



# *Seminari di Matematica*

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Ore 15:30 - Aula I

Dipartimento di Matematica

## **Orbital stability of ground states to Schrödinger equations with mass constraints**

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### **Abstract**

We discuss the existence of standing waves solutions to

$$i\partial_t \Phi - \Delta \Phi = F'(\Phi),$$

i.e.,  $\Phi(t, x) = e^{-i\lambda t} u(x)$ . Since the mass  $\int_{\mathbb{R}^N} |\Phi(t, x)|^2 dx$  and the energy  $\int_{\mathbb{R}^N} \frac{1}{2} |\nabla \Phi(t, x)|^2 - F(\Phi(t, x)) dx$  are conserved in time if  $F(\Phi) = F(|\Phi|)$ , such standing waves are sought as *normalised solutions*, i.e., solving

$$\begin{cases} -\Delta u + \lambda u = F'(u) \\ (\lambda, u) \in \mathbb{R} \times H^1(\mathbb{R}^N) \\ \int_{\mathbb{R}^N} |u|^2 dx = a^2 \end{cases}$$

for some prescribed  $a > 0$ , where  $\lambda$  is part of the unknown. We show that when the energy is bounded below over the set  $\{u \in H^1(\mathbb{R}^N) : \|u\|_{L^2(\mathbb{R}^N)} \leq a\}$  and its infimum is negative, the set of ground state solutions to the elliptic problem above is orbitally stable.