

MATEMATICA LEZIONE 6

giovedì 8 ottobre 2026 14:09

ESERCIZIO

Risolvere $\frac{1}{x^2-2x} = \frac{x}{x^2-4}$

c.e. $x^2 - 2x \neq 0$
 $x^2 - 4 \neq 0$

$$x^2 - 2x = x(x-2) = 0 \Leftrightarrow x = 0 \vee x = 2$$

$$x^2 - 2x \neq 0 \Leftrightarrow x \neq 0 \wedge x \neq 2$$

$$x^2 - 4 = (x-2)(x+2) \neq 0 \Leftrightarrow x \neq 2 \wedge x \neq -2$$

c.e.: $x \neq 0, x \neq 2, x \neq -2$

$$\frac{1}{x^2-2x} = \frac{x}{x^2-4}$$

$$\frac{1}{x(x-2)} = \frac{x}{(x-2)(x+2)}$$

moltiplico per
 $x(x-2)(x+2)$

$$x+2 = x \cdot x$$

$$x+2 = x^2$$

$$x^2 - x - 2 = 0$$

$$\Delta = 1 + 8 = 9$$

$$x_{1,2} = \begin{cases} \frac{1-3}{2} = -1 \\ \frac{1+3}{2} = 2 \end{cases}$$

$x = -1 \vee x = 2$ escluso per le c.e.

Unica sol. $x = -1$.

ESERCIZIO 2

$$\frac{2x}{x^2-1} \geq \frac{2x-1}{x-1}$$

$$\frac{2x}{x^2-1} - \frac{2x-1}{x-1} \geq 0$$

$$(x-1)(x+1)$$

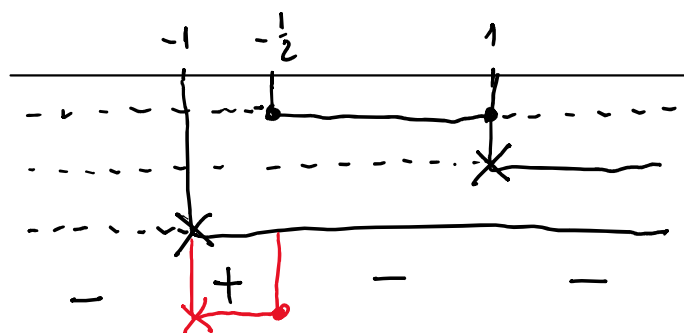
$$\frac{2x - (2x-1) \cdot (x+1)}{(x-1)(x+1)} \geq 0$$

$$\frac{2x - (2x^2 - x + 2x - 1)}{(x-1)(x+1)} \geq 0$$

$$\frac{2x - 2x^2 + x - 2x + 1}{(x-1)(x+1)} \geq 0$$

$$\frac{-2x^2 + x + 1}{(x-1)(x+1)} \geq 0$$

$\Delta = 1 + 8 = 9, x_{1,2} = \frac{-1 \pm 3}{-4} = \begin{matrix} 1 \\ -\frac{1}{2} \end{matrix}$

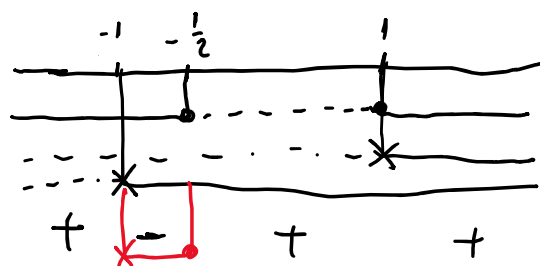


Soluzioni: $-1 < x \leq -\frac{1}{2}$.

