XMaths Workshop 2022

Università degli Studi di Bari Aldo Moro,
Dipartimento di Matematica

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Book of Abstracts

- **Giulia Bevilacqua**, Università di Pisa

  Title: *Looking for minimal surfaces: the Kirchhoff-Plateau problem*

  Abstract: Often used for children's enjoyment, soap bubbles are physical examples of the rich mathematical problem of minimal surfaces: they assume the shape of the least possible area, containing a given volume.
  In this talk I will present a variant of the Plateau problem, a centuries-old mathematical problem which asks if it exists a surface with minimal area spanning a given fixed curve. In contrast to the Plateau's problem, the Kirchhoff-Plateau one concerns the equilibrium shapes of a system in which a flexible filament has the form of a closed loop and it is spanned by a liquid film. Here, we model the filament as a Kirchhoff rod and the action of the spanning surface through the presence of the surface tension.
  The talk is based on some joint works with Luca Lussardi (Politecnico di Torino) and Alfredo Marzocchi (Università Cattolica del Sacro Cuore).

- **Alessandro Camasta**, Università degli Studi di Bari Aldo Moro

  Title: *The first shadows of Control Theory in the realm of Parabolic Equations: the tale of a degenerate operator in non divergence form that generated contraction semigroups*

  Abstract: In this talk we will present some motivations of Control Theory together with its preliminary methods to study Partial Differential Equations. Since many problems coming from Physics, Astronomy, Biology and Mathematical Physics are described by parabolic equations which admit different types of degeneracy, some recent remarkable developments on this topic will be presented. In particular, we will introduce the "right" functional spaces in which these problems are set. We will focus on the description of the main properties of the operator that governs these problems.
  These results are part of a joint work with Genni Fragnelli (Department of Ecology and Biology, Tuscia University).

- **Giacomo Cappiello**, Università degli Studi di Firenze

  Title: *SudoQ: a quantum variant of the Sudoku*

  Abstract: Many classical objects have their quantum counterpart. SudoQ, or quantum Sudoku, is a version of the popular game in which the integers are substituted by vectors from a finite dimensional Hilbert space. These vectors are constrained to form an orthonormal basis of the Hilbert space in every row, column and block.

  SudoQ has many applications in many quantum information protocols, such as quantum teleportation and dense coding. For this reason the study of its properties can be very important in the development of quantum algorithms.

  The aim of my research is the study of cardinalities of SudoQ, that is the number of different vectors it can contain, and conditions that determine a unique solution for a grid. In particular I concentrate on the 4×4 and 9×9 schemes.

- **Irene De Blasi**, Università degli Studi di Torino

  Title: *Examples of chaos in galactic billiards*

  Abstract: Studying dynamical systems coming from Celestial Mechanics, it is pretty common to come across *chaotic models*, that is, highly sensitive to initial conditions.

  In Mathematics, many different definitions of chaos, referred to different contexts, have been developed: in particular, in the late XX century Robert Devaney proposed the definition of *topological chaos*, based on the properties of sensitivity on the initial conditions, topological transitivity and density of the periodic orbits.

  In this talk a technique to prove the topological chaoticity of a complex model, based on the notion of *symbolic dynamics*, will be presented; the same technique is then applied to a brand new model coming from Celestial Mechanics, called *Refraction galactic billiard,* describing the motion of a particle in an elliptic galaxy with a Black Hole in its center.

- **Antonio De Carlo**, MER MEC S.p.A.

  Title: *Applications of Signal Analysis in railway safety*

  Abstract: Railway safety is a very important branch of railway engineering. In fact, while trains are travelling on the track there are many dangerous situations that can occur and that have to be avoided. For instance, the track geometry must be "good", meaning that a train should not travel, especially at high speed, on tracks which have developed over time some particular oscillations in their geometry that might potentially trigger dangerous vibrations in the vehicles.
  Another important field is the so called *Signalling*, whose purpose is to prevent the train from coming across obstacles on the track or dangerous situations in general.
  Also relevant is the *Vehicle Dynamics*, which is linked to both vehicle's safety (e.g. stability) and to passengers comfort.
  The common ground of these cases is the mathematical branch of *Signal Analysis* which allows us to acquire and understand the information that both the track and the train send us. The aim of this work is to show what kind of information we can detect and how we can interpret them through the instruments provided by Signal Analysis.

