Esercitazione 29/10/2026

giovedì 29 ottobre 2026 13:56

$$\frac{|1-2x|}{5-x} \leq 3+x$$

$$\frac{|1-2x|}{5-x} - 3 - x \leq 0$$

$$\frac{|1-2x| - (3+x)(5-x)}{5-x} \leq 0$$

$$\frac{|1-2x| - (15+2x-x^2)}{5-x} \leq 0$$

$$\frac{|1-2x| + x^2 - 2x - 15}{5-x} \leq 0$$

- $5-x > 0 \iff x < 5$
- $|1-2x| + x^2 - 2x - 15 \geq 0$

$$|1-2x| \geq -x^2 + 2x + 15$$

$\iff$ a) $1-2x \geq -x^2+2x+15$

$\vee$

b) $1-2x \leq +x^2-2x-15$

$|\alpha| \geq \beta$

$\iff \alpha \geq \beta \vee \alpha \leq -\beta$

a) $\iff x^2 - 4x - 14 \geq 0$

$$x_{\pm} = 2 \pm \sqrt{4+14} = 2 \pm \sqrt{18}$$

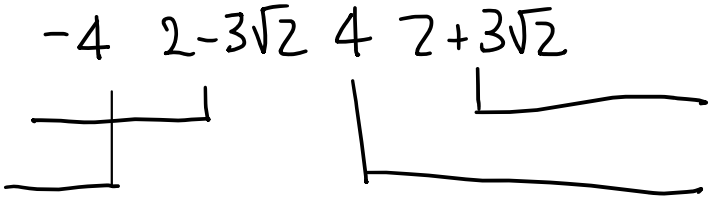
$$= 2 \pm 3\sqrt{2}$$

$x \leq 2-3\sqrt{2} \vee x \geq 2+3\sqrt{2}$

b): $1-7x \leq x^2-7x-15 \iff x^2 - 16 \geq 0$

$x^2 \geq 16$

$\iff x \leq -4 \vee x \geq 4$

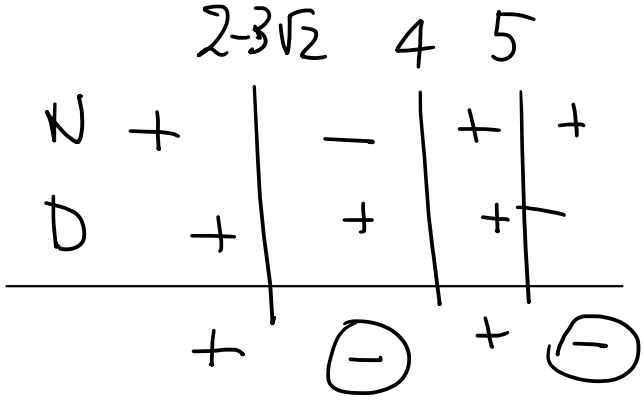


$N \geq 0 \iff x \leq 2-3\sqrt{2} \vee x \geq 4$

$2-3\sqrt{2} \quad 4 \quad 5$

$N \geq 0 \iff x \leq 2-3\sqrt{2} \vee x \geq 4$

$D > 0 \iff x < 5$



$2-3\sqrt{2} \leq x \leq 4 \quad \vee \quad x > 5.$

Strada alternativa:

$|1-2x| \geq -x^2+2x+15$

· se $-x^2+2x+15 \leq 0 \implies$ ok

$x^2-2x-15 \geq 0 \qquad x_{\pm} = 1 \pm \sqrt{1+15} = 1 \pm 4$

$x_{\pm} = 5/-3$

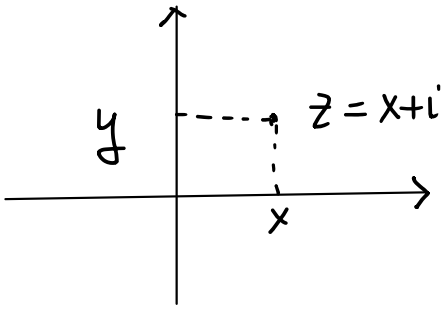
$\iff x \leq -3 \vee x \geq 5.$

$1-2x \geq -x^2+2x+15 \quad \vee \quad 1-2x \leq x^2-2x-15$

· $|x^{28} - 36 \log(x^2+1)| \geq -e^{x^7+8}$ sempre.

Numeri complessi:

$\mathbb{C} = \{x+iy : x, y \in \mathbb{R}\}$



, $x = Re(z)$, $y = Im(z)$

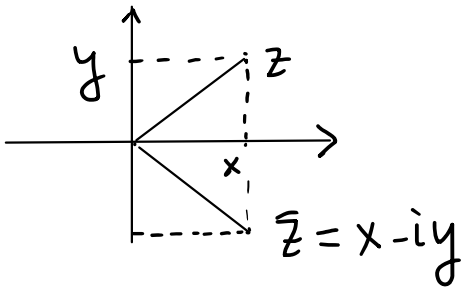
$\forall z \in \mathbb{C}, \exists! w \in \mathbb{C}: \; zw = wz = 1$

$w = \dfrac{\bar{z}}{|z|^2}$; $z = x+iy$, $w = \dfrac{x-iy}{x^2+y^2}$

rappresentazione algebrica (cartesiana).

