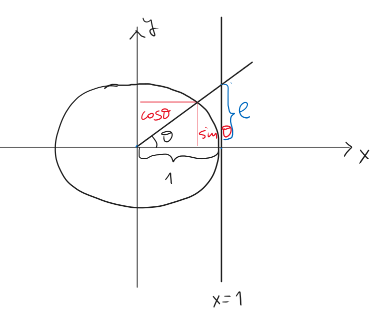
The geometric meaning of the tangent and cotangent:

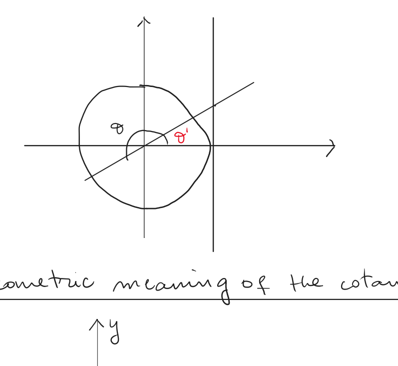
$\tan : \mathbb{R} \setminus \{\frac{\pi}{2} + k\pi : k \in \mathbb{Z}\} \to \mathbb{R}$

$x \longmapsto \tan x := \frac{\sin x}{\cos x}$

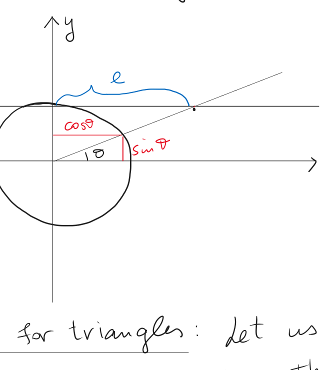


$y = \tan x$

geometric meaning:

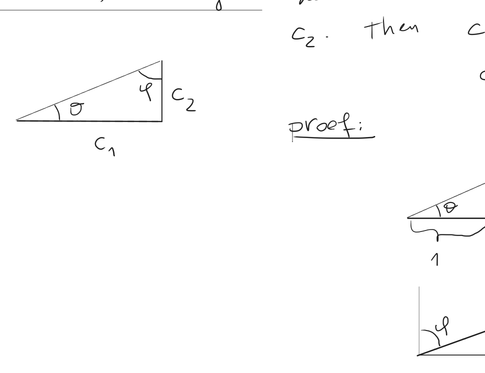


$\frac{\ell}{1} = \frac{\sin\theta}{\cos\theta} \Rightarrow \ell = \tan\theta$



$\tan\theta = \tan\theta'$

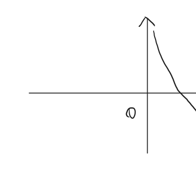
the geometric meaning of the cotangent:



$\frac{\ell}{1} = \frac{\cos\theta}{\sin\theta} \Rightarrow \ell = \cot\theta$

Theorems for triangles: Let us consider a rectangle triangle of legs c_1 and c_2. Then $c_2 = c_1 \tan\theta$

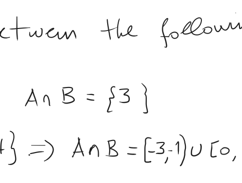
$c_2 = c_1 \cot\varphi$.



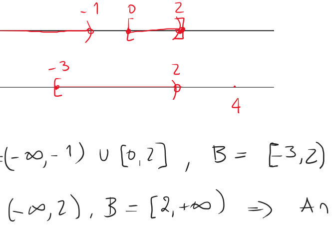
proof:

$\frac{c_2}{c_1} = \frac{\tan\theta}{1}$

$\Leftrightarrow \frac{c_1}{c_2} = \frac{1}{\tan\theta} = \cot\theta$



The geometric meaning of the cot also explains the graph of cot.

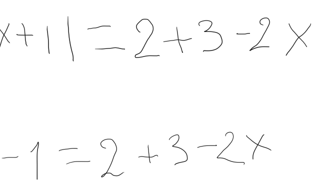


Exercises: (Sheet 2)

(1) Determine the intersection between the following sets.

