

## XMATHS WORKSHOP 2020

**Davide Lonigro (University of Bari):** Deviations from the exponential law for a generic quantum observable

The description of decay phenomena has been a central topic in quantum mechanics since its inception; in particular, a quantum mechanical description of exponential decay, which is ubiquitous in natural sciences, has been pursued for decades. However, a fundamental obstruction to quantum exponential decay comes from Fourier analysis: whenever the energy spectrum of a system is bounded from below, the survival probability of any pure state must decay slower than exponentially, for example with a power law, at sufficiently large times. In this talk we will generalize this argument by showing that, under the same assumptions, the average value of all quantum observables, whenever well-defined, cannot converge exponentially at large times to an extremal value of its spectrum. In all those cases, large-time deviations from the exponential behavior are to be expected.

**Luca Bellino (Polytechnic University of Bari):** Mathematical modeling of bio-inspired materials

Human kind has always tried to understand and find inspirations from the laws of Nature, a brilliant teacher and engineer. Investigate micro-systems and unveil the role of mechanical forces at such small scales is fundamental to correctly describe a variety of biological phenomena such as protein unfolding, DNA and RNA denaturation processes or brain axonal damage. We study these micro-mechanical models in a Statistical Mechanics framework with the aim of obtaining a multiscale description emerging from microscopic laws and leading to a macroscopic constitutive behavior. As a matter of fact, this is the turning point to both understand the effects of temperature and forces on these systems and correctly design new bio-inspired material.

**Felisia Angela Chiarello (Polytechnic University of Turin):** Multiscale control of generic second order traffic models by driver-assist vehicles

In this talk, we study the derivation of generic high order macroscopic traffic models from a follow-the-leader particle description via a kinetic approach. We introduce in the vehicle interactions a binary control modeling the automatic feedback provided by driver-assist vehicles and we upscale such a new particle description by means of an Enskog-based hydrodynamic limit. The resulting macroscopic model is a Generic Second Order Model (GSOM), which contains in turn a control term inherited from the microscopic interactions. We show that such a control may be chosen so as to optimise global traffic trends, such as the vehicle flux or the road congestion, constrained by the GSOM dynamics. By means of numerical simulations, we investigate the effect of this control hierarchy in some specific case studies, which exemplify the multiscale path from the vehicle-wise implementation of a driver-assist control to its optimal hydrodynamic design.

**Fabio Pizzichillo (Université Paris Dauphine):** Boundary value problems for 2-D Dirac operator on corner domains

This talk aims to present results on the self-adjoint extensions of Dirac operators on plane domains with corners in dimension two. We consider the case of general boundary conditions for a quantum dot (confined particle), and a the Lorentz scalar  $\delta$ -shell interaction. For these cases, we obtain explicitly the self-adjoint extensions of the operator. It turns out that the presence of corners typically spoils the elliptic regularity known to hold for smooth boundaries.

**Marco Gallo (University of Bari):** Fractional nonlinear Schrödinger equations with prescribed mass: story of a particle and its dream of a perfect weight

The search for standing waves in Schrödinger equations is often pursued with a prescribed frequency and a free mass. In this talk we will go in the opposite direction, focusing on some fractional nonlinear Schrödinger equation, where the mass of the particle, given by the  $L^2$ -norm, is prescribed; in such a case, the frequency becomes an unknown. Working in a variational setting, we will describe some geometric and analytic ideas modeled on the Pohozaev identity, which enable to treat the lack of regularity typical of the fractional framework. This talk is based on a joint work with Silvia Cingolani and Kazunaga Tanaka.

**Jacopo Schino (Polish Academy of Sciences):** When Schrödinger meets Maxwell

We demonstrate that, under cylindrical symmetry assumptions, the weak solutions to a curl-curl problem (which originates from Maxwell's equations in the vacuum) are in 1-to-1 correspondence with those to an elliptic Schrödinger equation with a singular potential, hence generalising a well-known result for classical solutions. Based on joint work with Michał Gaczkowski and Jarosław Mederski.