- $z = \dfrac{1+2i}{1+3i}$; chi è $\bar{z}$, $|z|^2$ e rappr. algebrica?

$z = x+iy \Rightarrow \bar{z} = x-iy$, $|z|^2 = x^2+y^2 = z \cdot \bar{z}$



$z \cdot \bar{z} = (x+iy)(x-iy) =$

$= x^2 - (iy)^2 + ixy - ixy$

$= x^2 - (-y^2) = x^2+y^2$

$z = \dfrac{1+2i}{1+3i} = (1+2i) \cdot \dfrac{1}{1+3i} = (1+2i)\dfrac{1-3i}{1^2+3^2}$

$z = \dfrac{1-3i+2i-6(i^2)}{10} = \dfrac{7-i}{10} = \dfrac{7}{10} - \dfrac{i}{10}$

$z = x+iy, \quad x = \dfrac{7}{10}, \; y = \dfrac{1}{10}$

$\bar{z} = 7 \ldots \ldots 1$

$\overline{z} = \frac{7}{10} + \frac{i}{10}$, $|z|^2 = \frac{49}{100} + \frac{1}{100} = \frac{1}{2} \iff |z| = \frac{1}{\sqrt{2}}$

$z = \frac{1-i}{1+i} = (1-i)\,\frac{1}{1+i} = (1-i)\,\frac{1-i}{1^2+1^2} = \frac{(1-i)^2}{2}$

$= \frac{1+(-i)^2-2i}{2} = \frac{1+i^2-2i}{2} = -\frac{2i}{2} = -i$

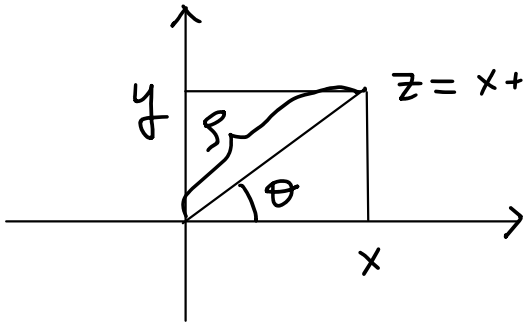
$\overline{z} = i$, $|z| = |-i| = 1$

$z = \frac{a}{b} = a \cdot \frac{1}{b} = a \cdot \frac{\overline{b}}{|b|^2}$

$b = x+iy$, $\frac{1}{b} = \frac{\overline{b}}{|b|^2} \iff b \cdot \frac{\overline{b}}{|b|^2} = 1$

$b \cdot \frac{\overline{b}}{|b|^2} = \frac{b\cdot\overline{b}}{|b|^2} = \frac{|b|^2}{|b|^2} = 1$

rappresentazione esponenziale-trigonometrica



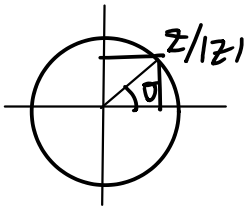
$\begin{cases} x = \rho\cos\theta \\ y = \rho\sin\theta \end{cases}$

$\iff \begin{cases} \rho = \sqrt{x^2+y^2} \\ \tan\theta = \frac{y}{x} \iff \theta = \arctan\frac{y}{x} \end{cases}$

(ρ,θ) = coordinate polari.

$z \neq 0$, $z = |z| \frac{z}{|z|} = \rho(\cos\theta + i\sin\theta)$

$\rho = |z|$, $\frac{z}{|z|} \in S^1 = \{|w|=1\} \Rightarrow \frac{z}{|z|} = (\cos\theta, \sin\theta)$



$\forall z \neq 0$.
$z = \rho(\cos\theta + i\sin\theta)$, $\rho > 0$, $\rho = |z|$, $\theta \in \mathbb{R}$ $[\theta \in [0, 2\pi)]$
forma trigonometrica.

Notazione: $e^{i\theta} := \cos\theta + i\sin\theta$

$z = \rho e^{i\theta}$, $\rho = |z|$, $\theta = \arctan\frac{y}{x}$, $z = x+iy$

<u>prodotti</u>: $z = \rho e^{i\theta}$, $z' = \rho' e^{i\theta'}$

$\Rightarrow z \cdot z' = \rho\rho' e^{i(\theta+\theta')}$.

