UNIVERSITÀ DEGLI STUDI DI BARI ALDO MORO

CONSIGLIO INTERCLASSE IN MATEMATICA

**COURSE OF STUDY** TWO-YEAR MASTER OF SCIENCE PROGRAMME IN MATHEMATICS

**ACADEMIC YEAR** 2023-2024

**ACADEMIC SUBJECT** AN INTRODUCTION TO OPERATOR ALGEBRAS

| **General information** | |
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| Programme year | Second |
| Term | Second semester (February 26, 2024 – May 31, 2024) |
| European Credit Transfer and Accumulation System credits (ECTS) | 4 |
| SSD | MAT/06 – Probability and Mathematical Statistics |
| Language | Italian |
| Mode of attendance | Not compulsory |

| **Lecturers** | | |
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| Name and surname | Stefano Rossi | Simone Del Vecchio |
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| Telephone | +39 080 544 2663 | |
| Department and office | Department of Mathematics room 14 second floor | Department of Mathematics room 24 second floor |
| Virtual meeting room | | |
| Web page | https://www.dm.uniba.it/it/members/rossi | https://www.dm.uniba.it/it/members/delvecchio |
| Office hours | By appointment via e-mail | |

| **Work schedule** | Total | Lectures | Hands-on learning (recitations) | Self-study |
|---|---|---|---|---|
| **Hours** | 92 | 32 | | 60 |
| **ECTS credits** | 4 | 4 | | |

| **Learning objectives** | |
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| | Getting familiar with the basics of C* and von Neumann algebras as a unified language of spectral theory and non-commutative probability. |

| **Course prerequisites** | |
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| | Acquaintance with elementary theory of Hilbert spaces, basics of Functional Analysis, Measure Theory |

| **Syllabus** | |
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| Course contents | 1. Banach algebras. Spectrum and resolvent of an element in a Banach algebra. Commutative Banach algebras. Spectrum of a commutative Banach algebra.<br>2. Commutative C*-algebras. Gelfand-Najmark theorem. Continuous functional calculus.<br>3. General C*-algebras: structure of the positive cone and states.<br>4. GNS representation, universal representation of a C*-algebra.<br>5. The topologies of B(H). Von Neumann density theorem. Kaplansky density theorem. |

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| | 6. Von Neumann algebras. Commutative von Neumann algebras.<br>7. Borel functional calculus. L^\infty functional calculus<br>8. Normal states on a von Neumann algebra. Gleason's theorem |
| Reference books | 1. Gert K. Pedersen, Analysis Now, Graduate Text in Mathematics 118, Springer.<br><br>2. M. Takesaki, Theory of Operator Algebras I, Springer. |
| Additional course materials | |
| Repository | |

| **Expected learning outcomes** | |
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| Knowledge and understanding | ○ The students will learn the language of abstract and concrete operator algebras. |
| Applying knowledge and understanding | ○ Capacity to employ abstract techniques to solve more specific problems from spectral theory of (normal) operators on a Hilbert space. |
| Soft skills | *Making judgements*:<br>By the end of the course the students will be able to:<br><br>○ make connections between the various topics covered in the course<br>○ tackle and solve simple concrete problems from spectral theory |
| | *Communication skills*:<br>By the end of the course the students will have learnt both languange and algebraic-analytical formalism needed to:<br><br>o expound their acquired knowledge<br>o analyze and solve problems |
| | *Learning skills*:<br>By the end of the course the students will be able to profitably read and understand textbooks and scientific articles on the topics covered in the course |

| **Teaching methods** | |
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| | Lectures and assignment of guided exercises |

| **Assessment** | |
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| Assessment methods | Oral exam, during which the committe will evaluate the comprehension of the theoretical topics presented in the lectures, and the capacity to solve simple abstract or concrete problems a |
| Evaluation criteria | • *Knowledge and understanding*:<br><br>o Learning the proof methods typical of Operator Algebra Theory<br>o Learning the main notions of Operator Algebras and Spectral Theory<br><br>• *Applying knowledge and understanding*: |

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| | ○ Being able to reduce a concrete problem (such as an integral equation or a Sturm-Liouville problem) to a suitable problem in spectral theory.<br><br>• *Making judgement*:<br>By the end of the course the students will be able to tackle and solve exercises in spectral theory<br><br>• *Communication skills*:<br>By the end of the course the students will have acquired the language and formalism needed to:<br><br>○ Show knowledge in public speaking<br>○ Solve problems<br><br>• *Learning skills*:<br>Reading and understanding textbooks or scientific articles on the topics |
| Grading policy | The minimum score to pass is 18/30, the maximum is 30/30. Evaluation derives from the criteria presented above. High-ranked evaluation will be achieved by any student showing deep knowledge of the topics and autonomy in solving problems. |

| **Further information** | |
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| | Attendance is highly recommended. |