oss Avremmo anche potuto cambiare il segno:

$$\frac{-2x^2 + x + 1}{(x-1)(x+1)} \geq 0$$

$$\frac{2x^2 - x - 1}{(x-1)(x+1)} \leq 0$$



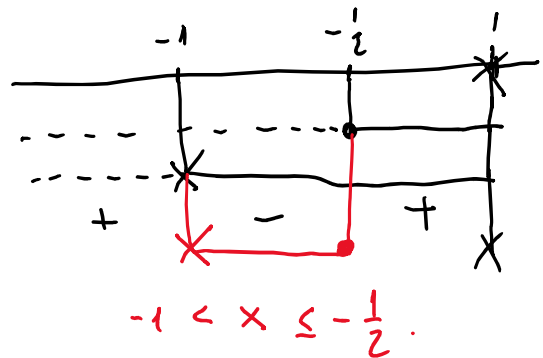
oss 2 Avremmo anche potuto semplificare un fattore $x-1$:

$$\frac{2x^2 - x - 1}{(x-1)(x+1)} \leq 0 \quad \xrightarrow{x_{1,2} = \begin{cases} -\frac{1}{2} \\ 1 \end{cases}} \quad -1 < x \leq -\frac{1}{2}$$

$$\frac{2(x + \frac{1}{2})\cancel{(x-1)}}{\cancel{(x-1)}(x+1)} \leq 0$$

c.e. $x \neq 1 \quad \wedge \quad x \neq -1$.

$$\frac{\cancel{2}(x + \frac{1}{2})}{x+1} \leq 0$$

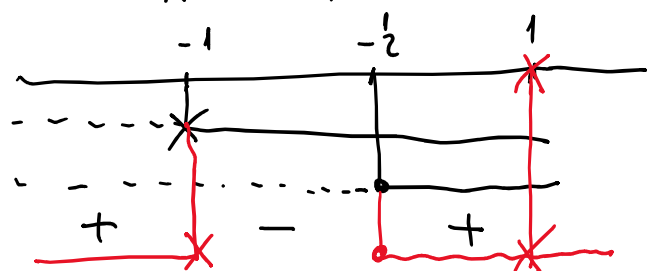


oss 3 Quando si semplificano dei termini come nell'esempio precedente, è importante ricordare di imporre le condizioni di esistenza. Ad esempio, modificando la disuguaglianza da risolvere;

$$\frac{2x^2 - x - 1}{(x-1)(x+1)} \geq 0$$

$$\frac{2(x + \frac{1}{2})}{x+1} \geq 0$$

c.e. $x \neq 1 \quad \wedge \quad x \neq -1$



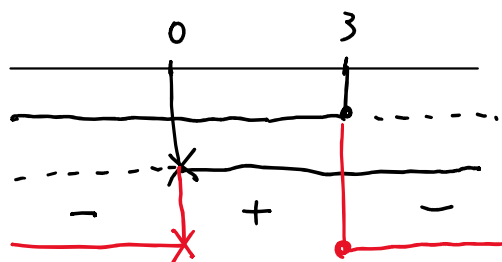
Sol: $x < -1 \quad \vee \quad -\frac{1}{2} \leq x < 1 \quad \vee \quad x > 1$

Attenzione: Anche nelle disuguaglianze semplici attenzione a non moltiplicare per il denominatore (a meno che non sia positivo)