$z^n = \rho^n e^{in\theta} = \rho^n(\cos(n\theta) + i\sin(n\theta))$, $\forall n \in \mathbb{N}$

$z = 1-i$, $z^4 = ?$

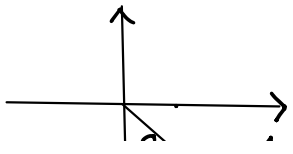
cartesiano $\rightsquigarrow$ polare.

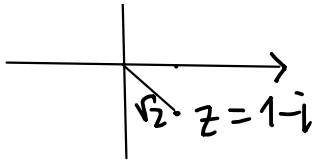
$z = 1-i = \sqrt{2}\left(\frac{1-i}{\sqrt{2}}\right) = \sqrt{2}\left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}\right)$

$|z| = \sqrt{1+1} = \sqrt{2}$

$\overset{!}{=} \sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)$

$= \sqrt{2}\left(\cos\left(\frac{7}{4}\pi\right) + i\sin\left(\frac{7\pi}{4}\right)\right)$





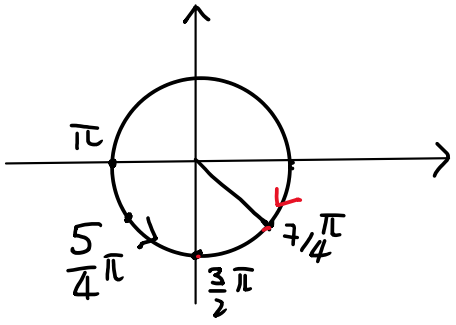
$= \sqrt{2}\left(\cos\left(\frac{7}{4}\pi\right) + i \sin\left(\frac{7\pi}{4}\right)\right)$

$\cos\theta = \frac{1}{\sqrt{2}}, \quad \sin\theta = -\frac{1}{\sqrt{2}}$

$\theta \in [0, 2\pi)$.

$$z = \sqrt{2}\, e^{i\frac{7}{4}\pi}$$

$$z^4 = (\sqrt{2})^4 e^{i\frac{7\pi}{4}\cdot 4} = 4\, e^{i7\pi} = 4\, e^{i\pi} = -4$$



Radici n-esime:

Siano $z, w \in \mathbb{C}$, $n \in \mathbb{N}$, $n \geq 2$. Diciamo che z è una radice n-esima di w se $z^n = w$.

Teorema fondamentale dell'algebra:

Sia p un polinomio di grado $n \geq 1$ con coefficienti complessi. Allora p ha esattamente n radici complesse (contate con molteplicità).

($\Leftrightarrow$ L'equazione $p(x) = 0$ ha esattamente n soluzioni (contate con molteplicità).

$p(x) = (x-1)^2$, $p(x) = 0 \Rightarrow x = 1$

$p(x) = (x-1)^2$, $p(x) = 0 \Rightarrow x = 1$

(1 sola radice con molteplicità 2).

$\Rightarrow$ $z^n = w$ ha esattamente n - soluzioni (contate con molteplicità).

In questo caso ci sono esattamente n soluzioni distinte, ciascuna con molt. 1.

$z = \rho e^{i\theta}$; $(\rho e^{i\theta})^n = w \iff \rho^n e^{in\theta} = w$

$w = r e^{i\alpha} \iff \rho^n e^{in\theta} = r e^{i\alpha}$

dove ρ e θ sono le incognite. (r, α = dati)

$$\begin{cases} \rho^n = r \\ e^{in\theta} = e^{i\alpha} \end{cases} \iff \begin{cases} \rho = r^{1/n} \\ n\theta = \alpha + 2k\pi \quad k \in \mathbb{Z} \end{cases}$$

$$\iff \begin{cases} \rho = r^{1/n} \\ \theta = \dfrac{\alpha + 2k\pi}{n} \end{cases}$$

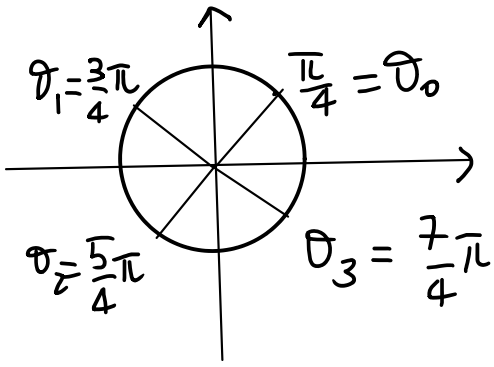
- $z^4 = -4$

$z = \rho e^{i\theta}$, $-4 = 4e^{i\pi}$

$\rho^4 e^{4i\theta} = 4e^{i\pi} \iff \begin{cases} \rho^4 = 4 \iff \rho = \sqrt{2} \\ e^{4i\theta} = e^{i\pi} \iff 4\theta = \pi + 2k\pi \end{cases}$

