UNIVERSITÀ DEGLI STUDI DI BARI ALDO MORO

DIPARTIMENTO DI MATEMATICA

# Seminari di Matematica

## A lagrangian approach for normalized solutions

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### Abstract

In this talk we consider the existence of positive solutions to the following nonlinear Schrödinger equations:

$$\begin{cases} -\Delta u + \mu u = g(u) \quad \text{in } \mathbf{R}^N, \\ \dfrac{1}{2}\displaystyle\int_{\mathbf{R}^N} u^2\,dx = m, \end{cases} \tag{$*$}$$

where $N \geq 2$, $g(s) \in C(\mathbf{R}, \mathbf{R})$, $m > 0$ are given and $(\mu, u) \in (0,\infty) \times H_r^1(\mathbf{R}^N)$ is unknown.

Usually this problem is approached through the constraint problem:

$$\mathcal{I}(u) = \frac{1}{2}\int_{\mathbf{R}^N} |\nabla u|^2\,dx - \int_{\mathbf{R}^N} G(u)\,dx : \mathcal{S} \to \mathbf{R},$$

where $G(s) = \int_0^s g(\tau), d\tau$ and $\mathcal{S} = \{u \in H_r^1(\mathbf{R}^N);\ \frac{1}{2}\int_{\mathbf{R}^N} u^2\,dx = m\}$. Here we introduce another approach using a lagrangian formulation of the problem $(*)$:

$$J_m(\mu, u) = \frac{1}{2}\int_{\mathbf{R}^N} |\nabla u|^2\,dx - \int_{\mathbf{R}^N} G(u)\,dx + \left(\frac{1}{2}\int_{\mathbf{R}^N} u^2\,dx - m\right) : (0,\infty) \times H_r^1(\mathbf{R}^N) \to \mathbf{R}\,.$$

Such an approach is introduced in [HT] for $L^2$-subcritical problems and [CGIT] for $L^2$-critical problems. Here we extend and apply it to $L^2$-critical problems.

This talk is based on joint works with S. Cingolani and M. Gallo.

References

[HT] J. Hirata and K. Tanaka, Adv. Nonlinear Stud., Vol. 9, Issue 2, 263–290 (2019).
[CGIT] S. Cingolani, M. Gallo, N. Ikoma, K. Tanaka, `arXiv:2410.23733v2` (2024).

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