

Luigi Accardi
(Roma Tor Vergata)

Markov-Dobrushin Inequality for Quantum Channels

Abstract: In the theory of classical Markov chains there are two techniques to guarantee that a Markov operator converges exponentially to equilibrium and to estimate the rate of convergence: one uses the spectral gap, the other one the Markov-Dobrushin (MD) inequality. The idea to introduce the quantum version of MD inequality was first advanced by F. Mukhammedov. We propose a different approach which is nearer to the original MD approach because the quantum MD constant we obtain has a natural probabilistic interpretation. Some examples and open problems will also be discussed. This is a joint work with Abdessatar Souissi and Yun Gang Lu

Raffaella Carbone
(Pavia)

On absorption problems for quantum channels

Abstract: We introduce a notion of absorption operators in the context of quantum Markov maps, generalizing the concept of absorption probability for classical Markov chains. We consider the problem for a quantum Markov evolution on a separable Hilbert space, both in discrete and continuous time: given an invariant domain, we define a well-behaving positive operator, which can correspond to classical absorption probabilities, and we study its structure and basic properties. We show how it can turn out to be useful in different contexts: in order to deduce some accessibility properties of quantum Markov channels, to describe the structure of preserved observables (especially in the case when the recurrent subspace is attractive), to study various kinds of asymptotic results. This work is in collaboration with Federico Girotti.

Simone Del Vecchio
(Roma Tor Vergata)

Ergodic properties of classical and quantum Anzai Skew-Products

Abstract: Anzai skew-products are a relatively simple class of dynamical systems, yet they are remarkable as they exhibit a wide range of ergodic behaviour. I will talk about the ergodic properties of a quantum version of Anzai skew-products, based on the noncommutative torus. Furthermore, in the context of classical skew-products, I will discuss the property of unique ergodicity w.r.t. the fixed point subalgebra, a natural generalization of unique ergodicity introduced by Abadie and Dykema in 2006. Based on joint work with Francesco Fidaleo, Luca Giorgetti and Stefano Rossi.

Franco Fagnola

(Politecnico di Milano)

Fibonacci oscillators as open quantum systems

Abstract: We consider an open quantum system with Hamiltonian H_S whose spectrum is given by a generalized Fibonacci sequence weakly coupled to a reservoir. We find the GKLS generator of the quantum Markov semigroup and characterize recurrent and transient regimes in terms of the parameters of the model.

If the system Hamiltonian is generic and there exists a faithful normal invariant state we show that convergence towards the invariant state is exponentially fast by computing explicitly the spectral gap under an additional assumption on the spectrum of H_S .

The talk is based on a joint work with Chulki Ko and Hyunjae Yoo.

Francesco Fidaleo

(Roma Tor Vergata)

Modular Spectral Triples and deformed Fredholm modules

Abstract: Due to possible applications to the attempt to provide a set of equations which unify the four elementary interactions in nature (the grand-unification) and in another, perhaps connected, direction in proving the long-standing, still unsolved, Riemann conjecture about the zeroes of the zeta-function, Connes' noncommutative geometry grew up rapidly in the last decades.

Among the main objects introduced (by A. Connes) for handling noncommutative geometry there are the so called spectral triples, encoding most of the properties enjoyed by the (quantum) "manifold" into consideration, and the associated Fredholm modules.

On the other hand, the so-called Tomita modular theory is nowadays assuming an increasingly relevant role for several applications in mathematics and in physics. Such a scenario suggests the necessary need to take the modular data into account in the investigation of quantum manifolds. In such a situation, the involved Dirac operators should be suitably deformed (by the use of the modular operator), and should come from non-type II_1 representations.

Taking into account such comments, we discuss the preliminary necessary step consisting in the explicit construction of examples of non type II_1 representations and relative spectral triples, called modular. This is done for the noncommutative 2-torus A_α , provided α is a (special kind of) Liouville number, where the nontrivial modular structure plays a crucial role.

For such representations, we briefly discuss the appropriate Fourier analysis, by proving the analogue of many of the classical known theorems in harmonic analysis such as the Riemann-Lebesgue lemma, the Hausdorff-Young theorem, the L^p -convergence results associated to the Cesaro means (i.e. the Fejer theorem) and the Abel means reproducing the Poisson kernel. We show how those Fourier

transforms "diagonalise" appropriately some examples of the Dirac operators associated with the previous mentioned spectral triples.

Finally, we provide a definition of a deformed generalisation of "Fredholm module", i.e. a suitably deformed commutator of the "phase" of the involved Dirac operator with elements of a subset (the so-called Lipschitz $*$ -algebra or Lipschitz operator system) which, depending on the cases under consideration, is either a dense $*$ -algebra or an essential operator system. We also show that all models of modular spectral triples for the noncommutative 2-torus mentioned above enjoy the property to being also a deformed Fredholm module. This definition of deformed Fredholm module is new even in the usual cases associated to a trace, and could provide other, hopefully interesting, applications. The present talk is based on the following papers:

[1] F. Fidaleo and L. Suriano: Type III representations and modular spectral triples for the noncommutative torus, *J. Funct. Anal.* 275 (2018), 1484-1531.

[2] F. Fidaleo: Fourier analysis for type III representations of the noncommutative torus, *J. Fourier Anal. Appl.* 25 (201), 2801-2835.

[3] F. Ciolli and F. Fidaleo: Type III modular spectral triples and deformed Fredholm modules, preprint.