(a) $A = [0,1] \cup \{3\}$, $B = [2,+\infty) \Rightarrow A \cap B = \{3\}$

(b) $A = (-\infty,-1) \cup [0,2]$, $B = [-3,2) \cup \{4\} \Rightarrow A \cap B = [-3,-1) \cup [0,2)$



(c) $A = (-\infty,-1) \cup [0,2]$, $B = [-3,2) \cup [4,+\infty) \Rightarrow A \cap B = [-3,-1) \cup [0,2)$

(d) $A = (-\infty,2)$, $B = [2,+\infty) \Rightarrow A \cap B = \emptyset$. (empty).

(2) Solve the following equations:

(a) $|x+1| + 2x - 3 = 2$

$|x+1| = 2+3-2x$

I

$-x-1 = 2+3-2x$

$-x+2x = 2+3+1$

$x_1 = 6$

II

$x+1 = 5-2x$

$3x = 4$

$x_2 = \frac{4}{3}$

(b) $2^{-x} - 2^x + 6 = 0$

$\frac{1}{2^x} - 2^x + 6 = 0$

$1 - (2^x)^2 + 6 \cdot 2^x = 0$

$2^x = y > 0$, $1 - y^2 + 6y = 0 \Leftrightarrow y^2 - 6y - 1 = 0$

$y_\pm = \frac{6 \pm \sqrt{36+4}}{2} = \frac{6 \pm \sqrt{40}}{2} = \frac{6 \pm \sqrt{4 \cdot 10}}{2} = \frac{6 \pm \sqrt{4} \cdot \sqrt{10}}{2} = \frac{6 \pm 2\sqrt{10}}{2} = \frac{\cancel{2}(3 \pm \sqrt{10})}{\cancel{2}} = 3 \pm \sqrt{10}$

$y_+ = 3 + \sqrt{10} > 0 > 3 - \sqrt{10} = y_-$

$3 = \sqrt{9} < \sqrt{10} \Leftrightarrow 3 - \sqrt{10} < 0$

$\} \Rightarrow 2^x = 3 + \sqrt{10} \Leftrightarrow x = \log_2(3+\sqrt{10})$

(c) $4x^4 + 11x^2 = 3$ $y = x^2$ $y^2 = x^4$

$4y^2 + 11y = 3$

$4y^2 + 11y - 3 = 0$ $a = 4$, $b = 11$, $c = -3$

$\frac{(-11) \pm \sqrt{(-11)^2 - 4 \cdot 4 \cdot -3}}{2a} = \frac{-11 \pm \sqrt{169}}{8} = \frac{-11 \pm \sqrt{169}}{8}$

$x^2 = \frac{-11 + \sqrt{169}}{8} \Leftrightarrow x = \pm\sqrt{\frac{-11+\sqrt{169}}{8}} = \pm\sqrt{\frac{1}{4}} = \pm\frac{1}{2}$

$4y^2 + 11y - 3 = 0$ $y_\pm = \frac{-11 \pm \sqrt{121+48}}{8} = \frac{-11 \pm \sqrt{169}}{8} = \frac{-11 \pm 13}{8} = \frac{2}{8} / -\frac{24}{8} = \frac{1}{4} / -3$

(d) $\left(\sqrt[3]{\frac{x^2+3x-1}{2x+1}} = 1\right)^3$

$\frac{x^2+3x-1}{2x+1} - 1 = 0$ $\frac{x^2+3x-1-(2x+1)}{2x+1} = 0$ $\frac{x^2+x-2}{2x+1} = 0$ $(x-1)(x+2)$ $(x = 1, -2)$

$\frac{x^2+3x-1}{2x+1} = 1$ $x^2+3x-1 = 2x+1$ $x \neq -\frac{1}{2}$

$x^2 + x - 2 = 0$ $x_\pm = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-1 \pm \sqrt{1+8}}{2} = \frac{-1 \pm \sqrt{9}}{2} = \frac{-1 \pm 3}{2} = 1/-2$

$a = 1$, $b = 1$, $c = -2$

$x^2 - x - 2 = a(x-x_-)(x-x_+) = (x-1)(x+2)$

(3) Find the solutions to the following inequalities:

(a) $|2x-1| \leq x+1$

I.

$2x-1 \leq x+1$

$2x - x \leq 1+1$

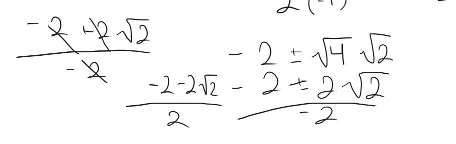
$x_1 \leq 2$

II

$-2x+1 \leq x+1$

$-3x \leq 0$

$x \geq 0$



$x \in [0;2]$.

