Esercitazione 22/10/2026

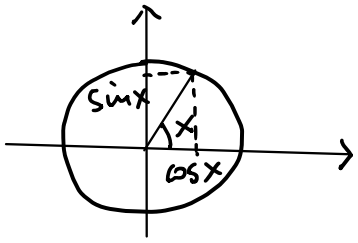
giovedì 22 ottobre 2026 13:57

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Matteo Rizzi, ufficio 9, II piano matematica.

- $\cos^2 x + \sin^2 x = 1 \quad , \forall x \in \mathbb{R}$.

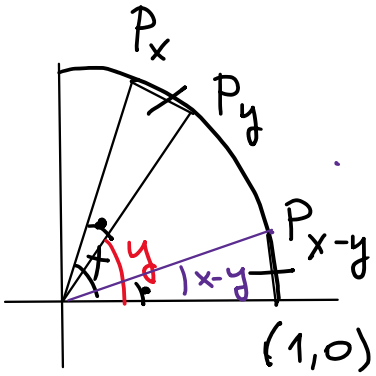


teorema di Pitagora

- $x, y \in \mathbb{R}$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$



$P_x = (\cos x, \sin x)$, $P_y = (\cos y, \sin y)$

$P_{x-y} = (\cos(x-y), \sin(x-y))$

$$|P_x - P_y|^2 = |P_{x-y} - (1,0)|^2$$

$$|(\cos x, \sin x) - (\cos y, \sin y)|^2 = |(\cos(x-y), \sin(x-y)) - (1,0)|^2$$

$$(\cos x - \cos y)^2 + (\sin x - \sin y)^2 = (\cos(x-y) - 1)^2 + \sin^2(x-y)$$

$$(\cos x-\cos y)^2+(\sin x-\sin y)^2=(\cos(x-y)-1)^2+\sin^2(x-y)$$

$$\cos^2x-2\cos x\cos y+\cos^2y+\sin^2x-2\sin x\sin y+\sin^2y=$$

$$=\cos^2(x-y)-2\cos(x-y)+1+\sin^2(x-y)$$

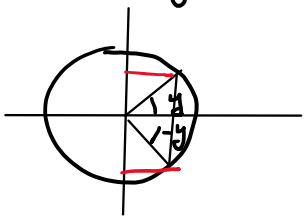
$$1+1-2\cos x\cos y-2\sin x\sin y=1+1-2\cos(x-y)$$

$$\cos(x+y)=\cos(x-(-y))=$$

$$=\cos x\cos(-y)+\sin x\sin(-y)$$

$$=\cos x\cos y-\sin x\sin y$$

$\cos(-y)=\cos y$



$$\sin\alpha=\cos\left(\frac{\pi}{2}-\alpha\right)$$

$$\sin(x+y)=\cos\left(\frac{\pi}{2}-(x+y)\right)=\cos\left(\frac{\pi}{2}-x-y\right)=$$

$$=\cos\left(\frac{\pi}{2}-x\right)\cos y+\sin\left(\frac{\pi}{2}-x\right)\sin y$$

$$=\sin x\cos y+\cos x\sin y$$

$$\sin(x-y)=\sin(x+(-y))=\sin x\cos(-y)+\cos x\sin(-y)$$

$$=\sin x\cos y-\cos x\sin y$$

- Duplicazione:

$$\sin 2x=\sin(x+x)=\sin x\cos x+\sin x\cos x$$

$$\boxed{\sin 2x=2\sin x\cos x}$$

$$\cos 2x = \cos(x+x) = \cos x \cdot \cos x - \sin x \cdot \sin x$$

$$\boxed{\cos 2x = \cos^2 x - \sin^2 x}$$

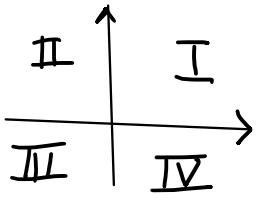
Remark: $\cos 2x = 1 - \sin^2 x - \sin^2 x = 1 - 2\sin^2 x$