Federico Girotti

(Pavia)

Large deviations, central limit and dynamical phase transitions in the atom maser

Abstract: The theory of quantum jump trajectories provides the framework for understanding dynamical phase transitions in open systems. A candidate for such transitions is the atom maser; although previous numerical results suggested that the "free energy" may not be a smooth function, in our talk we show that the atom detection counts satisfy a large deviation principle, and therefore we deal with a phase cross-over rather than a genuine phase transition. As a corollary, we obtain the central limit theorem for the counting process.

The proof relies on the analysis of a certain deformed generator whose spectral bound is the limiting cumulant generating function. The latter is shown to be smooth, so that a large deviations principle holds by the Gärtner-Ellis Theorem. One of the main ingredients is the Krein-Rutman theory which extends the Perron-Frobenius theorem to a general class of positive compact semigroups.

The talk is based on a joint work with R. Carbone, M. Guță and M. van Horssen.

Maria Elena Griseta

(STIIMA-CNR Bari)

Distributions for nonsymmetric monotone and weakly monotone position operators

Abstract: In this talk, we investigate the vacuum distribution, under an appropriate scaling, of a family of partial sums of nonsymmetric position operators on weakly monotone and monotone Fock spaces. We first consider the case of weakly monotone Fock space and show that any single operator has the vacuum law belonging to the free Meixner class. After establishing some relations between the combinatorics of Motzkin and Riordan paths, we give a recursive formula for the vacuum moments of the law of any finite sum. We also investigate the asymptotic measure for these sums, which turns out to belong to the free Meixner class, with an atomic and an absolutely continuous part, both explicitly computed. Finally, we briefly describe the case of monotone Fock space.

This is a joint work with V. Crismale and J. Wysoczański.

Damiano Poletti

(Politecnico di Milano)

Gaussian Quantum Markov Semigroups: a characterization and the study of the one-dimensional case

Abstract: Gaussian Quantum Markov Semigroups are related with gaussian states, that is states under which position and momentum operators are distributed according to a gaussian distribution. In the first part of the talk we will characterize gaussian QMSs as the only class of QMSs that preserve “gaussianity” of states. We will provide a Lindblad generator for them and we will give an explicit action on Weyl operators of a regular representation of the CCR. In the second part of the talk, that is based on a joint work with J. Agredo and F. Fagnola, we will deal with gaussian semigroups on the one-dimensional Fock Space. Here we can give a complete characterization of the irreducibility of gaussian QMSs, based solely on the parameters of the semigroup, and the characterization of existence and uniqueness of a normal invariant state for them.

Piergiacomo Sabino

(EON SE, Essen)

Normal Tempered Stable Processes and the Pricing of Energy Derivatives

Abstract: In this study we consider the pricing of energy derivatives when the evolution of spot prices is modeled with a normal tempered stable driven OrnsteinUhlenbeck process. Such processes are the generalization of normal inverse Gaussian processes that are widely used in energy finance applications. We first specify their statistical properties calculating their characteristic function in closed form. This result is instrumental for the derivation of non-arbitrage conditions such that the spot dynamics is consistent with the forward curve without relying on numerical approximations or on numerical integration. Moreover, we conceive an efficient algorithm for the exact generation of the trajectories which gives the possibility to implement Monte Carlo simulations without approximations or bias. We illustrate the applicability of the theoretical findings and the simulation algorithms in the context of the pricing of different contracts, namely, strips of daily call options, Asian options with European style and swing options. Finally, we present an extension to future markets

Emanuela Sasso

(Genova)

On the relationship between Covariance and the Decoherence free-subalgebra for a QMS

Abstract: We are interested to investigate if the presence of symmetries in the evolution of an open quantum system gives informations about the invariant subspaces. Unfortunately the answer is, in general, negative, but under suitable conditions we can observe that the representation through which we describe the symmetry determines a privileged family of orthogonal projections strongly correlated with the structure of the Decoherence-free sub-algebra and, as consequence, with harmonic projections.

Michael Skeide

(Campobasso)

CP-Semigroups and Dilations, Subproduct Systems and Superproduct Systems: The Multi-Parameter Case and Beyond

Abstract: These notes are the output of a decade of research on how the results about dilations of one-parameter CP-semigroups with the help of product systems, can be put forward to d -parameter semigroups –and beyond. While preliminary work on the two- and d -parameter case is based on the approach via the Arveson-Stinespring correspondence of a CP-map by Muhly and Solel [2002] (and limited to von Neumann algebras), here we explore consequently the approach via Paschke’s GNS correspondence of a CP-map [1973] by Bhat and Skeide [2000]. (A comparison is postponed to Appendix A (iv).)

The generalizations are multi-fold, the difficulties often enormous. In fact, our only true if-and only-if theorem, is the following: A Markov semigroup over (the opposite of) an Ore monoid admits a full (strict or normal) dilation if and only if its GNS-subproduct system embeds into a product system. Already earlier, it has been observed that the GNS-(respectively, the Arveson Stinespring) correspondences form a subproduct system, and that the main difficulty is to embed that into a product system. Here we add, that every dilation comes along with a superproduct system (a product system if the dilation is full). The latter may or may not contain the GNS-subproduct system; it does, if the dilation is strong – but not only. Apart from the many positive results pushing forward the theory to large extent, we provide plenty of counter examples for almost every desirable statement we could not prove. Still, a small number of open problems remains. The most prominent: Does there exist a CP-semigroup that admits a dilation, but no strong dilation? Another one: Does there exist a Markov semigroup that admits a (necessarily strong) dilation, but no full dilation?

This joint work with Orr Shalit