

Betül Tanbay

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Betül Tanbay was born in Istanbul and raised in Ankara. After her undergraduate studies at the Université Louis Pasteur in Strasbourg, France, she went to the United States where she gained a Master of Science in Mathematics and then a PhD in Mathematics at the University of California, Berkeley (1989). Betül is a full-time professor at the Boğaziçi University, Istanbul. She is the founding director of the Istanbul Center for Mathematical Sciences (IMBM), opened in 2006, where she conducted a leading doctoral network project by TÜBİTAK between 2008 and 2012. She has worked as an executive committee member, including as president, at the Turkish Mathematical Society. Betül was elected Vice-President of the European Mathematical Society for the 2019 – 2022 period, and after serving as a member of the Committee for Women in Mathematics at the International Mathematical Union she is now part of the Governing Board of the International Day of Mathematics.

- Research topics:
- Operator algebras
 - Set theory
 - Functional Analysis



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How and when did you choose to do mathematics?

The father of my two children, whom I've known since I was 14, claims that when we met I told him I would study mathematics. This is the only reference I have regarding this early decision. I have indeed always been curious about mathematics, and wanted to learn and understand more about numbers, vectors, structures, out of sheer curiosity. I also liked languages, and mathematics is a language with its syntax and semantics. When the syntax is built so well, there is a lot of space for the semantics to flourish. I find it interesting how different languages allow you to reach different depths. I was born in Istanbul, but we moved to Ankara in my childhood and I went to a French-speaking kindergarten very close to our house, which was part of the *École Française* where I did my primary school. There I was able to learn English and German as well. So I learned English in a French school and I am puzzled to see how difficult French people of my generation find it to speak English. Later on, in Berkeley, without telling my PhD advisors I also learned to speak Spanish.

As many youngsters of my generation who went to French schools, I was expected to go to France to study. It was the end



of the 70s, before the military coup in 1980, a time when tens of university students were being killed every day. So whenever they could, families tried to send their children abroad to study. At 17 years of age I went to Paris for the last year of high school, to the famous Lycée Janson de Sailly. My husband, who was my boyfriend at the time, had gone to a German high school in Istanbul and was studying in Karlsruhe. To be closer to him I moved to Strasbourg to the Université Louis Pasteur, which is now Strasbourg University. Upon graduating, I thought that if I studied the philosophy of mathematics, I would understand more about mathematics. That is how I decided to apply to Berkeley where there was a group in Logic and the Methodology of Science, founded by Alfred Tarski. At the beginning I felt totally lost: everything was so different compared to the Logic classes I had taken during my Bachelor degree in France. The Logic group was on the seventh floor of Evans Hall, where the Department of Mathematics occupied the top four floors. Its members were among the most brilliant researchers of their generation, but more than one of the members had spent time in a mental hospital. Too much abstraction can be damaging! There is a thin line between intelligence and madness.

Were you then encouraged by your family, friends or other people around you?

I can definitely say everyone was supportive. I come from a protected neighborhood of Istanbul and I had a happy childhood and youth. I was not pushed to make choices. I do not come from a family of academics: my father was a businessman, my mother did not work. However, my parents and grand-parents spoke foreign languages, one of my grand-fathers spoke seven. This has always been a sign of education in our country. My father told me “if you go abroad, travel and learn a language every year”, so there was no academic pressure. I am the only one of the three children my parents had chosen to study mathematics and to have opted for an academic career. My sister is a dancer, the best choreographer in Turkey; my brother studied computer engineering, and then went into business.

Did you come across obstacles in pursuing your career as a mathematician?

The first time I discovered the word “role model” was as a PhD student, in the US. I was asked who my role model was, and I didn’t understand what this meant. I later realized that in the US many girls were told that math was not for them. So

