UNIVERSITÀ DEGLI STUDI DI BARI ALDO MORO

CONSIGLIO INTERCLASSE IN MATEMATICA

COURSE OF STUDY	**TWO-YEAR MASTER OF SCIENCE PROGRAMME IN MATHEMATICS**
ACADEMIC YEAR	**2023-2024**
ACADEMIC SUBJECT	**ADVANCED COURSE IN MATHEMATICAL ANALYSIS 1**

General information	
Programme year	Second
Term	First semester (September 25, 2023 – December 22, 2023)
European Credit Transfer and Accumulation System credits (ECTS)	7
SSD	MAT/05 – Mathematical Analysis
Language	Italian
Mode of attendance	Not mandatory

Lecturers	
Name and surname	Giusi Vaira
E-mail	giusi.vaira@uniba.it
Telephone	+39 080 544 2706
Department and office	Department of Mathematics, room 16 fourth floor
Virtual meeting room	Microsoft Teams Profile: giusi.vaira@uniba.it
Web page	https://www.dm.uniba.it/it/members/vaira
Office hours	By appointment via e-mail

Work schedule				
	Total	Lectures	Hands-on learning	Self-study
Hours	175	56		119
ECTS credits	7	7		

Learning objectives

	To acquire knowledge and techniques of modern mathematical analysis, especially compact and locally compact topological spaces and continuous function spaces on them, compactness criteria in continuous function spaces, density theorem, Radon measure on locally compact spaces, Hausdorff measure and self-similar sets and basic notion of calculus of variations, especially for what concern finite perimeter sets and isoperimetric problems.

Course prerequisites	
	Mathematical knowledge which is usually acquired with a degree of L-35 class. Especially: classical analysis of one and several variables, metric spaces and Banach spaces, elements of general topology, abstract measure theory and integration.

Syllabus	
Course contents	**COMPACT AND LOCALLY COMPACT TOPOLOGICAL SPACES** Topological spaces. Compact and locally compact topological spaces. Alexandrov compactification. Urysohn theorem. The theorem on the finite partition of unity. Locally compact spaces countable at infinity. Locally compact spaces with countable bases. Separable topological spaces. **CONTINUOUS FUNCTION SPACES** The Banach space $C(X)$ of all (real-valued) continuous functions on a compact space X. The linear space of all continuous functions with compact support on a locally compact Hausdorff space. Continuous functions converging at infinity on a locally compact space. The Banach spaces $C_0(X)$ and $C_*(X)$ of all continuous functions vanishing at infinity, resp. converging at infinity, on a locally compact space X. **COMPACTNESS THEOREMS** Tychnoff theorem. Equicontinuous subsets of mapping. Equicontinuous subsets of linear mappings. Examples and properties. Uniformly convergent sequences of mapping and equicontinuity. The Ascoli-Arzelà theorem. Application to the study of integral operators. Compact maps. The Banach theorem on the weak* compactness of the closed balls of a separable Banach space.

	DENSITY THEOREMS Stone-Weierstrass type theorems for lattice subspaces and for sub-algebras of $C(X,\mathbb{R})$ and $C(X,\mathbb{C})$, X compact space. Stone-Weierstrass theorems in $C_0(X,\mathbb{R})$ and $C_0(X,\mathbb{C})$, X locally compact. Applications. **FIXED POINT THEOREMS** Introduction to fixed point theorems. Banach fixed point theorem. Brouwer fixed point theorem. Compact mappings between normed spaces. The Schauder fixed point theorem. Some applications to integral equations and to ordinary differential equations. The Leray-Schauder principle and a priori estimates. Applications to partial differential equations and to game theory. **POSITIVE LINEAR OPERATORS AND POSITIVE LINEAR FORMS ON CONTINUOUS FUNCTION SPACES; RADON MEASURES** Positive linear forms and positive linear operators on continuous function spaces. Positivity and continuity. Radon measures on locally compact spaces. Radon measures with finite support. Regularity and approximation theorems. The dual space of $C_0(X,\mathbb{R})$. **INTRODUCTION TO CALCULUS OF VARIATIONS** Direct methods of calculus of variations. Classical functional: Eulero-Lagrange equations, Du Bois-Reymon equation. Convessity methods. Fermat principle for geometric optics. Brachistocrone problem. Functional of the gradient. Functionals on Sobolev spaces: convessity and lower semicontinuity in $W^{1,p}$ Existence of minimum in $W^{1,p}$. Examples. Functions of bounded variations and properties. Finite perimeter sets. Properties of perimeter function. Isoperimetric inequality. Isoperimetric problems as variational problems when the minimum of the functional is achieved with symmetry properties. Rearrangments, symmetrizations and application to variational problems. Symmetrizations and partial differential equations.
Reference books	1. H. BAUER, Measure and Integration Theory, De Gruyter Series Studies in Mathematics, 26, De Gruyter & Co. Berlin, New York, 2001 2. G. CHOQUET, Lectures on Analysis, vol. I, W. A. Benjamin Inc., New York, 1969 3. L.C. EVANS, Par5al Differen5al Equa5ons, AMS, Providence, 1998 4. G. B. FOLLAND, Real analysis, J. Wiley & Sons Inc., New York, 1999 5. M. GIAQUINTA, S. Hildebrandt, Calculus of varia5ons I, Springer, 2006 6. W.RUDIN, Real and complex analysis, McGraw-Hill Inc., New York, 1987
Additional course materials	
Repository	Lecture notes and additional notes on the calculus of variations.

Expected learning outcomes	
Knowledge and understanding	Acquiring fundamental concepts and results in the setting of functional spaces and operator theory. Acquiring main tools and proof techniques.
Applying knowledge and understanding	The acquired theoretical knowledge finds many applications in several aspects of mathematics, including partial differential equations and related models.
Soft skills	*Making judgements*: Ability to analyze the consistency of the logical arguments used in a proof, the problem-solving skills and the ability to choose suitable mathematical tools consistent with the theoretical knowledge.
	Communication skills: Acquiring the mathematical language and the formalism necessary to read and understand textbooks, to explain the acquired knowledge, and to describe, analyze and solve problems
	Learning skills: Acquiring suitable learning methods, supported by consultation of texts and by solving exercises and problems related to the contents of the course.

Teaching methods	
	Lectures. Lessons will be held face-to-face.

Assessment	
Assessment methods	The final exam is based on an oral test in which it is possible to verify the theoretical and applied knowledge of the course.

Evaluation criteria	• *Knowledge and understanding*: Acquiring fundamental concepts and results in the setting of modern analysis. Acquiring main tools and proof techniques. • *Applying knowledge and understanding*: The acquired theoretical knowledge finds many applications in several aspects of mathematics, including partial differential equations and related models. • *Making judgement*: Ability to analyze the consistency of the logical arguments used in a proof, the problem-solving skills and the ability to choose suitable mathematical tools consistent with the theoretical knowledge. • *Communication skills*: Acquiring the mathematical language and the formalism necessary to read and understand textbooks, to explain the acquired knowledge, and to describe, analyze and solve problems. • *Learning skills*: Acquiring suitable learning methods, supported by consultation of texts and by solving exercises and problems related to the contents of the course.
Grading policy	During the oral exam the student must show mastery of the language, methodological accuracy an having acquired the fundamental notions and concepts of the course. To achieve a high mark, the student have developed independent judgement, skills and expository clarity. The exam is sufficient with 18/30 mark.

Further information	
	Attendance at lessons is strongly recommended.