$$\frac{3}{x} \leq 1$$

$$\frac{3}{x} - 1 \leq 0$$

$$\frac{3-x}{x} \leq 0$$



$x < 0 \quad \vee \quad x \geq 3.$

ESERCIZIO 3

$$\frac{x+9}{x^2+3} \geq \frac{9x-10}{2x^2-1}$$

Si può moltiplicare per x^2+3 perché $x^2+3 > 0 \quad \forall x \in \mathbb{R}$

$$x+9 \geq \frac{9x-10}{2x^2-1} (x^2+3)$$

$$x+9 - \frac{(9x-10)(x^2+3)}{2x^2-1} \geq 0$$

$$\frac{(x+9)(2x^2-1) - (9x-10)(x^2+3)}{2x^2-1} \geq 0$$

$$\frac{2x^3 + 18x^2 - x - 9 - (9x^3 - 10x^2 + 27x - 30)}{2x^2 - 1} \geq 0$$

$$\frac{-7x^3 + 28x^2 - 28x + 21}{2x^2 - 1} \geq 0$$

$$\frac{x^3 - 4x^2 + 4x - 3}{2x^2 - 1} \leq 0$$

$$p(x) = x^3 - 4x^2 + 4x - 3$$

possibili radici razionali: $\pm 1, \pm 3$

$$p(3) = 27 - 36 + 12 - 3 = 0$$

$$\begin{array}{c|ccc|c} & 1 & -4 & 4 & -3 \\ \hline 3 & & 3 & -3 & 3 \\ \hline & 1 & -1 & 1 & 0 \end{array}$$

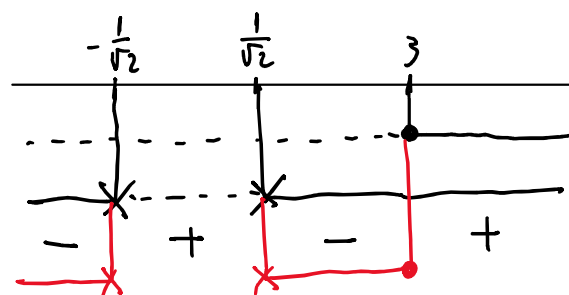
$$(x-3)(x^2 - x + 1)$$

$$\Delta = 1 - 4 = -3 < 0 \Rightarrow x^2 - x + 1 > 0 \quad \forall x$$

$$\frac{(x-3)(x^2 - x + 1)}{2x^2 - 1} \leq 0$$

$$\frac{x-3}{2x^2-1} \leq 0$$

radici: $\pm \frac{1}{\sqrt{2}}$



$$x < -\frac{1}{\sqrt{2}} \quad \vee \quad \frac{1}{\sqrt{2}} < x \leq 3$$

Esercizio 4

$$\frac{5x^4 + 14x^2 - 3}{1 - 24x^3} > 0$$

$$\bullet \quad 5x^4 + 14x^2 - 3 = 5t^2 + 14t - 3$$

$$t = x^2$$

$$t_{1,2} = \frac{-17 \pm \sqrt{49 + 15}}{5}$$

$$= \frac{-17 \pm 8}{5} \quad \begin{matrix} \swarrow \frac{1}{5} \\ \searrow -3 \end{matrix}$$

$$\begin{aligned} 5t^2 + 14t - 3 &= 5\left(t - \frac{1}{5}\right)(t + 3) \\ &= 5\left(x^2 - \frac{1}{5}\right)(x^2 + 3) \end{aligned}$$

$$\bullet \quad 1 - 27x^3$$

$$p(x) = 1 - 27x^3$$

$$p\left(\frac{1}{3}\right) = 1 - 27 \cdot \frac{1}{27} = 1 - 1 = 0.$$

-27	0	0	1
$\frac{1}{3}$	-9	-3	-1
-27	-9	-3	0

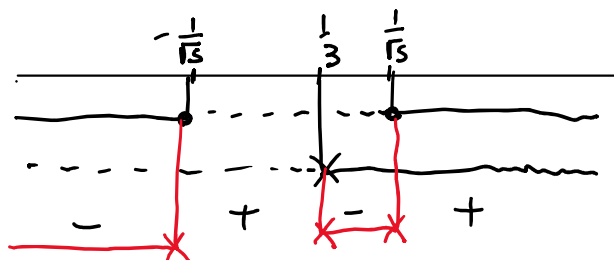
$$\begin{aligned} p(x) &= \left(x - \frac{1}{3}\right)(-27x^2 - 9x - 3) \\ &= -3\left(x - \frac{1}{3}\right)(\underbrace{9x^2 + 3x + 1}_{\Delta = 9 - 36 < 0}) \end{aligned}$$

