.1142/S0219025724500048>

- [2] M.E. Griseta, P. Zurlo, “C* -independence for \mathbb{Z}_2 -graded C*-algebras”, arXiv:2401.08293 [math.OA]
- [3] M.E. Griseta, “Uncoupled C*-independence for \mathbb{Z}_2 -graded C*-algebras”, in preparation
- [4] V. Crismale, S. Del Vecchio, M.E. Griseta, S. Rossi, “Non-commutative skew-product extension dynamical systems”, arXiv:2410.07255 [math.DS], accepted for publication on *Ergodic Theory and Dynamical Systems*