EDP e Dintorni X:
Tenth Meeting around PDE

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Speakers

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Elena Cordero (*University of Torino*)
Sandro Coriasco (*University of Torino*)
Piero D'Ancona (*Sapienza University of Rome*)
Francesco Fanelli (*Basque Center for Applied Mathematics*)
Claudia Garetto (*Queen Mary University of London*)
Vladimir Georgiev (*University of Pisa*)
Massimo Gobbino (*University of Pisa*)
Ning-An Lai (*Zhejiang Normal University*)
Sergio Polidoro (*University of Modena and Reggio Emilia*)
Hiroyuki Takamura (*Tohoku University*)
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Titles and abstracts

Alessia Ascanelli

Schwartz very weak solutions for Schrödinger type equations

This talk is about a joint work with Alexandre Arias Junior (University of São Paulo, Brasil), Marco Cappiello (University of Torino, Italy) and Claudia Garetto (Queen Mary University of London, UK). We focus on the Cauchy problem

$$\begin{cases} Su(t,x) = f(t,x), & t \in [0,T],\, x \in \mathbb{R}^n \\ u(0,x) = g(x), & x \in \mathbb{R}^n \end{cases} \tag{1}$$

for the Schrödinger-type operator

$$S = D_t + a(t) \sum_{j=1}^{n} D_{x_j}^2 + \sum_{j=1}^{n} a_j(t,x) D_{x_j} + a_0(t,x), \quad D = -i\partial,$$

where the leading coefficient $a(t)$ is a positive bounded function while for $j = 0, 1, \ldots, n$, we have $a_j(t,x) \in C([0,T]; \mathcal{S}'(\mathbb{R}^n))$ and, as usual, $\mathcal{S}'(\mathbb{R}^n)$ stands for the space of tempered distributions. For the initial data we assume $f(t,x) \in C([0,T]; \mathcal{S}'(\mathbb{R}^n))$ and $g \in \mathcal{S}'(\mathbb{R}^n)$.
Due to the remarkable Schwartz impossibility result on the multiplication of distributions, the product $a_j(t,x) D_{x_j} u(t,x)$ might be not defined if we look for solutions in the space $C([0,T]; \mathcal{S}'(\mathbb{R}^n))$. We thus need to define a suitable concept of solution; to do that, inspired by the approach of Garetto and Ruzhansky (Arch. for Rational Mech. and Anal. (2015)), we introduce the concept of Schwartz very weak solution and we prove that problem (1) admits a Schwartz very weak solution which is unique modulo negligible perturbations. We also show consistency with the classical theory in the case of regular coefficients and Schwartz Cauchy data.
The main idea of this approach is to replace the original equation with a family of regularised equations depending on a parameter ϵ, apply methods developed in the classical theory to prove existence and uniqueness of solutions to the regularised problems, and then investigate the ϵ-behaviour of the corresponding net of solutions.

Chiara Boiti

Construction of the log-convex minorant of a sequence $\{M_\alpha\}_{\alpha\in\mathbb{N}_0^d}$

(Joint work with David Jornet, Alessandro Oliaro and Gerhard Schindl)
For a sequence $\{M_p\}_{p\in\mathbb{N}_0}$ of real positive numbers (here $\mathbb{N}_0=\mathbb{N}\cup\{0\}$), its *associated function* is

$$\omega_M(t):=M_0\sup_{p\in\mathbb{N}_0}\log(t^p/M_p),\qquad t>0.$$

Mandelbrojt proved in [2] that if $\lim_{p\to+\infty}M_p^{1/p}=+\infty$, then

$$M_p=M_0\sup_{t>0}\frac{t^p}{\exp\omega_M(t)},\qquad p\in\mathbb{N}_0, \tag{1}$$

