EVOLUTION EQUATIONS and PDEs

on the occasion of the 60th birthday of

Gisèle RUIZ GOLDSTEIN

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Aula VI
Dipartimento di Matematica
Università degli Studi di Bari Aldo Moro

ABSTRACTS

UNIVERSITÀ DEGLI STUDI DI BARI ALDO MORO

DIPARTIMENTO DI MATEMATICA

Speakers

Francesco Altomare	*Università degli Studi di Bari Aldo Moro*
Giuseppe M. Coclite	*Politecnico di Bari*
Jerome A. Goldstein	*University of Memphis*
Davide Guidetti	*Università degli Studi di Bologna*
Sandra Lucente	*Università degli Studi di Bari Aldo Moro*
Giorgio Metafune	*Università del Salento*
Rosa Maria Mininni	*Università degli Studi di Bari Aldo Moro*
Diego Pallara	*Università del Salento*
Silvia Romanelli	*Università degli Studi di Bari Aldo Moro*
Cristian Tacelli	*Università degli Studi di Salerno*

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Anna Maria Candela	*Università degli Studi di Bari Aldo Moro*
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Rosa Maria Mininni	*Università degli Studi di Bari Aldo Moro*
Silvia Romanelli	*Università degli Studi di Bari Aldo Moro*

Program

09.30-10.00 *Welcome*

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10.00-10.20 **Jerome A. Goldstein**
The Mathematical Work of Gisèle Ruiz Goldstein

10.25-10.45 **Davide Guidetti**
On maximal regularity for the Cauchy–Dirichlet mixed parabolic problem with fractional time derivative

10.50-11.10 **Silvia Romanelli**
Wentzell boundary conditions in the research of Gisèle Ruiz Goldstein

11.15-11.35 *Break*

Chairman *Davide Guidetti*

11.35-11.55 **Rosa Maria Mininni**
Degenerate evolution equations in Mathematical Finance

12.00-12.20 **Giuseppe M. Coclite**
A model for the morphogens evolution in a growing tissue

12.25-12.45 **Cristian Tacelli**
Elliptic operators with unbounded diffusion, drift and potential terms

12.50-14.30 *Lunch*

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On a class of Fleming–Viot type differential operators on the unit hypercube

14.55-15.15 **Giorgio Metafune**
Weighted Rellich and Calderón–Zygmund inequalities

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15.40-16.00 **Diego Pallara**
Infinite dimensional gradient estimates for perturbed Ornstein–Uhlenbeck semigroups in convex sets

16.05-16.25 **Sandra Lucente**
Global existence and asymptotic profile for a class of nonlinear dissipative equations

16.30 *Closure*

Francesco Altomare
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On a class of Fleming–Viot type differential operators on the unit hypercube

The aim of the talk is to discuss some recent results obtained jointly with Mirella Cappelletti Montano and Vita Leonessa.

Of concern is a class of degenerate second–order elliptic differential operators, often referred to as Fleming–Viot type operators, acting in the framework of function spaces defined on the d–dimensional unit hypercube.

By making mainly use of techniques arising from approximation theory, it is shown that their closures generate positive contraction semigroups both in the space of all continuous functions and in weighted L^p–spaces.

In order to obtain a constructive approximation of such semigroups in terms of iterates of positive operators, a new sequence of polynomial type positive linear operators is introduced and studied. These operators generalize the Bernstein-Durrmeyer operators with Jacobi weights on [0,1].

As a consequence, after determining the unique invariant measure for the approximating operators and for the semigroups, some regularity properties as well as their asymptotic behaviours are established.

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A model for the morphogens evolution in a growing tissue

Recently Averbukh, Ben–Zvi, Mishra, and Barkai proposed a model for the regulation of growth and patterning in developing tissues by diffusing morphogens. The model consists of a free boundary nonhomogeneous Neumann problem with two equations, a diffusion one and a transport one.

We show that solutions of the underlying coupled system of nonlinear PDEs exist, are unique and are stable in a suitable sense.

The key tool in the analysis is the transformation of the underlying system to a porous medium equation.

These results were obtained in collaboration with M.M. Coclite (University of Bari) and S. Mishra (ETH-Zurich).