**Flavia Esposito (John Paul II Oncology Institute, Bari):** Nonnegative Matrix Factorization models for knowledge extraction from biomedical and other real world data

Inspect data for searching valuable information hidden in represents a key aspect in several fields. Fortunately, most of the available data presents an embedded mathematical structure which can be profitably exploited to better investigate latent patterns hidden in them. Dimensionality Reduction (DR) approaches represent one of the most suitable instrument to untangle latent information. In this work we consider Nonnegative Matrix Factorizations (NMFs), which prove to be the most effective among DR methods in analyzing real-life nonnegative data. Some variants of NMF will be also presented as minimization tasks to which regularization terms can be added in accordance to some additional characteristics. We present a new NMF model designed for biomedical data that incorporates specific biological proprieties as different constraints. Since NMF and its variants are daily used in several application domains, we conclude stressing how these techniques work in some real life applications, showing some original works related to the analysis of data from engineering field.

**Giuseppe Vacca (University of Milano-Bicocca):** Virtual Elements for a fluid-structure interaction problem

The Virtual Element Method (VEM) is a recent technology introduced in [Beiro da Veiga, Brezzi, Cangiani, Manzini, Marini, Russo, 2012, M3AS] for the discretization of partial differential equations. The VEM can be interpreted as a novel approach that generalizes the classical Finite Element Method to arbitrary even non-convex element-geometry. By avoiding the explicit integration of the shape functions that span the discrete Galerkin space and introducing a novel construction of the associated stiffness matrix, the VEM acquires very interesting properties and advantages with respect to more standard Galerkin methods, yet still keeping the same coding complexity. The present talk is both an introduction to the VEM for the Stokes equation, aiming at showing the main ideas of the method, and a brief look at some application to fluid-structure interaction problems. In the first part of the talk we will describe the basics of the divergence-free virtual elements for the Stokes equation. In the second part, we will present a first application of VEM for a fluid structure interaction problem. We study, both theoretically and numerically, the equilibrium of a hinged rigid leaflet with an attached rotational spring, immersed in a stationary incompressible fluid within a rigid channel.

**Mufutau Rufai (University of Bari):** One-step hybrid block method containing third derivatives and improving strategies for solving Bratu's and Troesch's problems

In this talk, I will introduce an efficient one-step hybrid block method which incorporates third derivatives for solv-

ing one-dimensional Bratu's and Troesch's problems. The fundamental properties of the proposed method will be theoretically analysed, and some improving strategies will be considered to get better performance of the technique. Some numerical experiments will be presented to show the efficiency and effectiveness of the proposed approach in comparison with other methods that appeared in the literature.

**Ilaria Castellano (University of Milano Bicocca):** The Five Ws for totally disconnected locally compact groups

According to the principle of the Five Ws, a report can be considered complete only if it answers five questions starting with the words Who, What, Why, When, Where. The aim of my talk is to report on the theory of locally compact groups acting on discrete structures; for example, the automorphism group of a regular tree. These groups are usually referred to as TDLC-groups, where the shorthand TDLC stands for totally disconnected and locally compact. With the solution of Hilbert's 5th Problem (1952), a structure theory for TDLC-groups appeared to be essential since it could be considered as the missing piece in the theory of general locally compact groups. The talk will be a soft introduction to the research activities on this class of topological groups.

**Antonio Macchia (Freie Universität Berlin):** Realizability problems for convex polytopes

A classical problem in Discrete Geometry concerns the realizability of a point configuration as a convex polytope. In 1922 Steinitz classified all possible realizations in dimension 3: a graph  $G$  is the edge graph of a 3-dimensional polytope if and only if  $G$  is simple, planar and 3-connected. However, already in dimension 4 the set of realizations of a polytope, known as realization space, can be arbitrarily complicated. Recently, we introduced the slack realization space, a new model of realization space which represents each polytope by its slack matrix, obtained by evaluating each facet inequality at each vertex. This model is inherently algebraic, since it arises as the positive part of a variety of a saturated determinantal ideal, and provides a new and powerful computational tool to study realizability problems. To this aim, we developed the SlackIdeals package for the software Macaulay2, that provides methods for creating and manipulating slack matrices and slack ideals of convex polytopes and matroids. Using this package we are able to certify in a fast and simple way the non-realizability of several quasi-simplicial spheres whose realizability was previously unknown. This talk is based on joint works with João Gouveia, Rekha Thomas and Amy Wiebe.