if and only if $\{M_p\}_{p\in\mathbb{N}_0}$ is logarithmically convex, i.e. $M_p^2\leq M_{p-1}M_{p+1}$ for $p\in\mathbb{N}$. However, condition (1) had never been generalized to the d-dimensional anisotropic case, since the classical coordinate-wise logarithmic convexity condition $M_\alpha^2\leq M_{\alpha-e_j}M_{\alpha+e_j}$, for $\alpha\in\mathbb{N}_0^d$, $1\leq j\leq d$, $\alpha_j\geq 1$ for a sequence $\{M_\alpha\}_{\alpha\in\mathbb{N}_0^d}$ is not sufficient to obtain the analogous of (1) for M_α. The reason is that this is a convexity condition on each variable separately and not on the globality of its variables. Assuming the stronger condition that $\{M_\alpha\}_{\alpha\in\mathbb{N}_0^d}$ is log-convex on the globality of its variables $\alpha\in\mathbb{N}_0^d$, in the sense that $\log M_\alpha=F(\alpha)$ for a convex function $F:[0,+\infty)^d\to\mathbb{R}$, we extend (1) to

$$M_\alpha=M_0\sup_{s\in(0,+\infty)^d}\frac{s^\alpha}{\exp\omega_{\mathbf{M}}(s)},\qquad\forall\alpha\in\mathbb{N}_0^d.$$

To obtain this result we construct the (optimal) convex minorant of a sequence $\{a_\alpha\}_{\alpha\in\mathbb{N}_0^d}$ (then $a_\alpha=\log M_\alpha$). The idea, in the one variable case, takes inspiration from the convex regularization of sequences of Mandelbrojt in [2], which was quite complicated and difficult to export to the more dimensional case. Our construction is made by taking the supremum of hyperplanes approaching from below the given sequence and leads to the notion of convexity for a sequence $\{a_\alpha\}_{\alpha\in\mathbb{N}_0^d}$ in the sense that $a_\alpha=F(\alpha)$ for a convex function $F:[0,+\infty)^d\to\mathbb{R}$. This condition gives the suitable notion of logarithmic convexity for a sequence $\{M_\alpha\}_{\alpha\in\mathbb{N}_0^d}$ as stated above.
This result is a very useful tool for working in the anisotropic setting, and we expect several applications, that could be object of future works.

REFERENCES

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Marco Cappiello

The Cauchy problem for p-evolution equations with variable coefficients in Gevrey classes

We study the Cauchy problem for a class of linear evolution equations of arbitrary order with coefficients depending both on time and space variables (t, x). Under suitable decay assumptions on the coefficients of the lower order terms for $|x|$ large, we prove a well-posedness result in Gevrey-type spaces.

This is a joint work with Alexandre Arias Junior (Universidade de São Paulo), Alessia Ascanelli (Università di Ferrara) and Eliakim Machado (Universidade Federal do Paraná).

Massimo Cicognani

Accelerated Gradient Descent in the Phase Recovery Problem

The problem of phase retrieval consists in recovering an unknown signal from the magnitude of its Fourier transform. This is an important task in many engineering fields, including imaging, electromagnetism, and audio signal processing. Introduced in 1984, the Griffin-Lim Algorithm (GLA) [1] is still one of the leading methods for phase retrieval. This celebrated double-projection method exploits the inherent redundancy of the short time Fourier transform (STFT). It operates by starting from an initial phase estimate and iteratively projects the resulting complex-valued matrix back and forth between time and time-frequency domains, regularizing the magnitude STFT to be equal to the target one at the end of each step. We prove that GLA is, in fact, a L-smooth Gradient Descent Algorithm (GDA), and we propose a converging accelerated method called Nesterov Accelerated Griffin-Lim Algorithm (NAGLA) following the most well known acceleration strategies of GDA introduced by Nesterov in his impactful 1983 paper [2].