$$\iff 2\sin^2 x = 1 - \cos 2x \iff \sin^2 x = \frac{1-\cos 2x}{2}$$

$$\iff \sin x = \pm\sqrt{\frac{1-\cos 2x}{2}}$$

$+$ se $x \in$ I o II

$-$ se $x \in$ III o IV



$$\boxed{\sin\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-\cos x}{2}}}$$ formula di bisezione del seno

$$\cos 2x = \cos^2 x - \sin^2 x = \cos^2 x - (1 - \cos^2 x)$$
$$= 2\cos^2 x - 1$$

$$\cos 2x + 1 = 2\cos^2 x \iff \frac{\cos 2x + 1}{2} = \cos^2 x$$

$$\iff \cos x = \pm\sqrt{\frac{1+\cos 2x}{2}}$$

$+$ se $x \in$ I o IV

$-$ se $x \in$ II o III

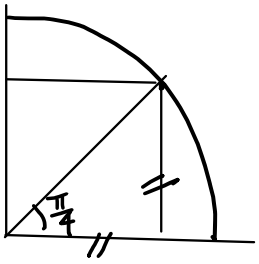
$$\boxed{\cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1+\cos x}{2}}}$$ formula di bisezione del coseno

Calcolo di seno - coseno di angoli "non noti":

Calcolo di seno - coseno di angoli "non noti":

$$\cos\left(\frac{\pi}{12}\right) = \cos\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = \cos\frac{\pi}{4}\cos\frac{\pi}{6} + \sin\frac{\pi}{4}\sin\frac{\pi}{6}$$

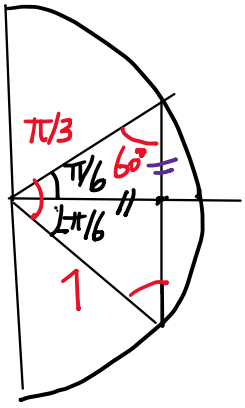
$$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}\,\frac{1}{2} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$



- $\sin^2\frac{\pi}{4} + \cos^2\frac{\pi}{4} = 1$
- $\sin^2\frac{\pi}{4} = \cos^2\frac{\pi}{4}$

$$\Leftrightarrow 2\cos^2\frac{\pi}{4} = 1$$

$$\Leftrightarrow \cos^2\left(\frac{\pi}{4}\right) = \frac{1}{2} \Leftrightarrow \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$



$$= = \frac{1}{2} = \sin\frac{\pi}{6}$$

$$\cos^2\left(\frac{\pi}{6}\right) = 1 - \sin^2\frac{\pi}{6} = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

$$\sin\left(\frac{\pi}{12}\right) = \sin\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = \sin\frac{\pi}{4}\cos\frac{\pi}{6} - \cos\frac{\pi}{4}\sin\frac{\pi}{6}$$

$$= \frac{1}{\sqrt{2}}\,\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}\,\frac{1}{2} = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

$$\tan(\pi/12) = \frac{\sin(\pi/12)}{\cos(\pi/12)} = \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$\cos\left(\frac{\pi}{8}\right) = \cos\left(\frac{1}{2}\cdot\frac{\pi}{4}\right) = \sqrt{\frac{1+\cos(\pi/4)}{2}} = \sqrt{\frac{1+1/\sqrt{2}}{2}}$$

$$= \sqrt{\frac{1}{2}\,\frac{\sqrt{2}+1}{\sqrt{2}}} = \sqrt{\frac{\sqrt{2}(\sqrt{2}+1)}{4}} = \frac{\sqrt{2+\sqrt{2}}}{2}$$

$$\sin\left(\frac{\pi}{8}\right) = \sqrt{\frac{1-\cos(\pi/4)}{2}} = \sqrt{\frac{1-1/\sqrt{2}}{2}} = \sqrt{\frac{1}{2}\,\frac{\sqrt{2}-1}{\sqrt{2}}}$$

$$= \sqrt{\frac{\sqrt{2}(\sqrt{2}-1)}{4}} = \frac{\sqrt{2-\sqrt{2}}}{2}$$

Formule di prostaferesi:

$$\sin\alpha+\sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\sin\alpha-\sin\beta = 2\sin\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$$

$$\cos\alpha+\cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

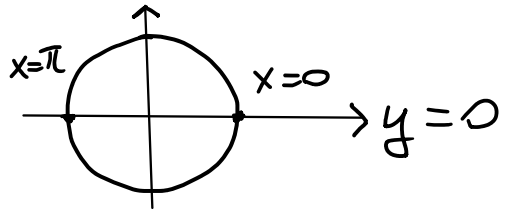
$$\cos\alpha-\cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