- **Arturo De Marinis**, Gran Sasso Science Institute (L'Aquila)

  Title: *Stable training of neural ordinary differential equations*

  Abstract: Neural ordinary differential equations (neural ODEs) are a new family of deep neural networks. Essentially, a neural ODE is a differential equation whose vector field is a neural network. Having a neural ODE as a part of a machine learning model makes the model more efficient than a standard one. Indeed, it is possible to train the neural ODE block of the model using the adjoint sensitivity method, which computes the gradients for the gradient descent method avoiding the computational cost of the classical backpropagation. Last but not least, it is possible to study the stability and the contractivity of the neural ODE block, being a differential equation, with the aim of designing training strategies to make the overall machine learning model robust and stable.

- **Grazia Gargano**, Università degli Studi di Bari Aldo Moro & Istituto Tumori Bari Giovanni Paolo II - IRCCS

  Title: *Low-rank in hematologic oncology: a NMF-based approach for gene signature identification to support mediastinal gray zone lymphoma diagnosis*

  Abstract: Mediastinal gray zone lymphoma (MGZL), a B-cell lymphoma with overlapping features between primary mediastinal B-cell lymphoma (PMBL) and classic Hodgkin lymphoma (CHL), represents a rare, diagnostically challenging entity. This lymphoma typically exhibits discordant morpho-phenotypical characteristics between CHL and PMBL, a high rate of diagnostic reclassification and consequent poor therapeutic outcomes. The diagnosis of MGZL is still challenging and largely based on the satisfaction of morphological and immunophenotypic criteria. Therefore, there is an urgent need for a deeper molecular characterization of MGZL to facilitate their diagnostic stratification and selection of optimal treatments. To accomplish this, we developed a computational framework by integrating transcriptome deconvolution methods and nonnegative constrained low-rank approximation techniques. Based on the hypothesis of a molecular collocation of MGZL between PMBL and CHL, we combined CIBERSORTx, a deconvolution algorithm for digital cytometry, with Nonnegative Matrix Factorization (NMF) to identify set of genes, related to both tumor and microenvironment, whose expression values discriminated these lymphoma subtypes. To potentiate the final translational value of the signature, an ensemble of filter-based features selection methods (Relief and Laplacian Score) allowed to lower the number of genes identifying a final balanced molecular signature. This signature is capable of distinguishing CHL from PMBL and placing MGZL within this spectrum, allowing to transcriptomically categorize these tumors as CHL-like or PMBL-like. If validated on larger cohorts, this signature could be useful to the design of a molecular assay easily transferable to routine clinical practice to support MGZL diagnosis.

  This is a joint work with the MIδAS research group, Department of Mathematics University of Bari Aldo Moro and Laboratory of Hematology of Istituto Tumori Giovanni Paolo II, Bari.

- **Giovanni Gentili**, Università di Torino

  Title: *An introduction to HKT geometry*

  Abstract: HKT manifolds were first introduced by physicists since they arise as internal spaces of some supersymmetric sigma model with Wess-Zumino term. From the mathematical standpoint, HKT manifolds represent in many ways the hypercomplex counterpart of the well-studied Kähler manifolds. We

will overview the basic relevant mathematical framework and present some of the features of hypercomplex and HKT geometry. In the final part of the talk, we will overview the current progress regarding the quaternionic Calabi conjecture posed by Alesker and Verbitsky in 2010.

- **Federico Girotti**, University of Nottingham

  Title: *Long–time behaviour of repeated indirect quantum measurements*

  Abstract: After a brief recap of the theory of measurements in quantum mechanics, I will introduce a class of stochastic processes (known as quantum trajectories) that models the sequence of outcomes of repeated indirect measurements performed on a quantum system and the conditional state of the system. Indirect measurements are relevant to study because in some practical situations it is unfeasible to perform direct measurements on the system and, in addition, they provide information on the system without affecting its state too heavily. From a mathematical point of view, such stochastic processes represent an interesting class including many relevant models (independent identically distributed random variables, Markov chains, hidden Markov models), but yet with enough structure and properties. I will present some results regarding the long-time behaviour of simple statistics shading some light on the ideas used to prove them and discussing how this affects the conditional state of the system.