REFERENCES

[1] D. Griffin, J. Lim, Signal estimation from modified short-time Fourier transform, *IEEE Transactions on acoustics, speech, and signal processing* **32**(2), (1984) 236–243
[2] Y.E. Nesterov, A method for solving the convex programming problem convergence rate $O(1/k^2)$, *Soviet Mathematics Doklady* **27**, (1983) 372–376

Elena Cordero

Latest developments on Hardy's Uncertainty Principle

Hardy's uncertainty principle is a classical result in harmonic analysis, stating that a function in $L^2(\mathbb{R}^d)$ and its Fourier transform cannot both decay arbitrarily fast at infinity. We extend this principle to the propagators of Schrödinger equations with quadratic Hamiltonians, known in the literature as metaplectic operators. These operators generalize the Fourier transform and have captured significant attention in recent years due to their wide-ranging applications in time-frequency analysis, quantum harmonic analysis, signal processing, and various other fields. However, the involved structure of these operators requires careful analysis, and most results obtained so far concern special propagators that can basically be reduced to rescaled Fourier transforms. The main contributions of this work are threefold: (1) we extend Hardy's uncertainty principle, covering all propagators of Schrödinger equations with quadratic Hamiltonians, (2) we provide concrete examples, such as fractional Fourier transforms, which arise when considering anisotropic harmonic oscillators, (3) we suggest Gaussian decay conditions in certain directions only, which are related to the geometry of the corresponding Hamiltonian flow.
This is a joint work with Gianluca Giacchi and Eugenia Malinnikova [1].

REFERENCES

[1] Cordero, E., Giacchi, G., and Malinnikova, E. (2024). Hardy's Uncertainty principle for Schrödinger equations with quadratic Hamiltonians. arXiv:2410.13818.

Sandro Coriasco

Global hypoellipticity for a class of evolution operators in time-periodic Sobolev-Kato spaces

We study the hypoellipticity properties of a class of time-periodic evolution operators, with coefficients globally defined on $\mathbb{R}^d$ and growing polynomially with respect to the space variable. To this aim, we introduce a class of time-periodic weighted Sobolev spaces, whose elements are characterised in terms of suitable Fourier expansions, associated with elliptic operators.

Piero D'Ancona

Magnetic dynamical uncertainty principles

A function and its Fourier transform can not be simultaneously localized beyond a certain threshold. Uncertainty principles measure this phenomenon in a quantitative way. Time slices of solutions of the Schrodinger and more general dispersive equations present the same property, and the corresponding quantitative versions are called dynamical uncertainty principles. In this talk I will describe some work in progress, in collaboration with Diego Fiorletta (Roma), concerning such principles for Schrodinger equations perturbed by an electromagnetic potential.

Francesco Fanelli

Hyperbolic effects in incompressible fluid mechanics

In this talk, we are interested in the well-posedness theory of a system of PDEs which describes the dynamics of a non-homogeneous fluid displaying *non-dissipative* viscosity effects. Examples of such fluids arise both in quantum and classical hydrodynamics. At the mathematical level, the non-dissipative nature of the viscosity is encoded by an odd term, dubbed precisely *odd viscosity* tensor. As the odd viscosity term involves higher order space derivatives of the velocity field and of the density, it is responsible for an apparent loss of regularity in the classical *a priori* estimates. In this talk, we show how to circumvent such a loss of derivatives and establish a well-posedness result in the framework of Sobolev (or, more generally, Besov) spaces of high enough regularity. The key is the identification of a suitable *effective velocity* in the model, which allows to highlight a hyperbolic structure underlying the system of equations.

The talk is based on joint works with *Rafael Granero-Belinchón* (Universidad de Cantabria), *Stefano Scrobogna* (Università degli Studi di Trieste) and *Alexis Vasseur* (University of Texas at Austin).

Claudia Garetto

On higher order hyperbolic equations with variable multiplicities

In this talk I will present some recent results on the C^∞ well-posedness of the Cauchy problem for higher order hyperbolic equations with variable multiplicities. I will discuss the connections with non-diagonalisable hyperbolic systems and how to deduce Levi conditions on the lower order terms. This work is joint with Bolys Sabitbek (QMUL).

