UNIVERSITÀ DEGLI STUDI DI BARI ALDO MORO

CONSIGLIO INTERCLASSE IN MATEMATICA

COURSE OF STUDY	**THREE-YEAR BACHELOR PROGRAMME IN MATHEMATICS**
ACADEMIC YEAR	**2023-2024**
ACADEMIC SUBJECT	**FUNCTIONAL ANALYSIS**

General information	
Programme year	Third
Term	Second semester (February 26, 2024 – May 31, 2024)
European Credit Transfer and Accumulation System credits (ECTS)	7
SSD	MAT/05 – Mathematical Analysis
Language	Italian
Mode of attendance	Not mandatory

Lecturers		
Name and surname	Giusi Vaira (instructor of record)	Marcello D'Abbicco
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Office hours	By appointment via e-mail	By appointment via e-mail

Work schedule				
	Total	Lectures	Hands-on learning (recitations)	Self-study
Hours	175	56		119
ECTS credits	7			

Learning objectives

	Acquiring language and basic tools concerning functional spaces, representation theorems, operator theory with applications to some classes of partial differential equations.

Course prerequisites	
	Mathematical knowledge which usually is required during the first two years of a degree ofL-35 class. Especially: classical analysis of one and several variables, normed spaces, general topology, linear algebra.

Syllabus	
Course contents	**1. NORMED SPACES** Normed spaces, convex sets, examples (L^p, ℓ^p, $\mathfrak{c}_0$, $\mathfrak{c}_c$, Hölder functions). Weak derivative and Sobolev spaces. Norm of bounded linear operators. Finite-dimensional normed spaces. Riesz lemma and Riesz theorem on the compactness of the unit ball. Space of bounded linear operators: completeness and characterization of continuity. **2. HANH-BANACH THEOREMS** Analytical and geometrical forms of Hanh-Banach theorem. Dual space and duality map. Minkowski functional. Bidual space and reflexive spaces. Closed subspaces of reflexive spaces. Reflexivity and separability of the dual. **3. WEAK AND WEAK * TOPOLOGIES** Initial and weak topologies. Weak convergence. Weak closure, and convexity. Weak* topology and convergence. Banach-Alaoglu-Boubaki Theorem. Helly lemma and Goldstine lemma. Kakutani theorem. Existence of minimizers for convex and lower semicontinuous functions in reflexive spaces. Separability and metrizability of the unit ball in the dual w.r.t. the weak * topology. Separability of the dual and metrizability of the unit ball w.r.t. the weak topology (with no proof). Milman-Pettis theorem. **4. L^PSPACES** Clarkson inequality and uniform convexity of L^p spaces (the proof is only given for $p \in [2,\infty)$). Reflexivity of L^p spaces. Properties of L^1 and L^∞. Dual space of $\mathfrak{c}_0$. Schur property for ℓ^1. **5. CONTINUOUS AND LINEAR OPERATORS** Neumann series. Banach-Steinhaus Theorem (the uniform boundedness principle). The effects of Banach-Steinhaus Theorem. The Open Mapping Theorem and the Closed Graph Theorem. Unbounded linear operator. Closed operators. Adjoint operators and properties. A characterization of operators with closed range.

	Operators with finite rank. Representation theorem and properties. The approximation of an operator and properties. The resolvent set, spectrum of and operator. Properties of the spectrum of a continuous and linear operator and of its adjoint. **6. COMPACT OPERATORS** Compact operators and properties. The approximation problem. Schauder Theorem. The spectrum of a compact operator. Fredholm operators. Fredholm Alternative Theorem. Compact embedding Theorems in Sobolev spaces. Eigenvalues and eigenfunctions of the Laplace operator in bounded domain with Dirichlet boundary condition and with Neumann boundary condition. **7. OPERATORS IN HILBERT SPACES** Orthornormal basis in separable Hilbert spaces. Hilbert-Schmidt operators and their representation. Hilbert-Schmidt operators as compact operators. Bounded and self-adjoint operators, monotone operators, idempotent operators and normal operators. Characterization of a self-adjoint and idempotent operator's. Characterization of a normal operator. Unbounded symmetric operators, self-adjoint and maximal monotone operators. Properties of a self-adjoint of the spectrum of a monotone and self-adjoint operator. Hilbert basis of eigenvectors of compact and self-adjoint operator. Spectral decomposition Theorem. Resolvent operator and Yosida approximation. Solution of evolution problems. Cauchy, Lipschitz, Picard Theorem. Hille-Yosida Theorem in Hilbert spaces. Applicational to evolution partial differential equations: heat and wave equations, reaction-diffusion system.
Reference books	H. Brezis, Analyse fonctionelle, Théorie et applications, 2e tirage, Masson 1987. H. Brezis, Functional Analysis, Sobolev Spaces and Partial Differential Equations, Springer, 2011.
Additional course materials	
Repository	Lecture notes.

Expected learning outcomes	
Knowledge and understanding	Acquiring fundamental concepts and results in the setting of functional spaces and operator theory. Acquiring main tools and proof techniques.
Applying knowledge and understanding	The acquired theoretical knowledge finds many applications in several aspects of mathematics, including partial differential equations and related models.

Soft skills	*Making judgements*: Ability to analyze the consistency of the logical arguments used in a proof, the problem-solving skills and the ability to choose suitable mathematical tools consistent with the theoretical knowledge.
	Communication skills: Acquiring the mathematical language and the formalism necessary to read and understand textbooks, to explain the acquired knowledge, and to describe, analyze and solve problems.
	Learning skills: Acquiring suitable learning methods, supported by consultation of texts and by solving exercises and problems related to the contents of the course.

Teaching methods	
	Lectures. Lessons will be held face-to-face.

Assessment	
Assessment methods	The final exam is based on an oral test in which it is possible to verify the theoretical and applied knowledge of the course.
Evaluation criteria	• *Knowledge and understanding*: Acquiring fundamental concepts and results in the setting of functional spaces and operator theory. Acquiring main tools and proof techniques. • *Applying knowledge and understanding*: The acquired theoretical knowledge finds many applications in several aspects of mathematics, including partial differential equations and related models. • *Making judgement*: Ability to analyze the consistency of the logical arguments used in a proof, the problem-solving skills and the ability to choose suitable mathematical tools consistent with the theoretical knowledge. • *Communication skills*: Acquiring the mathematical language and the formalism necessary to read and understand textbooks, to explain the acquired knowledge, and to describe, analyze and solve problems. • *Learning skills*: Acquiring suitable learning methods, supported by consultation of texts and by solving exercises and problems related to the contents of the course.
Grading policy	During the oral exam the student must show mastery of the language, methodological accuracy an having acquired the fundamental notions and concepts of the course. To achieve a high mark, the student have developed independent judgement, skills and expository clarity. The exam is sufficient with 18/30 mark.

Further information	
	Attendance at lessons is strongly recommended.