

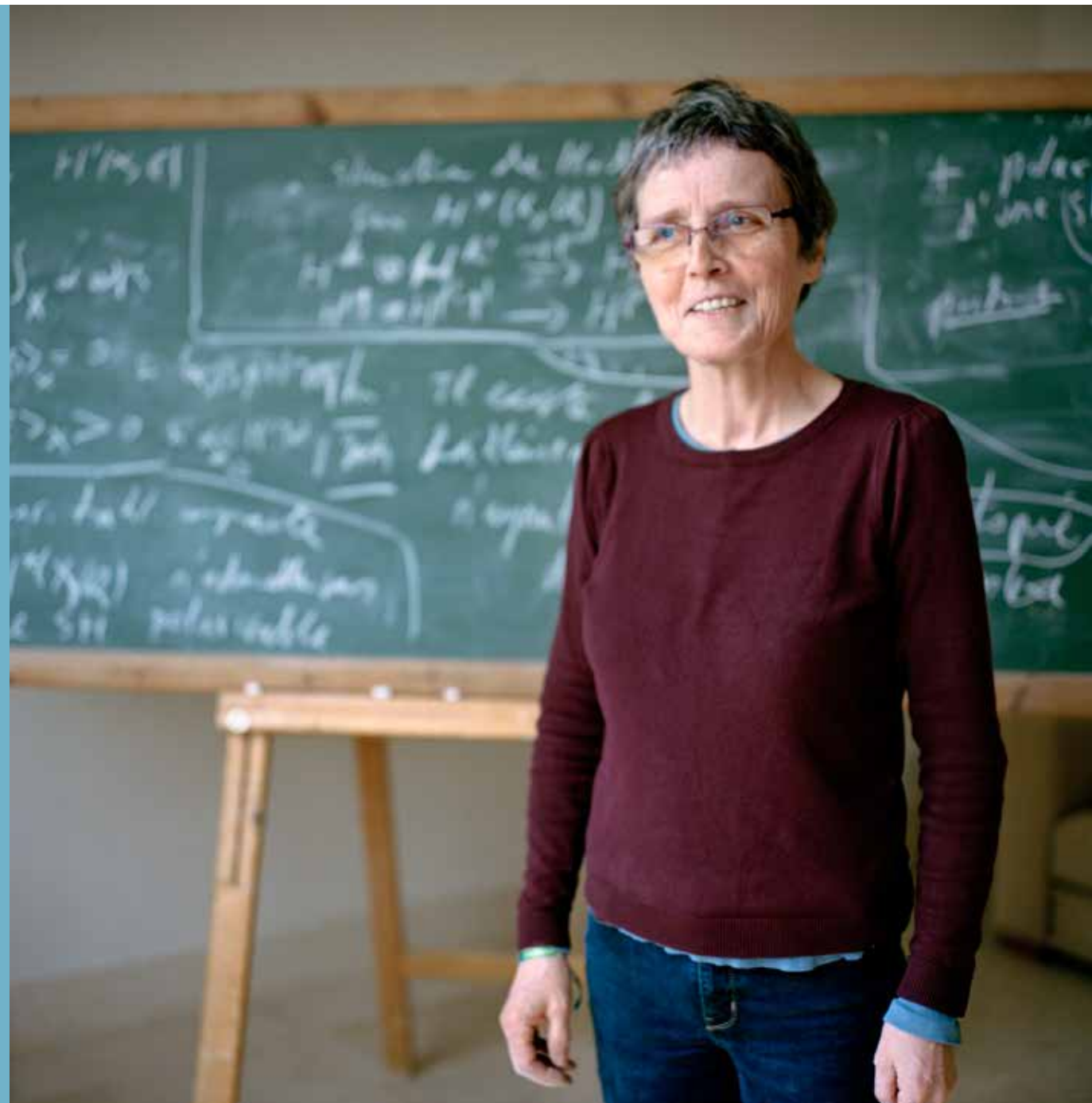
Claire Voisin

$$H^k(X_{an}, \mathbb{Q}) \otimes_{\mathbb{Q}} \mathbb{C} \simeq H^k(X_{an}, \mathbb{C}) \simeq H^k(X_{an}, \Omega_{X_{an}}^{\bullet}) \simeq H^k(X, \Omega_{X/K}^{\bullet}) \otimes_K \mathbb{C}$$

Claire Voisin obtained her PhD in 1986 under the supervision of Arnaud Beauville at the University of Orsay. She worked as a CNRS researcher at the University of Orsay, and later at the Mathematics Institute of Jussieu. From 2015 to 2020, she was Professor at the Collège de France. She was awarded at the Clay research award in 2008, the Heinz Hopf Prize in 2015, and the Shaw Prize in 2017.

