Equilibrium distributions in the Born-Infeld electrostatic theory

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Abstract

This talk reports on joint work with D. Bonheure (Univ. Libre de Bruxelles), P. D'Avenia (Politecnico di Bari), and A. Pomponio (Politecnico di Bari). Within the Born-Infeld electrostatic theory, the electrostatic potential generated by a probability measure ρ on the boundary $\partial\Omega$ of a domain $\Omega \subset \mathbb{R}^3$ is the unique minimizer ϕ_ρ of the Born-Infeld electrostatic action (with $b > 0$ a positive constant)

$$\mathcal{I}_\rho(\phi) = \int_{\mathbb{R}^3} \left(b^2 - b\sqrt{b^2 - |\nabla\phi|^2}\right) dx - \oint_{\partial\Omega} \phi d\rho.$$

Here ϕ runs through the set $\mathcal{X}$ of all $D^{1,2}(\mathbb{R}^3)$ functions with Lipschitz-constant less or equal b. For each electrostatic potential ϕ_ρ we can consider the Born-Infeld electrostatic energy $\mathcal{E}$ given by

$$\mathcal{E}(\phi_\rho) := -\mathcal{I}_\rho(\phi_\rho).$$

Among all possible charge distributions one can search for those distributions ρ^*, which create least-energy potentials. Such a minimizing distribution ρ^* (provided it exists) is called *equilibrium distribution*. Its corresponding potential ϕ_{ρ^*} is called an *equilibrium potential*. The main purpose of this talk is to provide the existence and the properties of the equilibrium distribution and the equilibrium potential. The four most important results are:

(i) equilibrium distributions exist;

(ii) the equilibrium potential ϕ_{ρ^*} is unique and takes a constant value λ^* in $\overline{\Omega}$;

(iii) if $\partial\Omega \in C^{2,\alpha}$ then also the equilibrium distribution ρ^* is unique and the equilibrium potential is a weak solution of the Euler-Lagrange equation associated with $\mathcal{I}_\rho$;

(iv) within the class of $C^{2,\alpha}$ domains, the ball is the unique member with equilibrium distribution being a constant multiple of the surface measure.

Similar results can be achieved for approximated electrostatic actions, where the action integrand $b^2 - b\sqrt{b^2 - |\xi|^2}$ (with ξ a placeholder for $\nabla\phi$) is replaced by its Taylor-polynomial.