La disequazione si scrive come:

$$\frac{\cancel{5}\left(x^2 - \frac{1}{5}\right)\cancel{(x^2 + 3)}}{-3\left(x - \frac{1}{3}\right)\cancel{(9x^2 + 3x + 1)}} > 0$$

$$\frac{x^2 - \frac{1}{5}}{x - \frac{1}{3}} \quad \textcircled{<} 0$$

(abbiamo moltiplicato per -3)



$$\sqrt{5} < 3$$

$$\frac{1}{3} < \frac{1}{\sqrt{5}}$$

$$x < -\frac{1}{\sqrt{5}} \quad \vee \quad \frac{1}{3} < x < \frac{1}{\sqrt{5}}$$

ESERCIZIO 5

$$|2x^2 - 1| - 2 = x$$

$$\begin{cases} 2x^2 - 1 \geq 0 \\ 2x^2 - 1 - 2 = x \end{cases}$$

$$\begin{cases} x \geq \frac{1}{\sqrt{2}} \vee x \leq -\frac{1}{\sqrt{2}} \\ 2x^2 - x - 3 = 0 \end{cases}$$

$$\Delta = 1 + 24 = 25$$

$$x_{1,2} = \frac{1 \pm 5}{4} = \begin{cases} -1 \\ \frac{3}{2} \end{cases}$$

$$\begin{cases} x \geq \frac{1}{\sqrt{2}} \vee x \leq -\frac{1}{\sqrt{2}} \\ x = -1 \vee x = \frac{3}{2} \end{cases}$$

entrambe le soluzioni
sono accettabili.

$$x = -1 \vee x = \frac{3}{2}$$

$$\text{Uniche soluzioni } x = -1 \vee x = \frac{3}{2}$$

$$\vee \begin{cases} 2x^2 - 1 < 0 \\ -2x^2 + 1 - 2 = x \end{cases}$$

$$\begin{cases} -\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}} \\ -2x^2 - 1 = x \end{cases}$$

$$2x^2 + x + 1 = 0$$

$$\Delta = 1 - 8 < 0$$

non ci sono
soluzioni.

ESERCIZIO 6

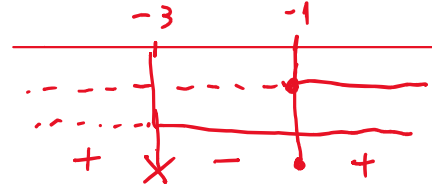
$$\left| \frac{x+1}{x+3} \right| = x$$

$$\begin{cases} \frac{x+1}{x+3} \geq 0 \\ \frac{x+1}{x+3} = x \end{cases}$$

Segno di $\frac{x+1}{x+3}$

V

$$\begin{cases} \frac{x+1}{x+3} < 0 \\ -\frac{x+1}{x+3} = x \end{cases}$$



$$\begin{cases} x < -3 \vee x \geq -1 \\ \frac{x+1}{x+3} = x \end{cases}$$

V

$$\begin{cases} -3 < x < -1 \\ -\frac{x+1}{x+3} = x \end{cases}$$

$$\begin{aligned} x+1 &= x(x+3) \\ x+1 &= x^2+3x \\ x^2+2x-1 &= 0 \\ x_{1,2} &= -1 \pm \sqrt{2} \end{aligned}$$

$$\begin{aligned} -(x+1) &= x^2+3x \\ -x-1 &= x^2+3x \\ x^2+4x+1 &= 0 \\ x_{1,2} &= -2 \pm \sqrt{3} \end{aligned}$$

$$\begin{cases} x = -1 - \sqrt{2} \vee x = -1 + \sqrt{2} \\ x < -3 \vee x \geq -1 \end{cases}$$

$$x = -1 + \sqrt{2}$$

$$\begin{cases} -3 < x < -1 \\ x = -2 - \sqrt{3} \vee x = -2 + \sqrt{3} \end{cases}$$

