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ABSTRACTS

Some recent results on approximations by Sampling type series

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In this talk, we deal with some recent results on approximation by generalized sampling series and exponential sampling series. In the first section of the talk, we talk about higher order Kantorovich forms of generalized sampling series and we present their convergence behaviors by means of pointwise and uniform convergence, a Voronovskaya type theorem and simultaneous approximation. The second section of the talk is devoted to convergence of generalized sampling series for functions belonging to weighted spaces of functions. We investigate pointwise and uniform convergence of generalized sampling series and their Kantorovich form, rate of convergence of both series via weighted modulus of continuity and a quantitative Voronovskaya type theorem and a quantitative result for difference of the series. In the last section, we consider generalized exponential sampling series and investigate their approximation properties in weighted spaces of functions. After giving a pointwise and uniform convergence result, we define a new weighted modulus of continuity and present some quantitative results for the series.

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On the spectral mapping theorem for the Weyl spectra

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This talk concerns the approximate point spectrum and the surjectivity spectrum of a Toeplitz operators T_Φ, defined on Hardy spaces $H^2(\mathbf{T})$ of the unit circle $\mathbf{T}$ of $\mathbb{C}$, where the symbol $\Phi \in L^\infty(\mathbf{T})$, or Φ is a continuous symbol. We also extend some results obtained by Farenick and W. Y. Lee in [4]. In particular, we consider Weyl type theorems and the spectral mapping theorem for the Weyl spectra.

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Solid hulls and cores of weighted H^∞-spaces

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We report on joint work with W. Lusky (Paderborn, Germany) and J. Taskinen (Helsinki, Finland).

We determine the solid hull and solid core of weighted Banach spaces H^∞_v of analytic functions f such that $v|f|$ is bounded, both in the case of the holomorphic functions on the disc and on the whole complex plane, for a very general class of strictly positive, continuous, radial weights v. Precise results are presented for concrete weights on the disc that could not be treated before. It is also shown that if H^∞_v is solid, then the monomials are an (unconditional) basis of the closure of the polynomials in H^∞_v. As a consequence H^∞_v does not coincide with its solid hull and core in the case of the disc. An example shows that this does not hold for weighted spaces of entire functions. Applications to spaces of multipliers will be mentioned.

Korovkin approximation of set-valued and vector-valued functions

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This talk is devoted to some aspects of Korovkin approximation theory connected with the approximation of operators between cones of Hausdorff continuous set-valued functions.

Some important consequences are also concerned with Korovkin approximation theory in spaces of continuous vector-valued functions.

The exposition starts with some classical results and continues with some later developments up to the most recent results.

In particular we consider a class of linear operators, the so-called convexity monotone operators, which allow to point out how the analysis of set-valued functions is strictly connected with that of vector-valued functions.

Finally, some interesting consequences concerning the classical Korovkin theory is also indicated.

The scientific achievements of Francesco Altomare

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Starting from 1977, Francesco Altomare has written 3 monographs (cfr. [1, 2, 3]) and more than 90 papers. He has been a source of inspiration for generations of mathematicians interested, among other things, in Real and Functional Analysis (in particular Choquet Integral Representation Theory, abstract Potential Theory and Harmonic Analysis), Operator Theory, Positive Semigroups Theory and Approximation Theory. He has, in fact, fully realized the deep connections among those topics and this original insight has been the guiding light for many of us who work in those fields.

This talk is devoted to enlightening those aspects of Altomare's work; in particular, I will present some results obtained during our almost 20 years long collaboration.

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Banach lattices and their dual spaces

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It is well-known that the dual of a Banach lattice is a Banach lattice. However, a Banach lattice may have isometric preduals which are not lattices: the space l_1 equipped with its natural norm in an example. We will show however that any separable isometric predual of an order-complete Banach lattice X is always contained in the space of order-continuous linear forms on X. The fine structure of Baire-mesurable functions on a Stonian compact space will be an useful tool.

Approximation processes for integrable functions

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During their scientific careers, professor Altomare and his collaborators dealt with several classes of approximation processes, investigating their properties and possible applications.

In this talk we focus on those ones that allow to work not only with continuous functions, but also with Lebesgue integrable functions.

The setting of these researches is mainly a convex compact set of a (not necessarily finite-dimensional) locally convex space, but there are studies even in the case of $\mathbb{R}^d$ $(d \geq 1)$ or unbounded real intervals. A further generalization for noncompact sets will be the subject of future works.

Below some of references on which the talk is based.