Research topics:

- Algebraic geometry
- Hodge theory
- Algebraic cycles and motives





When and how did you decide to pursue mathematics?

It took me a while to realise that the mathematics I was doing would become my whole life. I started doing mathematics seriously when I was in “classes préparatoires” and threw myself wholeheartedly into it without any specific goal in mind.

I owe my introduction to mathematics in part to my father, who completed his studies at the École Polytechnique, and with whom I sometimes discussed mathematics while I was still in high school. I have in particular a strong memory of my first confrontation with a conceptual proof of Bezout’s theorem, from M. Condamine and P. Vissio’s algebra book which had been left by my older brother (who later became a senior researcher in physics). I loved that book and the proof of Bezout’s theorem which it proposed, based on the concept of an ideal. I must have been 13 or 14 years old, and defended in writing this proof in front of my father who knew a different, more classical proof.

My mother was passionate about literature and the fine arts. I come from an old-fashioned family, typical of the post-war period. We were twelve children, three boys and nine girls, five of whom were born within one year of each other



just after the war. My mother followed her father, a general, from garrison to garrison, and she had to interrupt her studies just after the baccalaureate, which she passed before the war broke out. She remained quite bitter about having had to give up her studies and it was only much later, in her fifties, that she resumed her studies by taking courses at the École du Louvre.

Mine was an old-fashioned family where my parents, in a highly sexist fashion, gave all their attention to their sons and didn’t push their daughters to sit the baccalaureate. Four of the nine daughters didn’t make it to the baccalaureate, despite their grandfather having been a general and their father being a Polytechnician², a situation which in other circumstances could have led to very different futures for my sisters. My father, a little bit crazy and obsessed with the idea of wanting to pass on an intellectual heritage, deemed that only his sons were worthy of it. My second sister for example interrupted her studies in ninth grade and became a laboratory assistant. She only passed her baccalaureate later on, after she was married, and she then managed to become an architect. I get on quite well with most of my siblings, although we all suffered from this toxic environment. In a large family, children are typically born in “bundles”. I came in a bundle of three, between



an older brother and a younger sister. Neither of them had been particularly “academic” children, so my father turned his attention to me. In that way I took advantage of a vacant spot, playing the role of a “substitute son”. My mother retained a preference for my brother, who was one year older than me and shared her interest in literature and in particular in the study of Latin and Greek texts. So much so that upon her death, about a year ago, she made him her sole legatee, disinheriting her other children!

In such a dysfunctional family, I used to take refuge in mathematics, which like Ariane’s thread helped me escape the family labyrinth. I took refuge in a somewhat ethereal world. I think I was extremely lucky that I was able to build myself as a person through mathematics. I am sometimes surprised that I was able to escape from such chaos, and that my own

family functions well with my five children getting on very well with each other. I’ve come to the conclusion that it’s not that difficult to have a functional family. I left my parents’ home at the age of 17, and have only seen them occasionally since then. Because of the difficult financial situation my father found himself in after he was made redundant in the 70s, I benefitted from a national scholarship for my studies. This gave me a lot of autonomy very early on, both financially and in my educational choices. I enjoyed philosophy a lot thanks to a wonderful teacher I had in my final year of high school, and the depth of the subject attracted me for some time after. But when I discovered general topology and group theory, I realised that mathematics was made up of similarly deep structures, and I found them absolutely fascinating.

That being said, I still love poetry. I learned many poems by heart until I was 17, but found it more difficult from my mid-twenties.

Were you encouraged by your family, friends or other people around you?