(b) $|2x+1| \geq x^2$

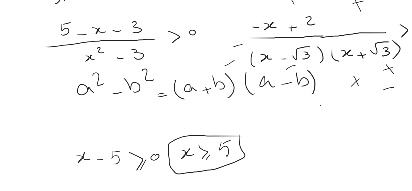
I. $2x+1 \geq x^2$

$-x^2 + 2x + 1 \geq 0$

$\frac{-2 \pm \sqrt{2^2 - 4(-1)(1)}}{2(-1)}$ $4+4$ $\frac{-2 \pm \sqrt{8}}{-2}$

$\frac{-\cancel{2} \pm 2\sqrt{2}}{-\cancel{2}}$ $\frac{-2-2\sqrt{2}}{2}$ $\frac{-2 \pm \sqrt{4}\sqrt{2}}{}$ $\frac{-2 \pm 2\sqrt{2}}{-2}$

$1 - \sqrt{2}$ or $1 + \sqrt{2}$

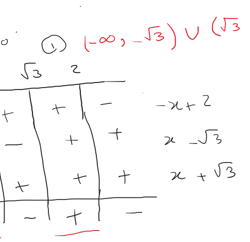


II. $-2x-1 \geq x^2$

$-x^2 - 2x - 1 \geq 0$ $4-4(1)$

$\frac{+2 \pm \sqrt{(-2)^2 - 4(-1)(-1)}}{2(-1)}$ $\frac{2 \pm \sqrt{0}}{-2}$

$\frac{2 \pm 0}{-2}$ $\frac{2}{-2}$ -1



(b)

$|2x+1| \geq x^2 \Leftrightarrow$ $2x+1 \geq x^2$ $x^2 - 2x - 1 \leq 0$ I

$2x+1 \leq -x^2$ $x^2 + 2x + 1 \leq 0$ II

$1-\sqrt{2} \leq x \leq 1+\sqrt{2}$

$x = -1$

$\Leftrightarrow (1-\sqrt{2} \leq x \leq 1+\sqrt{2}) \vee x = -1$

I $x^2 - 2x - 1 \leq 0$ $x_\pm = 1 \pm \sqrt{1+1} = 1 \pm \sqrt{2}$

(c) $\frac{|5-x|-3}{x^2-3} > 0$

$5 - x > 0 \to -x > -5$ $\boxed{x \leq 5}$

$\frac{5-x-3}{x^2-3} > 0$ $\frac{-x+2}{(x-\sqrt{3})(x+\sqrt{3})} > 0$

$a^2 - b^2 = (a+b)(a-b)$

$x - 5 \geq 0$ $\boxed{x \geq 5}$

(1) $(-\infty, -\sqrt{3}) \cup (\sqrt{3}, 2)$

	$-\sqrt{3}$	$\sqrt{3}$	2	
+	+	+	−	$-x+2$
−	−	+	+	$x-\sqrt{3}$
−	+	+	+	$x+\sqrt{3}$
+	−	+	−	

$\frac{x-5-3}{x^2-3} > 0$ $\frac{x-8}{x^2-3} > 0$

$(8, +\infty)$

	$-\sqrt{3}$	$\sqrt{3}$	8	
−	−	−	+	$x-8$
−	−	+	+	$x-\sqrt{3}$
−	+	+	+	$x+\sqrt{3}$
−	+	−	+	

$(-\infty, -\sqrt{3}) \cup (\sqrt{3}, 2) \cup (8, +\infty)$

alternative way

$\frac{|5-x|-3}{x^2-3} > 0$

. $N(x) = |5-x| - 3 > 0 \Leftrightarrow |5-x| > 3 \Leftrightarrow (5-x > 3) \vee (5-x < -3) \Leftrightarrow x < 2 \vee x > 8$

. $D(x) = x^2 - 3 > 0 \Leftrightarrow x < -\sqrt{3} \vee x > \sqrt{3}$

		$-\sqrt{3}$	$\sqrt{3}$	2	8
N	+	+	+	−	+
D	+	−	+	+	+
	+	−	+	−	+

$(-\infty, -\sqrt{3}) \cup (\sqrt{3}, 2) \cup (8, \infty)$