Exercises:

- $2^{-x}+2^{x}+6\geq 0 \quad \forall x\in\mathbb{R}$

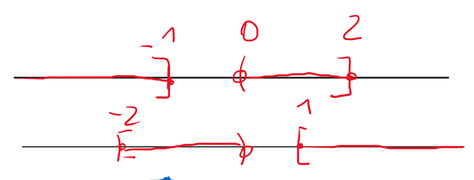
$2^{-x}>0 \;\; \forall x\in\mathbb{R};\; 2^{x}>0 \;\; \forall x\in\mathbb{R} \;\Rightarrow\; 2^{-x}+2^{x}+6>0 \quad \forall x\in\mathbb{R}$

- $\left|\frac{x^2-2}{x}\right|\leq 1$

$|a(x)|\leq b(x) \iff \underbrace{-b(x)\leq a(x)}\underbrace{\leq b(x)} \iff \begin{cases} a(x)\leq b(x) \\ -b(x)\leq a(x)\end{cases}$

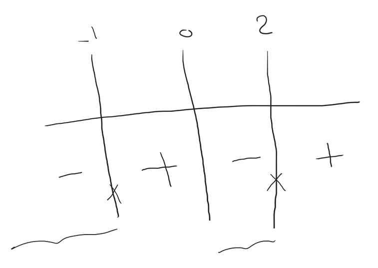
$\left|\frac{x^2-2}{x}\right|\leq 1 \iff -1\leq \frac{x^2-2}{x}\leq 1 \iff \begin{cases} \frac{x^2-2}{x}\leq 1 \\ \frac{x^2-2}{x}\geq -1\end{cases} \iff \begin{cases} \frac{x^2-2}{x}-1\leq 0 \\ \frac{x^2-2}{x}+1\geq 0\end{cases}$

$\begin{cases} \frac{x^2-2-x}{x}\leq 0 & (1) \\ \frac{x^2-2+x}{x}\geq 0 & (2)\end{cases} \quad \begin{cases} (-\infty,-1]\cup(0,2] \\ [-2,0)\cup[1,\infty)\end{cases}$



$\iff x\in[-2,-1]\cup[1,2]$

(1) $x=-1,2$, 0



-1		0		2		
−		−		−		+
−		+		+		+
−		−		+		+
−		+		−		+

$x-2>0 \iff x>2$

$x+1>0$

$x>0$

$N(x)=x^2-2-x=0 \iff x=-1 \vee x=2 \Rightarrow N(x)=(x-2)(x+1)$

$D(x)=x=0 \iff x=0$

$\frac{x^2-x-2}{x}\leq 0$ for $x\in(-\infty,-1]\cup(0,2]$

$p(x)=ax^2+bx+c=a(x-x_-)(x-x_+)$

$x_\pm=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

(2) $\frac{x^2-2+x}{x}\geq 0 \iff x\in[-2,0)\cup[1,\infty)$

$N(x)=x^2+x-2=0 \qquad x_\pm=\frac{-1\pm\sqrt{1-4(-2)}}{2}=\frac{-1\pm\sqrt{9}}{2}=\frac{-1\pm 3}{2}=1/-2$

$N(x)=(x-1)(x+2)$

$D(x)=0 \iff x=0$

-2		0		1		
−		−		−		+
−		+		+		+
−		−		+		+
→ −		+		−		+

$x-1>0$

$x+2>0$

$x>0$

- $\frac{-x^2+2x+2}{x+1}\geq 0 \iff \frac{x^2-2x-2}{x+1}\leq 0 \;\ldots\ldots$

$\left[\frac{a\cdot c}{b\cdot c}=\frac{a}{b},\; \forall c\neq 0\right]$

$\frac{ac}{bc}\geq 0 \iff \frac{a}{b}\geq 0$

- $\sqrt{x^2-4x}\leq x+3 \iff \left(\sqrt{x^2-4x}\right)^2\leq(x+3)^2 \iff \begin{cases} x^2-4x\leq(x+3)^2 \\ x^2-4x\geq 0\end{cases}$