**Eugenia Loiudice (Philipps Universität Marburg):** How to construct all metric f-K-contact manifolds

We show that any compact metric f-K-contact, respectively S-manifold is obtained from a compact K-contact, respectively Sasakian manifold by an iteration of constructions of mapping tori, rotations, and type II deformations.

**Dario Di Pinto (University of Bari):** Some global results on the submanifolds of Sasakian manifolds

In this talk we will deal with generic submanifolds of Sasakian manifolds and we will discuss two main results, based on the fact that such submanifolds are naturally endowed with a CR structure. The first one is a Frankel type theorem which provides sufficient condition in order to have non empty intersection between two generic submanifolds or a generic submanifold and an invariant submanifold. The second one is a non existence result of complete Einstein hypersurfaces tangent to the Reeb vector field of a regular Sasakian manifold which fibers onto a complex Stein manifold. - Based on joint work with prof. Antonio Lotta.

**Fabio Deelan Cunden (SISSA):** Airplane boarding, longest increasing subsequences and random matrices

In this talk I will give a brief introduction to the fascinating subject of the longest increasing subsequences of a random permutation. Starting from an innocent-sounding question about random permutations, we will be led on a journey touching enumerative and algebraic combinatorics, random matrices, statistical physics, and (time permitting) random topologies on surfaces.

**Giovanni Gramegna (University of Bari):** Generic aspects of the Resource Theory of Quantum Coherence

The class of incoherent operations induces a pre-order on the set of quantum pure states, defined by the possibility of converting one state into the other by transformations within the class. We prove that if two  $n$ -dimensional pure states are chosen independently according to the natural uniform distribution, then the probability that they are comparable vanishes as  $n$  goes to infinity. We also study the maximal success probability of incoherent conversions and find an explicit formula for its large- $n$  asymptotic distribution. Our analysis is based on the observation that the extreme values (largest and smallest components) of a random point uniformly sampled from the unit simplex are distributed asymptotically as certain explicit homogeneous Markov chains.

**Gianluca Orlando (Technical University of Munich):** Frustration in the antiferromagnetic XY spin system on the triangular lattice: a variational analysis

I will present the variational discrete-to-continuum analysis of the antiferromagnetic XY spin system on the triangular lattice, which favours anti-alignment on each pair of interacting spins. A configuration where every interaction is minimised is not achievable, leading to geometric frustration and to two distinct families of ground states. Through a Gamma-convergence approach, we shall study spin configurations with low energy, explaining what is the relevant variable in the system according to the energy scaling.

**Caterina Sportelli (University of Bari):** A minimax approach to a class of gradient-type quasilinear  $(p_1, p_2)$ -Laplacian system

The aim of this talk is to show that, under suitable assumptions, at least one nontrivial solution for a class of coupled gradient-type quasilinear systems involving  $(p_1, p_2)$ -Laplacian operators can be found. Here, some difficulties arise since we deal with coefficients depending also on the solution itself. Anyway, in spite of this hardship, we prove that, up to generalize some classical abstract tools, a variational approach still works. In particular, studying the interaction of two different norms in a suitable Banach space, we are able to find a nontrivial solution for our problem and, under assumptions of symmetry, infinitely many ones too.

**Fabio Difonzo (CTU in Prague):** A mass conservative quadrature-based scheme for numerical solutions to Richards' equation

We propose a new numerical scheme for solving Richards' equation within Gardner's framework and accomplishing mass conservation. In order to do so, we first integrate both in time and in space the mixed form of Richards' equation; then, resorting to Kirchhoff transform and exploiting specific Gardner model features reduce the high non-linear features of Richards' equation to a linear differential equation relative to the Kirchhoff transform variable which can be solved exactly, yielding the sought mass conservative numerical scheme.

**Laura Selicato (University of Bari):** Optimization Mechanism in Machine Learning and Data Science

Automatic learning research focuses on the development of methods capable of extracting useful information from a given dataset. A large variety of learning methods exists, ranging from biologically inspired neural networks to statistical methods. A common trait in these methods is that they are parameterized by a set of hyperparameters, which must be set appropriately by the user to maximize the usefulness of the learning approach. In this talk we review hyperparameter tuning and discuss its main challenges from an optimization point of view. We provide an overview on the most important approaches for hyperparameter optimization problem, comparing them in terms of advantages and disadvantages, focusing on Gradient-based Optimization