Equazioni trigonometriche:

- $\sin x = 0$

$x = 0+2k\pi \;\vee\; x = \pi+2k\pi$

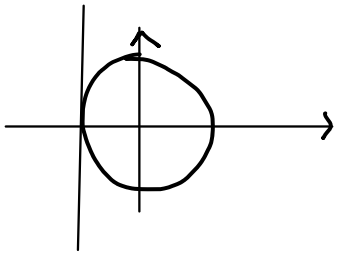
$\Leftrightarrow \quad x = k\pi, \quad k\in\mathbb{Z}.$



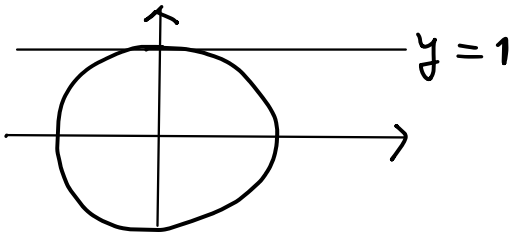
- $\cos x = -1$

$\Leftrightarrow \quad x = \pi+2k\pi, \quad k\in\mathbb{Z}$

· $\cos x = -1 \iff x = \pi + 2k\pi, \quad k \in \mathbb{Z}$

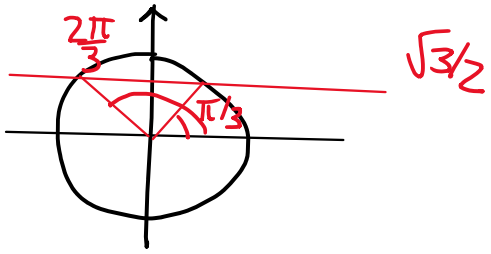


· $\sin x = 1$ $\qquad x = \frac{\pi}{2} + 2k\pi, \quad k \in \mathbb{Z}$



· $\sin x = 2$ impossibile.

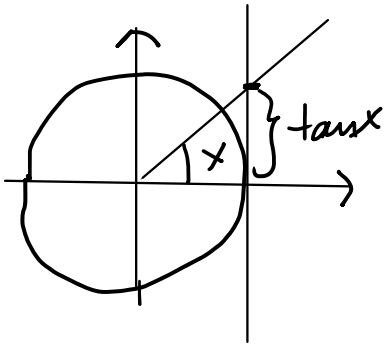
· $2\sin x = \sqrt{3}$ $\qquad \sin x = \frac{\sqrt{3}}{2}$



$\iff x = \frac{\pi}{3} + 2k\pi$

$\vee \quad x = \frac{2}{3}\pi + 2k\pi, \quad k \in \mathbb{Z}$

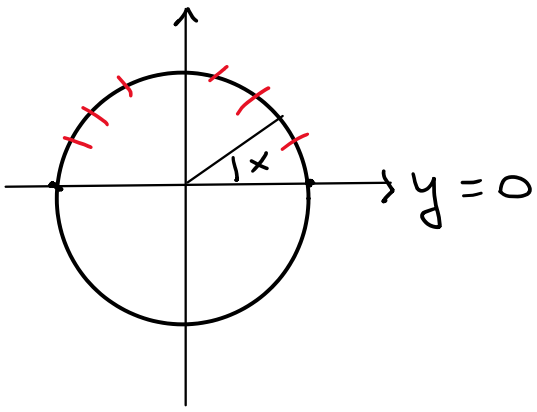
· $\tan x = 1$ $\qquad \iff x = \frac{\pi}{4} + k\pi, \quad k \in \mathbb{Z}$