Unica soluzione:

$$x = -1 + \sqrt{2}$$

il secondo sistema
non ha soluzioni

Nota: quando due condizioni sono legate da un $\vee$ è sufficiente che ne sia verificata una sola.
Se le condizioni sono messe a sistema, devono essere tutte verificate.

$$\begin{cases} x \geq 1 \vee x < 0 \\ x = 4 \end{cases}$$

$$\begin{cases} x \leq 3 \\ x \geq 1 \\ x = 4 \end{cases}$$

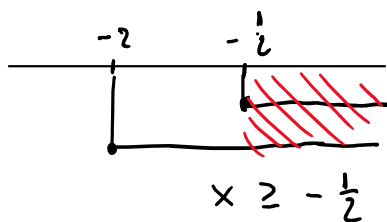
Esercizio 7

$$|2x + 1| \geq x - 1$$

Metodo 1:

$$\begin{cases} 2x + 1 \geq 0 \\ 2x + 1 \geq x - 1 \end{cases}$$

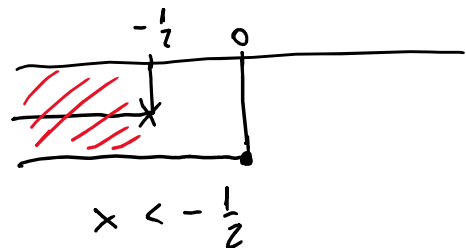
$$\begin{cases} x \geq -\frac{1}{2} \\ x \geq -2 \end{cases}$$



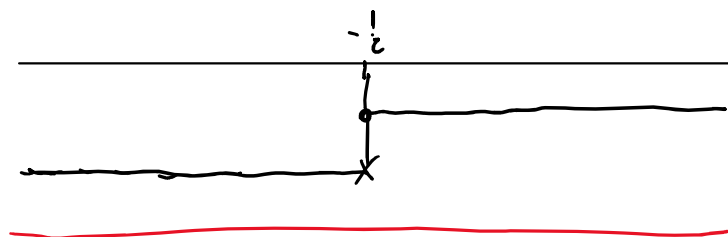
$$\vee \begin{cases} 2x + 1 < 0 \\ -2x - 1 \geq x - 1 \end{cases}$$

$$\begin{cases} x < -\frac{1}{2} \\ -3x \geq 0 \end{cases}$$

$$\begin{cases} x < -\frac{1}{2} \\ x \leq 0 \end{cases}$$



Soluzioni: $x \geq -\frac{1}{2} \quad \vee \quad x < -\frac{1}{2}$



unione

La disequazione è vera $\forall x \in \mathbb{R}$.

Metodo 2:

$$|2x + 1| \geq x - 1$$

$$|a| \geq b \Leftrightarrow a \geq b \vee a \leq -b$$

$$|a| \leq b \Leftrightarrow -b \leq a \leq b$$

$$\Leftrightarrow \begin{cases} a \leq b \\ a \geq -b \end{cases}$$

$$2x + 1 \geq x - 1$$

$$x \geq -2$$

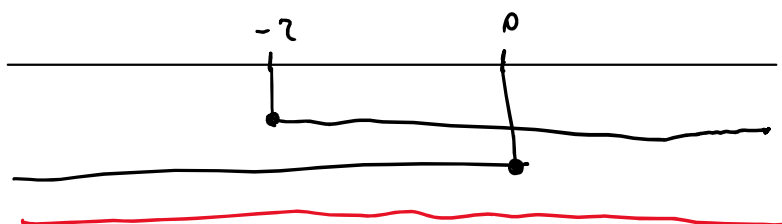
$$\vee \quad 2x + 1 \leq -x + 1$$

$$3x \leq 0$$

$$x \leq 0$$

$$x \geq -2$$

$$\vee \quad x \leq 0$$



Soluzioni: $\forall x \in \mathbb{R}$.