- **Marco Inversi**, University of Basel

  Title: *Weak-strong uniqueness and vanishing viscosity for incompressible Euler equations in exponential spaces*

  Abstract: This talk is devoted to the analysis of the Euler and the Navier-Stokes equations in the context of incompressible fluids. Despite their importance in modelling several natural phenomena, their rigorous mathematical study remains vastly incomplete. Indeed, even though these equations were proposed hundreds of years ago, major questions such as existence, uniqueness and smoothness of solutions presently remain extremely challenging open problems.

  We focus on the uniqueness of solutions to the incompressible Euler equations and on the inviscid limit of solutions to the Navier-Stokes equations. In the class of admissible weak solutions, we can prove a weak-strong uniqueness result for the incompressible Euler equations assuming that the symmetric part of the gradient belongs to $L^1_{\mathrm{loc}}([0,+\infty); L^{\exp}(\mathbb{R}^d;\mathbb{R}^{d\times d}))$, where $L^{\exp}$ denotes the Orlicz space of exponentially integrable functions. Moreover, under the same assumptions on the limit solution to the Euler system, we can obtain the convergence of vanishing-viscosity Leray-Hopf weak solutions to the Navier-Stokes equations.

- **Alessandro Lupoli**, TU München

  Title: *Signal Processing: One-Bit Sigma-Delta Modulation*

  Abstract: The study of bandlimited signals on manifolds has become increasingly important for engineering applications.
  Reconstruction of signals by replacing sampled values with elements of a finite alphabet can also have important implications in the data science world.
  The first part of the talk will deal with some of the known results on 1-bit Sigma-Delta quantization, a well-known method for both analog-to-digital (A/D) conversion and the Digital Halftoning problem. The analysis focuses on quantization schemes, their modifications and how to adapt them so that they

are effective for manifolds.
Finally, new basic problems related to quantization on the n-dimensional torus will be presented.

- **Luciano Maino**, University of Bristol

Title: *Look up and Project: Abelian Varieties and Post-Quantum Cryptography*

Abstract: Recent developments in quantum computing are undermining the security of the current public-key infrastructure. The need for new quantum-resistant cryptography is becoming tangible. In particular, we would like to have cryptosystems that not only can run on ordinary laptops but are also resistant to the quantum threat. Amongst the solutions suggested, we can find isogeny-based cryptography. Isogeny-based cryptography relies on the hardness of computing special maps - called isogenies - between two given elliptic curves defined over finite fields. Arguably, the most influential cryptosystem in this field is Supersingular Isogeny Diffie-Hellman (SIDH).
In this talk, I will narrate the bitter-sweet story of SIDH: from being a candidate in the NIST Post-Quantum standardisation process to its recent break. En passant, I will tell you how we exploited the computation of isogenies between higher-dimensional Abelian varieties to recover isogenies between Abelian varieties of dimension 1, i.e. elliptic curves. Also, I will discuss potential future directions for higher-dimensional isogeny-based cryptography.
This talk is loosely based on joint work with Chloe Martindale.

- **Claudio Mele**, Università del Salento

Title: *Estimates between time-frequency representations and Donoho-Stark type uncertainty principles*

We give different estimates between Lebesgue norms of quadratic time-frequency representations (short-time Fourier transform, spectrogram, Rihaczek, and so on). We show that, in some cases, it is not possible to have such bounds in classical L^p spaces, but the Lebesgue norm needs to be suitably weighted. From such estimates, we deduce uncertainty principles of Donoho-Stark type, saying, roughly speaking, that if the content of a time-frequency representation in a Lebesgue measurable set $E \subset \mathbb{R}^{2N}$ is greater than (a fraction of) the whole content of another time-frequency representation, then the set E must have sufficiently large measure.

References

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[2] D. L. Donoho, P. B. Stark, *Uncertainty principles and signal recovery*, SIAM J. Appl. Math. **49** (3), (1989), 906–931.
[3] K. Gröchenig, *Foundations of Time-Frequency analysis*, Birkhäuser, Boston, 2001.