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The Mathematical Work of Gisèle Ruiz Goldstein

We briefly review the major contributions of Gisèle Goldstein to various areas of Applied Analysis, including
(1) Rigorous Thomas–Fermi theory of ground state electron densities in quantum theory,
(2) Wentzell boundary conditions in linear and nonlinear PDE,
(3) Cahn–Hilliard equations and related problems in Continuum Mechanics,
(4) Instantaneous blowup,
(5) Asymptotics of energy associated with dissipative waves, and
(6) Financial mathematics.

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On maximal regularity for the Cauchy–Dirichlet mixed parabolic problem with fractional time derivative

In this seminar we illustrate some results of maximal regularity for the Cauchy–Dirichlet mixed problem, with a fractional time derivative of Caputo type in spaces of continuous and Hölder continuous functions.

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Global existence and asymptotic profile for a class of nonlinear dissipative equations

We consider the class of evolution equation

$$u_{tt} + (-\Delta)^{2\theta} u + 2\mu(-\Delta)^{\theta} u_t + |u_t|^{p-1} u_t = 0, \quad t \geq 0,\ x \in \mathbb{R}^n,$$

with $\mu > 0$ and $\theta > 0$. Assuming

$$p > 1 + 4\theta/n,$$

in space dimension $n \in (2\theta, 4\theta)$, we discuss

- global existence in the energy space $H^{4\theta} \times H^{2\theta}$, with additional L^1 regularity,
- optimal decay estimates, i.e. the decay rate is the same of the corresponding linear problem.

For $\mu \geq 1$ we find the self–similar asymptotic profile of the solutions, by combining Riesz potential with the solution of

$$v_t + a(-\Delta)^{\theta} v = 0,$$

for suitable $a > 0$.

References

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Weighted Rellich and Calderón–Zygmund inequalities

In 1956, Rellich proved the inequalities

$$\left(\frac{N(N-4)}{4}\right)^2 \int_{\mathbb{R}^N} |x|^{-4}|u|^2 dx \le \int_{\mathbb{R}^N} |\Delta u|^2 dx$$

for $N \neq 2$ and for every $u \in C_c^\infty(\mathbb{R}^N \setminus \{0\})$. These inequalities have been then extended to L^p–norms: in 1996, Okazawa proved the validity of

$$\left(\frac{N}{p}-2\right)^p \left(\frac{N}{p'}\right)^p \int_{\mathbb{R}^N} |x|^{-2p}|u|^p dx \le \int_{\mathbb{R}^N} |\Delta u|^p dx$$

for $1 < p < \frac{N}{2}$ showing also the optimality of the constants.

Weighted Rellich inequalities have also been studied. In 1998, Davies and Hinz obtained for $N \ge 3$ and for $2 - \frac{N}{2} < \alpha < \frac{2}{p'}$

$$C^p(N,p,\alpha) \int_{\mathbb{R}^N} |x|^{(\alpha-2)p}|u|^p dx \le \int_{\mathbb{R}^N} |x|^{\alpha p}|\Delta u|^p dx \tag{1}$$

with the optimal constants $C^p(N,p,\alpha) = \left(\frac{N}{p} - 2 + \alpha\right)^p \left(\frac{N}{p'} - \alpha\right)^p$. Later Mitidieri showed that (1) holds in the wider range $2 - \frac{N}{p} < \alpha < \frac{N}{p'}$ and with the same constants.
In recent papers Ghoussoub and Moradifam and Caldiroli and Musina improved weighted Rellich inequalities for $p = 2$ by giving necessary and sufficient conditions on α for the validity of (1) and finding also the optimal constants $C^2(N,2,\alpha)$. In particular (1) is verified for $p = 2$ if and only if $\alpha \neq N/2 + n$, $\alpha \neq -N/2 + 2 - n$ for every $n \in \mathbb{N}_0$.

We show that (1) holds if and only if $\alpha \neq N/p' + n$, $\alpha \neq -N/p + 2 - n$ for every $n \in \mathbb{N}_0$. Moreover, we use Rellich inequalities to find necessary and sufficient conditions for the validity of weighted Calderón–Zygmund estimates when $1 < p < \infty$

$$\int_{\mathbb{R}^N} |x|^{\alpha p}|D^2 u|^p dx \le C \int_{\mathbb{R}^N} |x|^{\alpha p}|\Delta u|^p dx \tag{2}$$

for $u \in C_c^\infty(\mathbb{R}^N \setminus \{0\})$.