· Disequazioni goniometriche:

· $\sin x \geq 0$ $\qquad 0 + 2k\pi \leq x \leq \pi + 2k\pi$

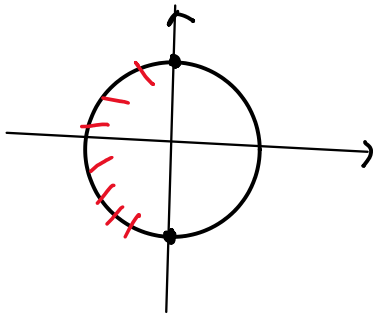
· $\sin x \geq 0$



$0 + 2k\pi \leq x \leq \pi + 2k\pi$

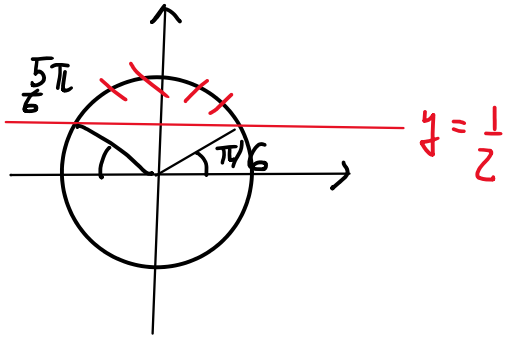
$2k\pi \leq x \leq \pi + 2k\pi$

· $\cos x < 0$



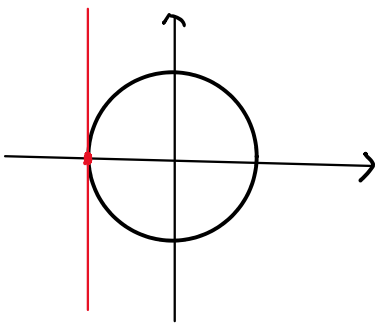
$\frac{\pi}{2} + 2k\pi < x < \frac{3}{2}\pi + 2k\pi$

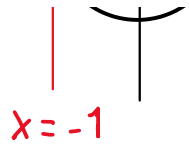
· $\sin x > \frac{1}{2} \iff \frac{\pi}{6} + 2\pi k < x < \frac{5}{6}\pi + 2\pi k, \quad k \in \mathbb{Z}.$



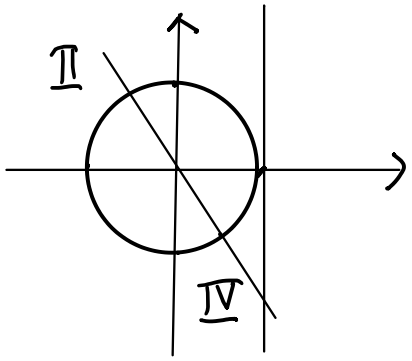
· $\cos x < -1$ Impossibile.

· $\cos x \leq -1 \iff x = \pi + 2k\pi \quad k \in \mathbb{Z}$





· $\tan x \leq 0$



$\frac{\pi}{2} + k\pi < x \leq \pi + k\pi$

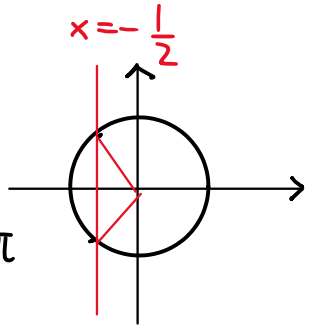
· $\sin x + \sin 2x = 0$

$\sin x + 2\sin x \cos x = 0 \iff \sin x (1 + 2\cos x) = 0.$

$\iff \sin x = 0 \quad \vee \quad 1 + 2\cos x = 0$

$\iff x = k\pi \quad \vee \quad \cos x = -\frac{1}{2}$

$\iff x = k\pi \quad \vee \quad x = \frac{2}{3}\pi + 2k\pi \vee x = \frac{4}{3}\pi + 2k\pi$