$$\begin{cases} e^{4i\theta} = e^{i\pi} \iff 4\theta = \pi + 2k\pi \end{cases}$$

$$\iff \begin{cases} \rho = \sqrt{2} \\ \theta_k = \frac{\pi}{4} + \frac{k\pi}{2} \qquad k \in \mathbb{Z} \end{cases}$$



$$z_0 = \sqrt{2}\, e^{i\pi/4}$$

$$z_1 = \sqrt{2}\, e^{i\frac{3\pi}{4}}$$

$$z_2 = \sqrt{2}\, e^{i\frac{5\pi}{4}}$$

$$z_3 = \sqrt{2}\, e^{i\frac{7\pi}{4}}$$

$$z^3 = (1+i)^5 = \left(\sqrt{2}\left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)\right)^5 = 2^{5/2}\left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)^5$$

$$z^3 = 2^{5/2}\left(e^{i\pi/4}\right)^5 \qquad \cos\alpha = \sin\alpha = \frac{1}{\sqrt{2}}$$

$$z^3 = 2^{5/2}\, e^{i\frac{5\pi}{4}} \qquad \alpha = \frac{\pi}{4}$$

- $z = \rho e^{i\theta} \Rightarrow (\rho e^{i\theta})^3 = 2^{5/2}\, e^{i\frac{5\pi}{4}}$

$$\rho^3 e^{i3\theta} = 2^{5/2}\, e^{i\frac{5\pi}{4}}$$

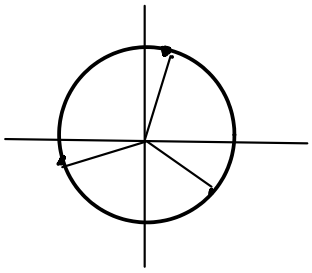
$$\begin{cases} \rho = 2^{5/2 \cdot 1/3} = 2^{5/6} \\ e^{i\cdot 3\theta} = e^{i\frac{5\pi}{4}} \end{cases} \iff 3\theta = \frac{5\pi}{4} + 2\pi k$$

$\Rightarrow \quad \theta_k = \frac{5}{12}\pi + \frac{2k\pi}{3} \qquad k \in \mathbb{Z}$

$\theta_0 = \frac{5}{12}\pi$, $\quad \theta_1 = \frac{5}{12}\pi + \frac{2\pi}{3} = \frac{5+8}{12}\pi = \frac{13\pi}{12}$

$\theta_2 = \frac{5}{12}\pi + \frac{2\cdot 2}{3}\pi = \left(\frac{5}{12} + \frac{16}{12}\right)\pi = \frac{21\pi}{12}$

$\frac{29}{12} = \frac{5}{12} + 2$



$z_0 = 2^{5/6} e^{i\frac{5\pi}{12}}$, $z_1 = 2^{5/6} e^{i\frac{13}{12}\pi}$

$z_2 = 2^{5/6} e^{i\frac{21}{12}\pi}$

$w = \frac{3+i}{1-i}$; forma algebrica, calcolare $(w-3i)^3$

$w = (3+i)\ \frac{1}{1-i} = 3+i \cdot \frac{1+i}{2} = \frac{3+3i+i-1}{2} = \frac{2+4i}{2}$

$w = 1+2i$

$\Rightarrow (w-3i)^3 = (1+2i-3i)^3 = (1-i)^3$

$= \left(\left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}\right)\sqrt{2}\right)^3 = \left(\sqrt{2}\, e^{i\frac{7\pi}{4}}\right)^3$

$= 2^{3/2} e^{\frac{21}{4}\pi i}$

$w = \dfrac{3-2i}{2+3i}$; forma algebrica, radici cubiche.

$w = (3-2i)\dfrac{1}{2+3i} = (3-2i)\dfrac{2-3i}{4+9} = \dfrac{6-9i-4i-6}{13}$

$w = -i$

$z^3 = -i \iff \rho^3 e^{3i\theta} = e^{\frac{2\pi}{3}}$

$\rho = 1, \quad e^{3i\theta} = e^{\frac{2\pi}{3}} \iff 3\theta = \dfrac{2\pi}{3} + 2k\pi$

$\iff \theta = \dfrac{2}{9}\pi + \dfrac{2k\pi}{3}$

$\theta_0 = \dfrac{2}{9}\pi, \quad \theta_1 = \dfrac{2\pi}{9} + \dfrac{2}{3}\pi = \dfrac{8}{9}\pi$

$\theta_2 = \left(\dfrac{2}{9} + \dfrac{4}{3}\right)\pi = \dfrac{14}{9}\pi$

$z_0 = e^{i\frac{2\pi}{9}}, \quad z_1 = e^{i\frac{8\pi}{9}}, \quad z_2 = e^{i\frac{14}{9}\pi}$

$w = \dfrac{8+i}{2-3i}$ forma algebrica, $(w-1)^6$