We use spectral arguments reducing the inequality above to a spectral inequality for a degenerate operator, whose spectrum can be explicitlly determined. Rellich and Calderón–Zygmund inequalities can be applied to show generation properties of the operator $|x|^\alpha L$ above and to characterize its domain.

Most of the content of these lectures is based on joint work with Chiara Spina and Motohiro Sobajima.

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Degenerate evolution equations in Mathematical Finance

In this talk I will discuss some recent results concerning evolution equa- tions associated to second order differential operators with degeneracy at the boundary of the space domain involving general boundary conditions. This kind of operators naturally arise in Mathematical Finance to derive the price of zero–coupon bonds under a stochastic short–rate interest or of European options on risk assets with stochastic volatility ([2], [4]).

Our main focus will be on existence, uniqueness and regularity of solutions on suitable function spaces, and approximation formulas.

Our results were obtained in joint works with A. Canale, G. Ruiz Goldstein, J.A. Goldstein, A. Rhandi and S. Romanelli.

References

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Infinite dimensional gradient estimates for perturbed Ornstein–Uhlenbeck semigroups in convex sets

Let X be a separable Hilbert space endowed with a non–degenerate centred Gaussian measure γ and let λ_1 be the maximum eigenvalue of the covariance operator associated with γ. The associated Cameron–Martin space is denoted by H. For a sufficiently regular convex function $U : X \to \mathbb{R}$ and a convex set $\Omega \subseteq X$, we set $\nu = \mathrm{e}^{-U}\gamma$ and we consider the semigroup $(T_\Omega(t))_{t\geq 0}$ generated by the self-adjoint operator defined via the quadratic form

$$(\psi, \varphi) \mapsto \int_\Omega \langle D_H\psi, D_H\varphi\rangle d\nu,$$

where ψ, φ belong to $D^{1,2}(\Omega, \nu)$, the Sobolev space defined as the domain of the closure of the gradient operator along H in $L^2(\Omega, \nu)$.

We prove pointwise gradient estimate for $(T_\Omega(t))_{t\geq 0}$. In particular, we show that

$$|(D_H T_\Omega(t)f)(x)|_H^p \ \leq \ \mathrm{e}^{-p\lambda_1^{-1}t}(T_\Omega(t)|D_H f|_H^p)(x), \qquad t > 0, \ \nu\text{-a.e., } x \in \Omega,$$

for any $p \in [1, +\infty)$ and f smooth enough and we deduce some relevant consequences of the previous estimate, such as logarithmic Sobolev inequality, Poincaré inequality and some improving summability properties of $(T_\Omega(t))_{t\geq 0}$. Finally we investigate on the asymptotic behaviour of $T_\Omega(t)$ as $t \to +\infty$.

The results have been obtained in collaboration with L. Angiuli and S. Ferrari.

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Wentzell boundary conditions in the research of Gisèle Ruiz Goldstein

In the last twenty years many results concerning Wentzell boundary conditions and their generalizations were obtained by Gisèle Ruiz Goldstein, also in collaboration with other coauthors.

In this framework a relevant role is played by her paper concerning derivation and physical interpretation of them, which is enlightening also for the applications. See G.R. Goldstein, Derivation and physical interpretation of general boundary conditions, *Adv. Diff. Eq.* **11** (2006), 457-480.

After a short overview on the above topics, we will focus on the most recent results.

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Elliptic operators with unbounded diffusion, drift and potential terms

We prove that the realization A_p in $L^p(\mathbb{R}^N)$, $1 < p < \infty$, of the elliptic operator

$$A = (1 + |x|^\alpha)\Delta + b|x|^{\alpha-1}\frac{x}{|x|} \cdot \nabla - c|x|^\beta$$

with domain $D(A_p) = \{u \in W^{2,p}(\mathbb{R}^N) \,|\, Au \in L^p(\mathbb{R}^N)\}$ generates a strongly continuous analytic semigroup $T(\cdot)$ provided that $\alpha > 2$, $\beta > \alpha - 2$ and any constants $b \in \mathbb{R}$ and $c > 0$.

This generalizes the recent results in [1] and in [2].

Moreover we give estimates of the heat kernel associated to the semigroup $T(\cdot)$.

Joint work with S.E. Boutiah, F. Gregorio and A. Rhandi.

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