it was rather late that I discovered women could experience discrimination in math. I can’t think of a single anecdote involving discrimination during my studies in Turkey or in France, even though most of the professors in Strasbourg were men. I do recall one professor making a negative remark when I told him that I would be going to Berkeley for my PhD. He was a geometer and said that I was not dedicated enough. Maybe he was right as far as geometry is concerned. I am of the generation who was taught “les maths modernes”: we learnt about vector spaces in high-school but had never seen a triangle, which meant that when we started geometry at university we were all lost! As a PhD student I had the fortune and the misfortune of hitting a very hard problem, the *Kadison-Singer problem*, posed in 1959. My adviser was Robert Solovay, the famous set-theorist, a great mathematician and a wonderful person who knew a lot of analysis, but his expertise was not in the area of the problem I was considering. He just kept on saying it was solvable. In 1979, Joel Anderson had brought down the Kadison-Singer problem to what is since then called the Anderson’s paving conjecture. Since I was unaware of his results, I was rediscovering small corollaries of his work without re-

alising. I never thought I would cry while reading an article, but this happened to me on a plane when I discovered that the result I had shown, thinking it was original, had been proved some ten years earlier. There was also some joy in my tears of despair, as a prominent mathematician had proven my results and published them. But this meant that my thesis would take longer! In the long-run, working on such a hard problem had a positive influence on my scientific life. I had to learn a lot since the problem, which long remained an open conjecture (it was solved in 2013), evolved over time. It guided my research for most of my career, so for me the initial difficulty turned into a blessing. I like to turn obstacles into opportunities and I have a tendency to think I am lucky. I probably inherited some of my mother’s extremely positive mindset. On the other hand, speaking of obstacles, the political situation in Turkey plays a role in the lives of academics, and it played a role in mine, even though not to the extent it has for many people of my generation. Many of my classmates went to jail and endured torture in the early eighties because they had been members of left-wing movements. Changes in the university system after the 1980 military coup had a durable negative impact on research. A



smaller but recent incident occurred during the coup attempt in 2016, when all academics were banned from leaving the country, which prevented me from attending the EMS meeting in Berlin where I was one of the candidates for the election of the Executive Committee. In the last three years, hundreds of academics have lost their jobs for having signed a peace petition. I am sure that one day this period will be considered as a detrimental period for academic life. It has caused a very unfortunate brain drain.

In retrospect, are you happy to have chosen mathematics or do you have some regrets?

What are for you the joys of mathematics? What are for you the hardships of mathematics?

I have never regretted choosing mathematics even a second of my life. I don’t recall a single moment of regret. I felt, and still feel, that it is a difficult, serious and very beautiful subject, which sometimes can be draining. But isn’t life also like that? Mathematics is hard in the sense that it does not allow laziness or sloppiness, it is demanding and requires dedication. Mathematics fills life up, it gives you strength. When you feel pessimistic about the world, mathematics still holds truth. It is not affected by climate change, by quarrels, by

politics or by economics. It is not so much a matter of being good at math or not, it is a matter of loving it. Also, it becomes more beautiful when you can share the curiosity you have and the work you do.

I was quite good at high school, but after that I never felt special. The fact that I was at Berkeley showed that things were going rather well, and my references were decent, but I personally felt that I was average. I also prefer to be in places where I feel average. Often I am reluctant to call myself a mathematician, as I don't think I devote myself enough to math. Although I have been taught constantly these past thirty years, I have not done research continuously. Today I don't consider myself to be an active researcher. I know enough to know it is not enough. I do not sleep and wake up with a math problem in mind ... I certainly like to teach, and the more I teach the more I understand. Something I am really interested in is repetition. Artists, musicians and dancers rehearse and repeat. In French the word "répéter" means to repeat and also to rehearse. Research also means a lot of repetition, you repeat and you repeat again and at some point something new comes out of this repetition. The feeling of stagnation, that you are in a vicious circle, is preparation for creation ... How does it happen that at night-time

nothing seemed to work, yet as you wake up things are suddenly crystal clear? I find this kind of phenomenon very exciting. A process which might disappear with computers. I am not so sure that this artificial intelligence will bring only prosperity, I am somewhat wary of the impact it will have on humankind. Mathematics is extremely aesthetic and very carefully built. For me it is human creation, which is divine in nature. It opens skies but it also reminds you of your limits. Our ambition for this amazing human construction was impacted at the beginning of the 20th century by the failure of the ambitious Russell project. Gödel's incompleteness theorem forced us to be more humble by showing us that we can't control nature as much as we claim. Even in mathematics, which we create ourselves, we are not omnipotent. Mathematics indeed makes you humble. I had the luck of meeting many mathematicians around the world, and to see that being a great mathematician comes with quite a bit of modesty. A lack of modesty raises doubts in me. Creating may have a stronger effect on the male ego, women are biologically used to the idea of creating, so maybe our ego doesn't inflate as much since we have been creating for as long as we exist!