ESERCIZIO 8

$$\frac{|2x - 5| - 2}{3 - x} \geq 0$$

• Numeratore:

$$|2x - 5| - 2 \geq 0$$

$$|2x - 5| \geq 2$$

qui possiamo usare il 1° o il 2° metodo.

$$2x - 5 \geq 2$$

$$2x \geq 7$$

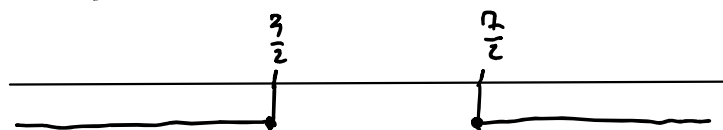
$$x \geq \frac{7}{2}$$

$$\vee$$

$$2x - 5 \leq -2$$

$$2x \leq 3$$

$$x \leq \frac{3}{2}$$



soluzioni di $N \geq 0$

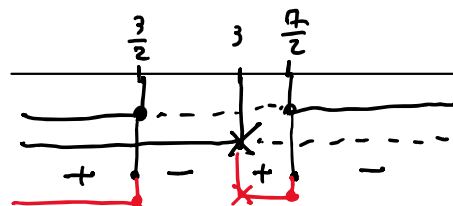
Segno del denominatore:



• Denominatore: $3 - x$



• Segno della frazione:



$$x \leq \frac{3}{2} \quad \vee \quad 3 < x \leq \frac{7}{2}$$

ESERCIZIO 9

$$\frac{4x - 2}{2 - |4 - x|} \leq 0$$

N: $4x - 2 \geq 0 \Leftrightarrow x \geq \frac{1}{2}$

D: $2 - |4 - x| \geq 0$
 $|4 - x| \leq 2$

$$-2 \leq 4 - x \leq 2 \Leftrightarrow \begin{cases} 4 - x \leq 2 \\ 4 - x \geq -2 \end{cases}$$

$$\Leftrightarrow \begin{cases} x \geq 2 \\ x \leq 6 \end{cases}$$

$2 \leq x \leq 6$

Segno di D:

Segno della frazione:

$N \leq 0$

$$\frac{1}{2} \leq x < 2 \quad \vee \quad x > 6.$$

ESERCIZIO 10

$$\bullet \quad \frac{|5x+1| - 3}{x^2 + 2x - 8} < 0$$

$$N: |5x+1| - 3 > 0$$

$$|5x+1| > 3$$

$$5x+1 > 3$$

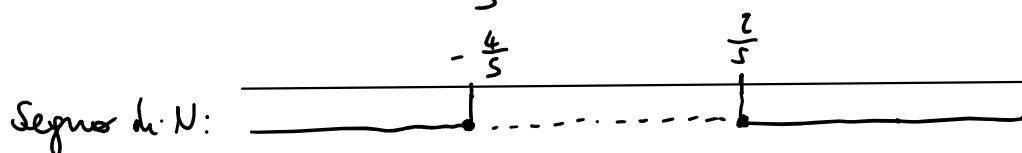
$$5x > 2$$

$$x > \frac{2}{5}$$

$$\vee \quad 5x+1 < -3$$

$$5x < -4$$

$$x < -\frac{4}{5}$$

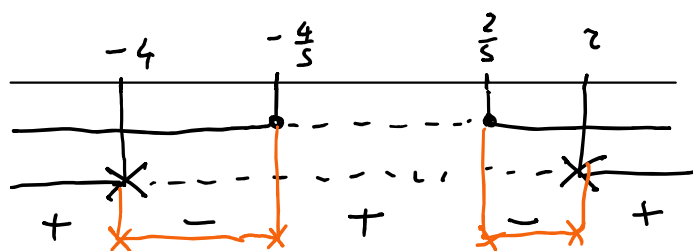


$$D: x^2 + 2x - 8 > 0$$

$$x_{1,2} = -1 \pm 3 = \begin{cases} 2 \\ -4 \end{cases}$$



Fratture:



$$-4 < x < -\frac{4}{5}$$

$$\vee \quad \frac{2}{5} < x < 2.$$