My mathematics teacher in Math Sup at Louis le Grand, Denis Monasse, gave a course³ which I found absolutely superb. It was astounding! For the first time, I started to really work hard in my mathematics classes. Before the “classes préparatoires”, I had never taken any notes in class. That year I started to take careful and structured notes in a notebook. In my Maîtrise and DEA, I took wonderful courses such as those of Joseph Le Potier in algebraic topology and of Roger Godement in algebra. The concept of the fundamental group, the theory of cohomology, and later Riemannian



geometry delighted me. When I was taking these courses, I felt like I was in my element. Just like the character from “The Drunken Boat” who lets himself drift away and, upon reaching the open sea, becomes one with the water: “And from then on I bathed in the Poem of the Sea”. I really enjoyed seeing entire theories unfold before my eyes, and it gave me the impression that something was being built inside my mind. It’s a pity that what was asked of us at the time was mainly just routine.

Did you encounter any obstacles on your way to becoming a mathematician?

I completed my thesis in one year at the age of 24. It was a very good thesis and for a while after I defended it I felt that what I was doing was not as good as the results from my thesis. As I completed my PhD thesis very quickly, I did not have enough time to mature

mathematically. I lacked exposure to mathematical culture and I hadn’t attended enough seminars. I was very much on my own, and it took me a while to find my bearings. It was two years after my thesis before I started again to produce what I considered to be good maths. For me, mathematics is an extremely demanding discipline, one which does not tolerate mediocrity. You have to give the best of yourself, to be original. In mathematics there needs to be surprised and wonder, it’s not enough to just do the obvious.

I once wrote an article which was not particularly original. It was related to mirror symmetry, and in it I gave a rigorous proof of a physicist’s calculation. The article disappointed me and the experience drove me to turn to other problems. At the end of a research project, I am always happy with the idea of being able to embark on a

new project. That being said, I do often feel somewhat rejected by mathematics. From one research project to the next, I have a tendency to think that because I’m not coming up with any ideas, I’m finished. This feeling of distress is something I don’t really have any control over, and yet, as my son who is also a mathematician tells me, I have always managed to get myself going again. To which I add: “up until now...”. Very often I work alone. But, since I have been in an ERC with Debarre, Huybrechts⁴ and Macri, I have discovered collaborative work and I really appreciate the collective synergy that comes out of it. However I also like the more introspective side of mathematics, that feeling of wandering alone in the desert. With collaborative work, things are very different, we discuss things together, it’s a way of doing mathematics that is more controlled, less stressful. You

can't be both the manager and the employee at the same time, it feels like a small company. I find it hard to discuss maths in the very early stages of a project, but I absolutely love communicating mathematics once things are clear to me. Showing my true self in all its messiness does not come easily to me. I couldn't do research with someone sitting next to me, it is too intimate.

In hindsight, are you happy to have chosen mathematics or do you have any regrets? For you, what are the joys of mathematics, what are the difficulties?

Mathematics is a very big part of my life, it is through mathematics that I have gained self-confidence. It has allowed me to become a strong person. My personality, my psyche, my social and intellectual beings have all been built around mathematics. It is the only way I have of holding myself together. So I don't regret it at all. Motherhood has not hindered my mathematical career. My husband, who is also a mathematician, and I shared the load of taking care of our five children. We were well organised and made the most of the free time we each had. My pregnancies were even periods of renewal for me. I didn't do much during the pregnancies. I could give a graduate

course or write a book, but I didn't prove any significant results. I was focused on my unborn child and mathematics did not interest me so much, so I was less involved with it. After each birth, I would dive back into mathematics with great enthusiasm. I would start working again as soon as I could and turn to new projects. Perhaps I had saved up intellectual energy during the pregnancies and grown in confidence. In hindsight though, I can see that my best maths was done before my first child was born and five years after my last child was born. I do however regret the period (2016–2020) during which I was a professor at the Collège de France⁵. It was not a good period in my life. I had been approached by Alain Connes and Jean-Christophe Yoccoz and was taking over from Don Zagier, who could only keep his position at the Collège de France if he gave up his position at the Max Planck Institute in Bonn, something which he was not prepared to do. During this period at the Collège de France, I was in the media relatively frequently and had to answer questions from journalists. This led me to reflect on why I do mathematics and I firmly believe that we carry out a fundamental intellectual activity. We develop the ability to construct beautiful objects, such as the

