ACIPDif22

School-Workshop on Analysis, Control & Inverse Problems for Diffusive Systems with Application to Natural and Social Sciences

Bari, July 18-22, 2022

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ABSTRACTS - MINI COURSES

Control, inverse problems and real-life phenomena

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The aim of these lectures is to present several techniques frequently used to solve controllability and/or inverse problems for PDE's with origin in physics and other sciences. In particular, we will deal with equations of the Navier-Stokes kind, free-boundary models, tumor growth systems, etc. We will speak of bi-optimal and hierarchic control, exact controllability to the trajectories, time optimal control and geometric parameter identification. Theoretical and numerical aspects will be considered. Among others, the tools will include local and global Carleman inequalities, fixed-point and local inversion theorems, least square formulations and finite element methods.

Well-posedness and stability of nonlinear abstract evolution equations with time delays

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Let $\mathcal{H}$ be a fixed Hilbert space with norm $\|\cdot\|$, and consider an operator $\mathcal{A}$ from $\mathcal{H}$ into itself that generates a C_0-semigroup $(S(t))_{t\geq 0}$ that is exponentially stable, i.e., there exist two positive constants M and ω such that

$$\|S(t)\|_{\mathcal{L}(\mathcal{H})} \leq Me^{-\omega t}, \ \forall t \geq 0.$$

Consider the evolution equation with delay

$$\begin{cases} U_t(t) = \mathcal{A}U(t) + k\mathcal{B}U(t-\tau) & \text{in } (0,+\infty) \\ U(0) = U_0, \ \mathcal{B}U(t-\tau) = f(t), & \forall t \in (0,\tau), \end{cases} \tag{1}$$

where $\tau > 0$ is the time delay, k is a real parameter, $\mathcal{B}$ is a fixed bounded operator from $\mathcal{H}$ into itself, the initial datum U_0 belongs to $\mathcal{H}$ and $f \in C([0,\tau];\mathcal{H})$.

Time delay effects often appear in many applications and physical problems. On the other hand, it is well-known (cf. [4, 5, 6]) that they can induce some instability. Then, we are interested in giving an exponential stability result for such a model under a suitable *smallness condition* on the delay term $k\mathcal{B}$.

For some particular examples of (1) (see e.g. [2, 9, 3, 1]) the exponential stability had been proved, under appropriate conditions on the delay feedback $k\mathcal{B}$, the proof being from time to time quite technical because some observability inequalities or perturbation methods are used.

More recently, we have considered delay problems in an abstract framework and obtained general stability results [7, 8]. In this course, starting from (1), we will analyze the semilinear model

$$\begin{cases} U_t(t) = \mathcal{A}U(t) + G(U(t)) + k\mathcal{B}U(t-\tau) & \text{in } (0,+\infty) \\ U(0) = U_0, \ \mathcal{B}U(t-\tau) = f(t), & \forall t \in (0,\tau), \end{cases}$$

where $G : \mathcal{H} \to \mathcal{H}$ satisfies some Lipschitz conditions.

We will prove an exponential stability result under a suitable assumption between the constant k and the constants M, ω, τ, the norm of $\mathcal{B}$ and the nonlinear term G. We give a direct proof, simpler with respect to the ones used previously for particular models, by using the so-called Duhamel's formula (or variation of parameters formula), some inductive arguments and step by step procedures. Moreover, we emphasize the generality of our abstract result. Indeed, it applies to every model in the previous abstract form when the operator $\mathcal{A}$ generates an exponentially stable semigroup.

As a further generalization, we will consider the model (cf. [8])

$$\begin{cases} U_t(t) = \mathcal{A}U(t) + \sum_{i=1}^{I} F_i(U(t), U(t-\tau_i)) \quad \text{in } (0, +\infty) \\ U(t-\tau) = U_0(t), \quad \forall t \in (0, \tau), \end{cases}$$

where I is a positive natural number, $\tau_i > 0,\ i = 1, \cdots, I$, are time delays, $\tau = \max_{1 \leq i \leq I} \tau_i$, and the nonlinear terms $F_i : \mathcal{H} \times \mathcal{H} \to \mathcal{H}$ satisfy some Lipschitz conditions.

Some applications of the abstract results to models in population dynamics will be also described. In particular, we will consider models for single species and competition models for two species. Finally, some recent developments will be discussed.

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Inverse problems for time-fractional diffusion-wave equations

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One should be concerned with many kinds of the diffusion of substances in heterogeneous media, and when one considers diffusion of contaminants such as Caesium-137, such studies may be very serious issues for public safety. In the diffusion in heterogeneous media, it is often observed that profiles of density of substances indicate large deviation from the classical diffusion profiles. Therefore, more suiable model equations are demanded and time-fractional diffusion-wave equations is one hopeful model equation and call great attention.

Anomalous diffusion is characterized by slow diffusion and long-tail profile, and for fractional equations we can prove slow decay for large time, less smoothing property of distribution, etc. Thus the time-fractional diffusion-wave equation is very probable model and is given as follows:

$$\partial_t^\alpha u(x,t) - Au(x,t) = F(x), \quad x \in \Omega,\ 0 < t < T \tag{1}$$

with initial and boundary conditions. Here we describe most basic form of the equation and will study more gerenal forms. Here Ω is a bounded domain in $\mathbb{R}^d$, $d \in \mathbb{N}$, $-A$ is a uniform elliptic operator of the second order. Moreover ∂_t^α with $\alpha \in (0,2) \setminus \{1\}$, is a Caputo type of fractional derivative: for $0 < \alpha < 1$, it is defined by

$$\partial_t^\alpha w(t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t (t-s)^{-\alpha} \frac{dw}{ds}(s) ds$$

if $w \in W^{1,1}(0,T)$, where Γ is the gamma function.

Introducing time-fractional diffusion equations for modelling actual anomalous diffusion, we should first identify parameters governing diffusion. As such parameters, we refer for example to the orders of time-derivative, source terms. This is nothing but inverse problems.

The main purpose of this mini-course is to study several kinds of inverse problems for time-fractional diffusion-wave equations (1). With suitable measurement data of solution u to (1), we mainly study the following kinds of inverse problems:
(I) Determination of orders α and related quantities.
(II) Determination of $\mu(t)$ or $f(x)$ in the case where a source $F(x,t)$ is modelled in a form $F(x,t) = \mu(t)f(x)$.
(III) Backward problems in time: for fixed $T > 0$ determine

$$\begin{cases} u(\cdot,0) \quad \text{by } u(\cdot,T) \text{ if } 0 < \alpha < 1, \\ \{u(\cdot,0),\, \partial_t u(\cdot,0)\} \quad \text{by } \{u(\cdot,T),\, \partial_t u(\cdot,T)\} \text{ if } 1 < \alpha < 2. \end{cases}$$

Our main interests are the uniqueness and the stability for these inverse problems.

It is important that which kind of data we should adopt for the uniqueness for inverse problems. Adequate choices of data essentially depend on qualitative properties of initial boundary value problems for time-fractional diffusion-wave equations. Thus our studies for inverse problems should be grounded on the direct problems, and we will explain also necessary knowledge of direct problems.

We will discuss the inverse problems in a self-contained way on the basis of recent results by me and my colleagues (the exact references are given later). We mostly need underground levels of mathematical analysis.

The planned contents are

1. Introduction: physical and mathematical backgrounds
2. Initial boundary value problems: eigenfunction expansion, Mittag-Leffler functions, properties of the solutions
3. Uniqueness for inverse problems of determining orders and source terms
4. Stability and uniqueness for backward problems
5. Optimal control problem

Some general references are listed below.

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