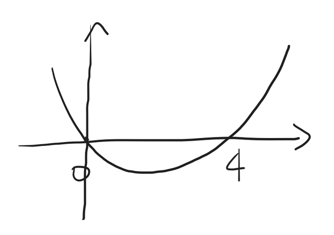
$\iff \begin{cases} \cancel{x^2}-4x\leq \cancel{x^2}+9+6x \\ x^2-4x\geq 0\end{cases} \iff \begin{cases} -9\leq 10x \\ x(x-4)\geq 0\end{cases} \iff \begin{cases} x\geq -\frac{9}{10} \\ x(x-4)\geq 0\end{cases} \iff \begin{cases} x\geq -\frac{9}{10} \\ x\leq 0 \vee x\geq 4\end{cases}$

$(a+b)^2=(a+b)(a+b)=a^2+\underbrace{ab+ba}_{2ab}+b^2=a^2+2ab+b^2$

$\iff x\in\left[-\frac{9}{10},0\right]\cup[4,\infty)$

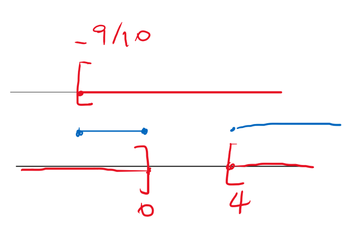
$x(x-4)\geq 0 \iff x\leq 0 \vee x\geq 4$

$x(x-4)=0 \iff x=4 \vee x=0$

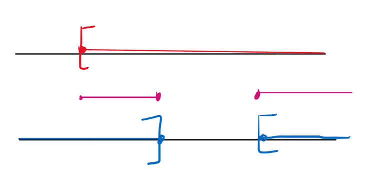


0		4		
−		−		+
−		+		+
+		−		+

$x-4$



$\left(x\geq -\frac{9}{10}\right)\wedge(x\leq 0 \vee x\geq 4) \iff -\frac{9}{10}\leq x\leq 0 \;\vee\; x\geq 4$



Exercise 4: Say if each of the following functions is injective or surjective. In case they are bijective, determine their inverse.

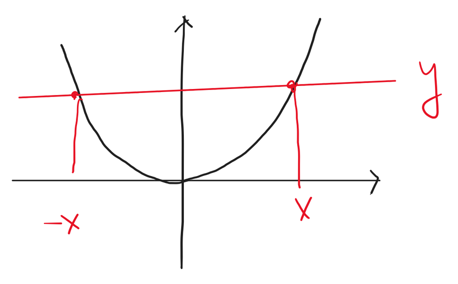
- $f:\mathbb{R}\to\mathbb{R}$, $x\mapsto f(x)=x^3$. is surjective and injective. $\Rightarrow f$ is bijective $\Rightarrow f$ is invertible

We saw that $\forall y\in\mathbb{R},\ \exists!\, x\in\mathbb{R}: x^3=y$ (lesson about powers).

What is f^{-1}? $\quad f^{-1}(y)=y^{1/3}=\sqrt[3]{y}$. (lesson about powers).

- $f:\mathbb{R}\to[0,\infty)$, $x\mapsto f(x)=x^2$ is surjective but not injective

$\forall y\in[0,\infty),\ \exists x\in\mathbb{R}: x^2=y$ ($\Rightarrow f$ is surjective) but such a point x is not unique [if $x^2=y \Rightarrow (-x)^2=y$]



- $f:(0,\infty)\to\mathbb{R}$, $x\mapsto \log x$ is surjective and injective $\Rightarrow f$ is bijective

what is f^{-1}? $\quad e^x$

$a\in(0,1)\cup(1,\infty),\ x\in\mathbb{R},\quad a^x=y\in(0,\infty) \iff x=\log_a y \quad [a=e]$