notion of a group. We are the guardians of treasures, akin to a cultural heritage. Mathematics is an essential activity that begins with language, concentration and the requirement to prove things rigorously. It is a truly human process. At the Collège de France, I couldn't stand the professors' "closed circle". I found the atmosphere there very artificial and intellectually hollow. Interacting with all these people in the final stages of their careers, full of conceit, made me feel uncomfortable. I don't belong to that world. It was not a question of not feeling legitimate in this position at the Collège, a position I think I deserved as much as my other colleagues. As a mathematician, you can't be sure of yourself because you are always aware that you are nothing in comparison to the field that you work in, and to the history of your discipline. I don't put myself on a pedestal, doing mathematics feels more like being in quicksand. Moreover, teaching at the Collège de France caused me anxiety: giving a course consisting of only a small number of lectures (15 hours) to the general public is a perilous exercise which was highly stressful for me and hindered my research. I have never been as exhausted as I was after giving a lecture at the Collège de France.



It was the horizontal structure of the world of the Collège de France that bothered me. In mathematics we need a vertical structure that brings together different generations, mathematicians at different stages in their scientific development. We need seminars that offer a tremendous potential for communication between generations of scientists. The horizontal structure of the Collège de France hinders intergenerational exchanges. So I returned to the CNRS (Centre National de Recherche Scientifique), not without some administrative hassle, after tiptoeing out of the Collège de France so as not to make any waves.

What would you recommend to a young mathematician who came knocking on your office door to ask for advice about her scientific career?

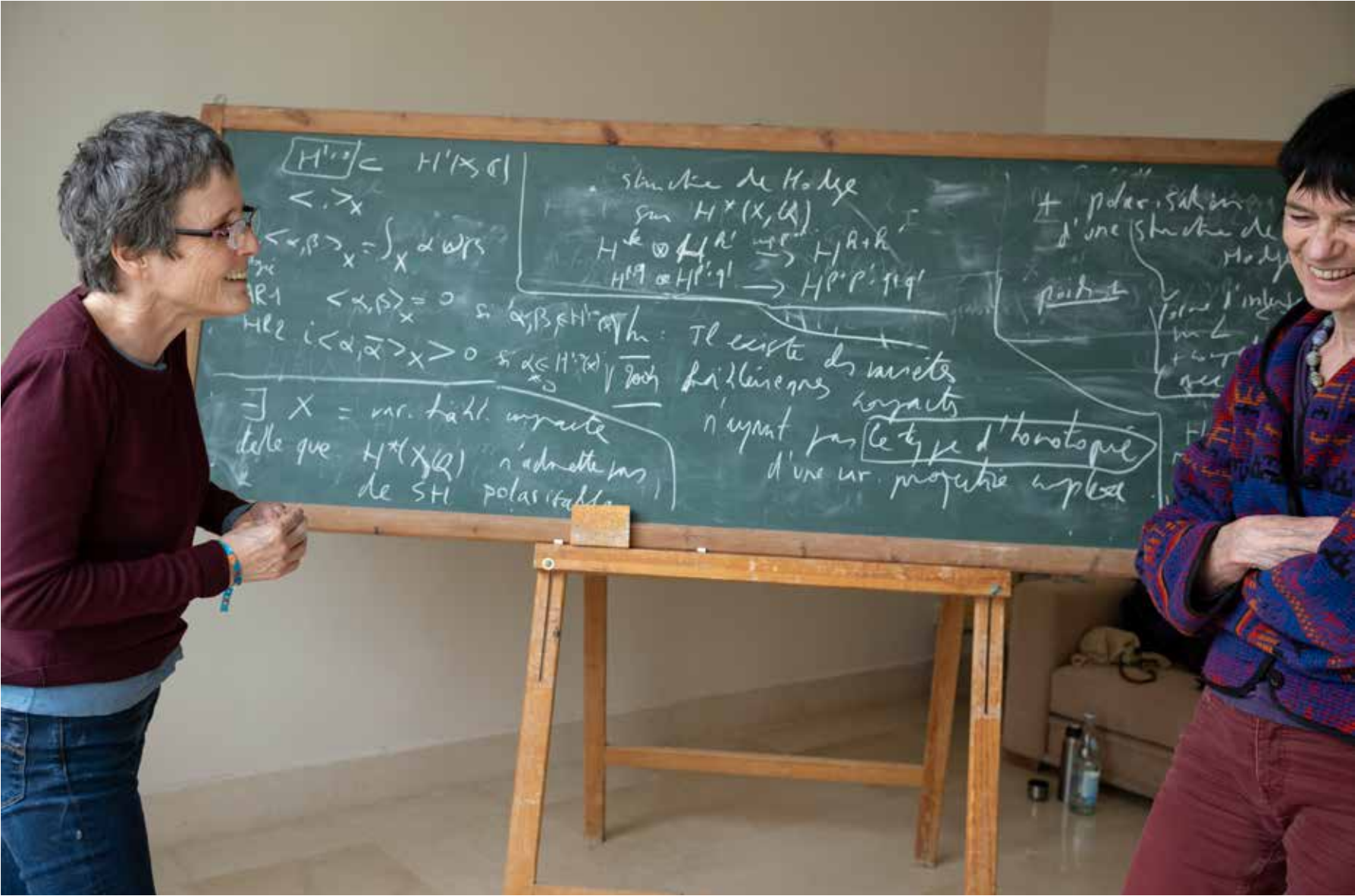
I would be delighted to meet a woman who would like to do maths, but I would be wary of giving advice to anyone. I have a strong sense that each person is their own self and needs to make do with who they are. We have to be authentic, and embrace our originality. Being ourselves is the only thing we have. We definitely should not try to copy others, which is why I don't agree with the expression "role model". You have to look inside yourself.