- **Federica Mennuni**, Università degli Studi di Bari Aldo Moro

Title: *Generalized quasilinear elliptic equations in* $\mathbb{R}^N$

Abstract: In this talk, I will present a new result on the existence of nontrivial weak bounded solutions to the quasilinear elliptic equation

$$-\mathrm{div}(a(x,u,\nabla u)) + A_t(x,u,\nabla u) + V(x)|u|^{p-2}u = g(x,u) \qquad \text{in } \mathbb{R}^N$$

where $p > 1$, $N \geq 2$ and $V : \mathbb{R}^N \to \mathbb{R}$ is a given Lebesgue measurable and positive function. Moreover, we have supposed that $A : \mathbb{R}^N \times \mathbb{R} \times \mathbb{R}^N \to \mathbb{R}$ is a given C^1-Carathéodory function which grows as $|\xi|^p$,

with partial derivatives $A_t(x,t,\xi) = \frac{\partial A}{\partial t}(x,t,\xi)$, $a(x,t,\xi) = \nabla_\xi A(x,t,\xi)$, and also that $g : \mathbb{R}^N \times \mathbb{R} \to \mathbb{R}$ is a given Carathéodory function which satisfies a suitable sub-p-linear growth condition. Since the coefficient of the principal term $A(x,u,\nabla u)$ depends on the solution and its derivatives, we have aimed to investigate the interaction of two different norms in a suitable Banach space with the aim of obtaining a good variational formulation. Thus, using approximation arguments on bounded domains, more precisely on spheres of center the origin and increasing radius, we have established the existence of a nontrivial weak bounded solution as the limit of a sequence of solutions defined on the spheres. Finally, slightly stronger assumptions on functions A and g have allowed us to prove the existence of a positive solution to our problem.

The presented results are part of the joint work with Addolorata Salvatore.

- **Nicola Picoco**, Università degli Studi di Bari Aldo Moro

  Title: *Cayley-Bacharach property: from the ancient geometry to the modern algebraic geometry*

  Abstract: The Cayley–Bacharach condition is a very classical property that found its roots in ancient and classical geometry. In this talk we retrace the main steps through the history that have led to the modern formulation. We show how being Cayley–Bacharach with respect to the complete linear system of hypersurfaces of given degree forces a set of points in the projective space to lie on a reduced curve of low degree. In particular, starting from a result for points in the projective plane due to Lopez and Pirola, we present some partial extensions to any $\mathbb{P}^n$. Moreover, in a joint work with Francesco Bastianelli, we study the Cayley–Bacharach condition for points on Grassmannians; as any point on Grassmannians parametrizes a linear subspace of $\mathbb{P}^n$, we rephrase the Cayley–Bacharach condition as a property for linear subspaces and we prove that this property affects their geometry. Namely, we get an upper bound for the dimension of the linear span of linear subspaces satisfying Cayley–Bacharach conditions.

- **Giovanni Russo**, Università degli Studi dell'Aquila

  Title: *GKM actions on cohomogeneity one manifolds*

  Abstract: GKM theory is about understanding topological and geometric aspects of spaces with torus-actions in terms of purely combinatorical data, namely graphs. We first give an introduction to GKM theory via essentials on group theory and explicit examples. Then we illustrate the above idea on homogeneous and cohomogeneity one spaces. The results in the cohomogeneity one case are based on a joint work with Oliver Goertsches and Eugenia Loiudice from Philipps-Universität Marburg.

- **Angelina Zheng**, Università degli Studi Roma Tre

  Title: *Cohomology of moduli spaces of trigonal curves*

  Abstract: The moduli space of algebraic curves is a central object in algebraic geometry. The idea behind this space is that it answers to a classification problem, by allowing us to classify algebraic curves up to isomorphisms. Nonetheless, the geometry of this space is rather abstract and subtle, and only few general statements are known.

  The aim of this talk is to discuss some moduli spaces of genus g curves, such as trigonal curves, from the point of view of one of the most important topological invariants, their rational cohomology.

  In general, their full cohomology ring is still unknown, but we have a complete description for some values of g and we will provide a description for their stable cohomology ring.