What would you recommend to a young woman in your country wanting to start a career in mathematics?

In Turkey the central university entrance exam is tough, so people don't choose what they want but rather what they get. Hence having studied undergraduate mathematics is not necessarily the result of a will to do so. Also, as a result of the marketing world we living, people are not honest enough towards themselves and towards others, so I would try to help the student decide what she is good at, where she is not good enough at, and to make an honest evaluation of herself. I would try to check for consistency between her intention and her achievements, without expecting her to have obtained excellent results. From undergraduate to graduate school, passion is essential, you can always catch up even if your past marks were not so good. So rather than looking at her marks, I would look for some serious intent, some sign of passion, and if I found that then I would urge her to try. You can easily tell whether or not there is a serious intent. I think I am an experienced teacher. I don't expect my students to become mathematicians, but I want them to acquire a certain amount of respect towards their work. The first time I met



the piano teacher with whom I wanted to study, Jeanne Stark-lochmans, I warned her that I only played the piano “for fun” to which she replied that “one can be serious and have fun”. I have learnt a lot from her. Today’s young generation communicates by means that do not require much thoroughness, they do not care to read full statements. In class, I often have the theorem we are going to prove read out loud by a student. By doing maths, they acquire good reading, writing and listening skills. Although mathematicians are not the most sociable people and don’t usually give good talks, they can convey these skills to students. Also, developing analytical thinking is essential for every human being.

Could you describe your favourite personal achievement(s) in your career as a mathematician?

It is a pleasure to be cited by the people who made the biggest contributions to my subject. Some even added adjectives like “elegant” while citing my work. I had always said that I would be happy if the problem I was working on was solved during my lifetime. And it was. That was satisfying enough for me. Publishing makes mathematics very competitive, and you probably need to be more competitive than

I am. I don’t like competition, instead I like sharing ideas, understanding more, learning and finding out more. I play tennis, but don’t have the urge to win the game. Yet I get upset if I feel I could have done better than I actually did.

Another favourite achievement of mine is what I feel when I read student evaluations. They make me think I should continue.

Could you describe your field of research to a non-expert?

The problem I work on is about what one can know about a big set, given a certain amount of information about a smaller set contained in it. It is about how much the information regarding the small set extends to the bigger set. It is called an *extension problem*, and such problems occur in many areas of mathematics. There are a priori different ways of “extending while preserving some properties”. My precise question is whether this extension is unique, namely whether there is only one way or several ways of doing so. This specific extension problem, which has to do with functions, called states, was raised by Dirac for physical reasons. It turns out that it is equivalent to a much simpler problem, involving matrices, like the 9×9 matrices in the Sudoku game. A problem stemming from physics be-

comes a mathematical problem concerning functions, to finally reduce to a paving problem about matrices, which are simpler objects than general functions. This transformation process from the initial question posed by Dirac in the 40’s to the solution of the paving problem of matrices in 2013 took several decades. And the solution did not use any of all this, but rather polynomials! I think this kind of translation of problems into other seemingly unrelated problems can impress the non-expert too.

Would you like to add something?

I think one of the big problems with mathematics is secondary education. Mathematics should be seen as a game by children, how can it become a nightmare? You really have to work hard to turn such a beautiful subject into a nightmare. Learning to write and read does turn into a nightmare, so why does math? The fact that it gets harder as you go along cannot be the only explanation. Mathematics education alienates people; you know that music is beautiful but you don’t know that mathematics is beautiful. It might be hard to play a Beethoven sonata, but you can enjoy playing an instrument without having to reach that level. If you had parents warning you that music is difficult but very important, then you

would end up hating music. Mathematics is more accessible than people think. I sometimes teach the fundamental theorem of calculus to people who have left school forty years ago, and they love it, regretting that they couldn’t learn it like this before. I compare mathematics to art a lot, to languages as well, and I also like to compare

mathematics to law. Although people trained in law like to create a language which puts it out of reach for most people, it is fascinating to think about syntax and semantics in law. Though maybe in mathematics we are more concerned with being correct, whereas in law, the priority is being right.