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Mathematics As A Natural Bridge Between Humanism and Science

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Mathematics has often played a central role within ancient societies, thanks to religious rites, astronomic speculations, rhythms of nature, as well to various needs of everyday life. It was quite natural, therefore, the coming into being of an ideal "bridge" between humanism and science. A startling example of this can be found in Dante's Comedy, through his concrete description of the old vision of the universe, efficiently expressed by the Arts of the Quadrivium: Geometry, Arithmetic, Astronomy, Music. A radical change started with Galileo, soon after the success of his new scientific method, which gave rise to a progressive separation between the "two cultures". In the 20th century, Mathematics had to face deep internal crises. Nevertheless, being rather close to the "spirit of time", that had the benefic effect of stimulating the interest and the appreciation by poets, writers, thinkers and artists and, in so doing, the image of Mathematics as a "bridge" between the two cultures clearly re-emerged. The present talk aims to illustrate briefly the long and intense path outlined above, and possibly to put to the fore the richness and fascination of Mathematics.

Perturbations beyond Schauder

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Nonuniformly elliptic and parabolic problems are a classical topic since the pioneering work of Finn, Gilbarg, Ladyzhenskaya, Uraltseva, Trudinger, Serrin, Ivanov, Ivochkina. There has been an intensive activity in proving regularity results for solutions since the 60s. Such problems play also an important role in the Calculus of Variations, where they arise in connection to the Euler-Lagrange equations of functionals with non-standard growth conditions. A basic issue that has remained open is the validity of Schauder estimates, whose lack is connected with the failure of classical perturbation methods in the nonuniformly elliptic case. In this talk I will present the first solution to this problem, providing an approach to Hölder continuity which is alternative to the classical ones based on perturbations. From joint work with Cristiana De Filippis (Torino).

Hardy-type inequalities as a source of inspiration for 100 years

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First I present some Pers(s)onally chosen results about the remarkable prehistory and history of Hardy-type inequalities from G. H. Hardy's early discoveries around 1920 till 2017 ee e.g. [1], [2] and the references therein). After that I present some examples of interesting newest results in this area, which I hope can give inspiration and possibilities for new directions of further developments and applications of this fascinating subject. The content of the lecture is clearly influenced by my visit to Italy 2009 to a conference organized by Professor Altomare.

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Markov semigroups and Bernstein-Schnabl operators: work of Altomare and some consequences

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(Joint work with Ana Maria Acu.) Bernstein-Schnabl operators occupy a central place in the work of Altomare. In a series of papers he clarified and fruitfully used their relationship with Markov semigroups and related objects. A first comprehensive presentation of such results was given in the monograph [1]. The state of art around 2014 was described in [2]. Meanwhile, the study of convexity preserving properties of Bernstein-Schnabl operators led to some problems solved recently with a variety of analytic and stochastic methods: for details and references see [3] and [4].

This talk gives a brief account of all these topics.

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Evolution equations of Cox-Ingersoll-Ross type and their associated (C_0) semigroups

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We will consider the following initial value problem

$$(1) \qquad \frac{\partial u}{\partial t} = \nu^2 x \frac{\partial^2 u}{\partial x^2} + (\gamma + \beta x)\frac{\partial u}{\partial x} - rxu, \ t \geq 0, x \geq 0,$$

$$(2) \quad u(0,x) = f(x), \ x \geq 0,$$

where $\nu > 0, \gamma > 0$, $\beta \in \mathbf{R}$ and $r \geq 0$. Our main aim is to present the results obtained in [2, 3] concerning the existence of a (C_0) semigroup governing (1)-(2) in the space $C[0,\infty]$ of all continuous functions f in

$[0,\infty)$ with finite limit at ∞, equipped with the sup norm, or in its subspace $C_0[0,\infty) := \{f \in C[0,\infty] : \lim_{x\to\infty} f(x) = 0\}$, or in the weighted space

$$C_{s,0}[0,\infty) := \{f \in C[0,\infty) : \frac{f(x)}{1+x^s} \in C_0[0,\infty)\} \quad s > 0,$$

equipped with the weighted norm $\|f\|_s := \sup_{x\geq 0} \frac{|f(x)|}{1+x^s}$.

Observe that, under suitable assumptions about the financial markets and for $r = 1$, $f(x) \equiv 1$, the solution u of (1)-(2) represents the price of a discount bond with face value 1 at the maturity, according to the results by J.C. Cox, J.E. Ingersoll, and S.A. Ross in [1], where $\sqrt{2}\nu$ is the volatility of the stochastic interest rate.
Other aspects dealing with possible approximations of the solutions and the nonautonomous version of (1)-(2) will be mentioned.

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A survey about anisotropic parabolic operators

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We will speak about evolution anisotropic operators of the type

$$u_t = \sum_{i=1}^{N} Div(|Du|^{p_i-2}Du).$$

The first results on such class operators were found in the eighties, but still, especially for the rough regularity, the theory is incomplete and fragmented.

Some recent approximation results with applications

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I will present some approximation result for a family of operators and discuss some application.