Vladimir Georgiev

On damped wave equation in dimension 1: global solution and decay in Fujita subcritical case

We consider the Cauchy problem:

$$\begin{cases} \partial_t^2 u + \partial_t u - \Delta u = |u|^p, & (t,x) \in (0,\infty) \times \mathbb{R}^n, \\ u(0,x) = u_0(x), & x \in \mathbb{R}^n, \\ \partial_t u(0,x) = u_1(x), & x \in \mathbb{R}^n. \end{cases} \tag{1}$$

The aim is to estimate the decay rate of u with $n = 1$ in the L^∞ framework under the pointwise initial condition

$$u_0 \leq 0, \quad u_1 \leq -u_0/2. \tag{2}$$

We remark that the local well-posedness of (1) is known.
The main result is the following. Let $u_0 \in C^2 \cap W^{2,1} \cap W^{2,\infty}$ and $u_1 \in C^1 \cap L^1 \cap L^\infty$ satisfy (2). Then the corresponding solution u to (1) satisfies

$$\sup_{t \geq 0} (1+t)^{1/(p-1)} \|u(t)\|_{L^\infty} \leq \infty$$

if $p > 5/3$.
The talk is based on collaboration with Kazumasa Fujiwara.

Massimo Gobbino

Resonance effects for wave equations with time-dependent coefficients

We consider abstract linear wave equations where either the propagation speed or the damping term is time-dependent. It is well-known that oscillations in these coefficients can affect the asymptotic behavior of solutions, and even their existence. In this talk, we review some of the known results in this area, emphasizing the role of resonance phenomena. In particular, we discuss recent progress in the dissipative case, showing that oscillations that are either too fast or too slow have no effect on the decay rate of solutions. However, oscillations resonant with one of the elastic system's frequencies can significantly alter the decay rate. We conclude by highlighting some open problems. (This talk is based on joint works with Marina Ghisi.)

Ning-An Lai

Morawetz type estimate for wave equations and its application

We would like to talk about some Morawetz type estimates for the wave equations and some applications will be discussed.

Sergio Polidoro

Taylor formulas for Nonlocal Kinetic Equations

We consider spaces of Hölder continuous functions suitable for studying the regularity theory for non local kinetic operators acting on the phase space. We first recall the geometrical setting where the regularity theory of the analogous purely differential operators is usually studied. Then we state and discuss an intrinsic Taylor-like formula taylored on the relevant non-Euclidean geometry. This result has been proved in a joint work in collaboration with Maria Manfredini and Stefano Pagliarani.

Hiroyuki Takamura

Recent progress on the general theory and its optimality for 1D nonlinear wave equations

In this talk, the recent progress on nonlinear wave equations in one space dimension will be presented. More precisely, the so-called combined effect plays a key role in the analysis on model equations which improves the general theory for nonlinear wave equations, expected complete more than 30 years ago.

Giulio Tralli

Duality methods for caloric functions in convex domains

In this talk we deal with first-order and second-order derivative estimates of Hamilton-Li-Yau type for solutions to the classical heat equation with homogeneous Neumann conditions on convex domains. We first discuss linear and nonlinear methods to obtain such estimates in the whole space and in bounded domains. Then we focus on a new approach which exploits an interplay between the Bernstein method and an adjoint method à la Evans and which relies on contractivity properties for advection-diffusion equations. The talk is based on a joint work with A. Goffi.

Dimiter Vassilev

An overdetermined fractional Serrin problem

A version of a singular fractional Serrin problem and its solution will be presented. This is a joint work with Nicola Garofalo (Arizona State University)

Eugenio Vecchi

Mixed local-nonlocal singular problems

In this talk I will focus on boundary value problems of mixed type, meaning that the leading operator is given by the superposition of $-\Delta$ and $(-\Delta)^s$ with $s \in (0,1)$, in presence of singular nonlinearities of the form $u^{-\gamma}$ with $\gamma > 0$. I will discuss a few regularity results in the spirit of Boccardo and Orsina and multiplicity results of positive solutions when a critical power term is added to the singular nonlinearity.
The talk is mainly based on a joint work with S. Biagi (Politecnico di Milano).

Nicola Visciglia

Construction of a sequence of invariant measures for the complex valued mKdV

We discuss the Cauchy theory associated with the mKdV equation for complex valued functions. Then we construct for the corresponding dynamic a sequence of invariant weighted Gaussian measures supported on Sobolev spaces more and more regular. This is a joint work with C. Kenig, A. Nahmod., N. Pavlovic, G. Staffilani