One of my nephews, François Espéret, is a writer and man of letters, about 45 years old. He is a real character. For example he left the École Normale Supérieure⁶, where he was taking a history course, to become a policeman. He was then recruited by the Council of Paris, and later took up a position at Gallimard where he contributed to editing the recently published book “Récoltes et Semailles”⁷. In an interview with Léa Salamé⁸ who was commenting on his originality, he replied: “I am my own benchmark”. That’s exactly what I feel. I would say to the young woman who knocks on my door that it is important to embrace your personality, whether you are a mathematician or not. If we want to build our personality within the world of mathematics, it is important to know what is within our own selves.

Could you describe your field in a few words that are intelligible to a layman?

When I was doing my Master’s degree and the DEA⁹, I was torn between symplectic, Riemannian and algebraic geometry. In the end, it was the latter that won: if you like differential geometry and you’re not so good at analysis, you do algebraic geometry. I have never had to prove that a function is continuous, since in algebraic geometry functions are just

polynomials. Many structures in algebraic geometry rely on concepts from differential geometry, like the notions of tangent and cotangent bundles for example. Algebraic geometry brings together my favourite areas. It brings in topology, which I really enjoyed during my studies, through the notion of cohomology, but also Riemannian geometry, symplectic geometry and analytic geometry. It even involves commutative algebra, which appears when you study the set of all polynomials that vanish on a variety. The point of view of the Grothendieck school is to make algebraic geometry self-sufficient by importing beautiful concepts from related disciplines, such as étale cohomology being the algebro-geometric version of Rham cohomology, or the GAGA comparison theorems relating analytic and algebraic geometry. Being self-sufficient whilst at the same time engaging in some back and forth with the outside world, I think that’s the way I like to function.



1 Elite school for future engineers after high school.
2 Student at the École Polytechnique, see footnote 1.
3 Mathématiques Supérieures is the first level at preparatory school, which prepares for the entrance examinations

for elite high schools such as the École Normale Supérieure and the École Polytechnique, see footnote 2. The Lycée Louis le Grand is one of the most renowned preparatory schools in France.
4 A research unit funded by a research grant.

5 An interdisciplinary institution in Paris dating back to the 16th century and which encompasses both teaching and research.
6 An elite high school for future high school teachers or university professors.
7 A two-volume book which encom-

passes many of the writings of the famous mathematician Alexander Grothendieck.
8 A famous journalist on France Inter.
9 Diplôme d’études approfondies.