# Mini-Workshop on Nonlinear PDEs

## January 16, 2025

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| 15:00 - 15:50 | Serena Dipierro | *Phase transitions and a conjecture by De Giorgi.* |
| 15:50 - 16:10 | Break | |
| 16:10 - 17:00 | Stefano Nardulli | *Multiplicity of solutions to the multiphasic Allen–Cahn–Hilliard system with a small volume constraint on closed parallelizable manifolds.* |
| 17:00 - 17:50 | Gaetano Siciliano | *Critical points with prescribed energy: an approach with the Lusternik-Schnirelmann theory.* |

# Abstracts

### Phase transitions and a conjecture by De Giorgi

Serena Dipierro
*University of Western Australia*

In this talk we discuss the theory of phase transitions under various perspectives, presenting the classical theory and recent developments, also in connection with a famous conjecture by Ennio De Giorgi.

### Multiplicity of solutions to the multiphasic Allen–Cahn–Hilliard system with a small volume constraint on closed parallelizable manifolds

Stefano Nardulli
*Universidade Federal do ABC*

We prove the existence of multiple solutions to the Allen-Cahn-Hilliard (ACH) vectorial equation (with two equations) involving a triple-well (triphasic) potential with a small

volume constraint on a closed parallelizable Riemannian manifold. More precisely, we find a lower bound for the number of solutions depending on some topological invariants of the underlying manifold. The phase transition potential is considered to have a finite set of global minima, where it also vanishes, and a subcritical growth at infinity. Our strategy is to employ the Lusternik–Schnirelmann and infinite-dimensional Morse theories for the vectorial energy functional. To this end, we exploit that the associated ACH energy $\Gamma$-converges to the weighted multi-perimeter for clusters, which combined with some deep theorems from isoperimetric theory yields the suitable setup to apply the photography method. Along the way, the lack of a closed analytic expression for the multi-isoperimetric function for clusters imposes a delicate issue. Furthermore, using a transversality theorem, we also show the genericity of the set of metrics for which solutions to the ACH system are nondegenerate. This a joint work with J.H. Andrade, J. Conrado, P. Piccione, and Reinaldo Resend Oliveira.

## Critical points with prescribed energy: an approach with the Lusternik-Schnirelmann theory

Gaetano Siciliano
*Università degli Studi di Bari Aldo Moro*

In the talk we discuss the existence of multiple critical points for abstract energy functionals depending on a parameter under the constraint of fixed energy level. We use the Lusternik-Schnirelmann Theory and the method of nonlinear generalized Rayleigh quotient developed by Il'yasov to obtain general results which are then applied to some elliptic PDEs giving existence of solutions and bifurcation results. Joint paper with Humberto Ramos Quoirin (Universidad Nacional de Córdoba, AG) and Kaye Silva (Universidade Federal do Goiás, BR).