UNIVERSITÀ DEGLI STUDI DI BARI ALDO MORO

CONSIGLIO INTERCLASSE IN MATEMATICA

COURSE OF STUDY	**TWO-YEAR MASTER OF SCIENCE PROGRAMME IN MATHEMATICS**
ACADEMIC YEAR	**2023-2024**
ACADEMIC SUBJECT	**CRITICAL POINT THEORY**

General information	
Term	Second semester (February 26, 2024 – May 31, 2024)
European Credit Transfer and Accumulation System credits (ECTS)	7
SSD	MAT/05 – Mathematical Analysis
Language	Italian
Mode of attendance	Not mandatory

Lecturer	
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Department and office	Department of Mathematics, room 10 fourth floor
Virtual meeting room	Microsoft Teams codice 7vl9dpm
Web page	https://www.dm.uniba.it/en/members/salvatore
Office hours	By appointment via email

Work schedule				
	Total	Lectures	Hands-on learning (recitations)	Self-study
Hours	175	56		119
ECTS credits	7	7		

Learning objectives	
	Knowledge of variational and topological methods in the study of nonlinear problems, especially topological degree and index theories. Applications to the study of some semi-linear elliptic problems.

Course prerequisites	
	Mathematical knowledge which usually is acquired during the first three years of a degree of L-35 class. Especially: classical mathematical analysis of one and several variables, general topology, linear algebra, Hilbert spaces and Lp spaces.

Syllabus	
Course contents	**Topological degree and applications** Topological degree for continuous functions in finite dimension: axiomatic definition and properties. Construction of the topological degree. Brower fixed point Theorem. Retraction Theorem. Borsuk Theorem. Topological linking, definition and examples: mountain pass linking, multidimensional mountain pass linking, saddle linking. Topological degree in infinite dimension. Schauder fixed point Theorem. Retraction Theorem. Other fixed point Theorems. **Index theory**

	Index theory in topological spaces. Axiomatic definition. Lusternik-Schnirelmann category: definition, examples and properties. Krasnoselski genus: definition, examples and properties. Relationship between the degree and the category of a set. Index theory related to a group of unitary transformations on a Hilbert space. S1- index. **Abstract theorems of critical points existence and applications** Generalities on the Palais-Smale condition. Deformation Lemma. Linking Theorem and applications to the study of some elliptic problems with sublinear or superlinear growth. Linking Theorem for strongly indefinite functionals. Application to the study of a first order hamiltonian system. **Abstract theorems of critical points multiplicity and applications** Deformation lemma for functionals compatible with an index theory. Abstract theorems of critical points multiplicity for functionals bounded from below and compatible with an index theory. Abstract theorems using the Lusternik-Schnirelmann category and applications. Study of a nonlinear eigenvalue problem. Abstract theorems of critical points multiplicity for functionals even and bounded from below. Applications to some symmetric elliptic equations. Symmetric mountain pass theorem and symmetric multidimensional mountain pass theorem. Pseudo-index theory. An abstract theorem of critical points multiplicity for functionals even and unbounded from below. Applications to some superlinear or asymptotically linear elliptic problems. Multiplicity results for strongly indefinite S1 -invariant functionals (sketch of the proof). Application to the study of symmetric first order hamiltonian system.
Reference books	P. H. Rabinowitz, Minimax methods in critical point theory with applications to differential equations, CBMS Regional conference Series in Applied Mathematics, 65 (1986). J.T. Scharwtz, Nonlinear Functional Analysis, Gordon & Breach, New York (1969). M. Struwe, Variational methods, Fourth edition, Springer- Berlin (2008).
Additional course materials	
Repository	

Expected learning outcomes	
Knowledge and understanding	Knowledge of basic and advanced tools in the study of variational problems.
Applying knowledge and understanding	The acquired theoretical knowledge is useful in the study of many nonlinear variational problems.
Soft skills	*Making judgements*: Problem solving skills should be supported by the capacity in evaluating the consistency of the found solutions with the theoretical knowledge.
	Communicated skills: Students should acquire the mathematical language and formalism necessary to read and comprehend textbooks, to explain the acquired knowledge and to describe, analyze and solve problems.
	Learning skills: Acquisition of suitable learning methods, supported also by consultation of the texts and by solution of problems suggested during the course.

Teaching methods	
	The mode of delivery of the teaching is frontal. Lessons are held in presenced.

Assessment

Assessment methods	The oral exam includes an interview on the main topics covered in the course.
Evaluation criteria	• *Knowledge and understanding*: acquisition and mastery of the definitions and theoretical results presented in the course and of the related demonstration techniques. • *Applying knowledge and understanding*: ability to apply the acquired knowledge to the study of non linear variational problems. • *Making judgement*: critical approach to concepts, ability to choose theoretical results and solution techniques for the study of nonlinear variational problems. • *Communication skills*: : mastery of the specific mathematical language, quality of exposition. • *Learning skills*: ability to organize knowledge, critical reasoning and possible independent study.*Knowledge and understanding*:
Grading policy	The student takes the oral test which is passed if the grade obtained is greater than or equal to 18/30. The student must show mastery of the language, methodological rigor and have acquired the fundamental notions and concepts of the course. Assessment is based on the achievement of the intended learning objectives. To achieve a high evaluation, the student must have developed independent judgment and adequate capacity for argumentation and exposition.

Further information	
	Class attendance is strongly recommended.