· $\cos^2 x + \sin x = 1$

$1 - \sin^2 x + \sin x = 1 \iff \sin x - \sin^2 x = 0$

$\iff \sin x (1 - \sin x) = 0 \iff \sin x = 0 \vee 1 - \sin x = 0$

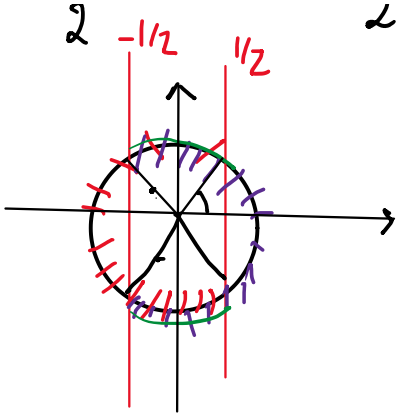
$\iff x = k\pi \vee \sin x = 1 \iff x = k\pi \vee x = \frac{\pi}{2} + 2k\pi.$

· $3\cos^2 x - \sin^2 x \leq 0 \iff 3\cos^2 x - (1 - \cos^2 x) \leq 0$

$\iff 4\cos^2 x - 1 \leq 0 \iff \cos^2 x \leq \frac{1}{4} \iff$

$-\frac{1}{2} \leq \cos x \leq \frac{1}{2} \qquad \left[t^2 \leq \frac{1}{4} \iff -\frac{1}{2} \leq t \leq \frac{1}{2} \right]$





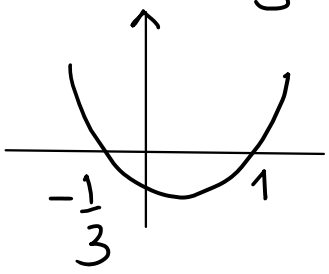
$\Leftrightarrow \quad \cos x \le \frac{1}{2} \wedge \cos x \ge -\frac{1}{2}$

$\frac{\pi}{3} + 2\pi k \le x \le \frac{2}{3}\pi + 2\pi k \vee \frac{4}{3}\pi + 2\pi k \le x \le \frac{5}{3}\pi + 2\pi k$

$3\cos^2 x - 2\cos x - 1 > 0 \qquad \cos x = t$

$3t^2 - 2t - 1 > 0 \quad \Leftrightarrow \quad t < -\frac{1}{3} \vee t > 1$

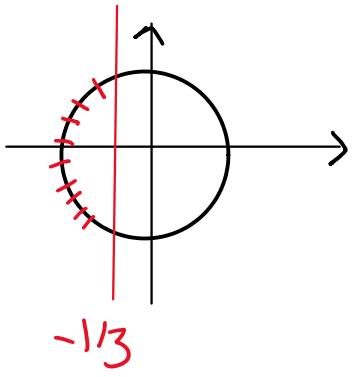
$t_{\pm} = \frac{1 \pm \sqrt{1+3}}{3} = \frac{1 \pm 2}{3} = 1 / -\frac{1}{3}$



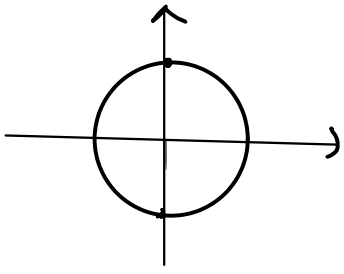
$\Leftrightarrow \quad \cos x < -\frac{1}{3} \vee \cos x > 1$

$\Leftrightarrow \quad \arccos\left(-\frac{1}{3}\right) + 2\pi k < x < 2\pi - \arccos\left(-\frac{1}{3}\right) + 2\pi k$

$k \in \mathbb{Z}.$



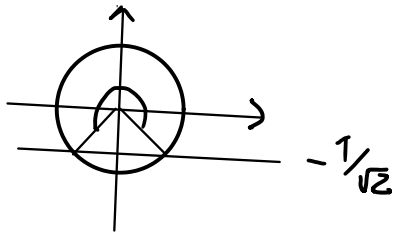
$\frac{2\sin x + \sqrt{2}}{\cos x} \le 0$



- $\cos x = 0 \quad \Leftrightarrow \quad x = \frac{\pi}{2} + \pi k, \quad k \in \mathbb{Z}.$

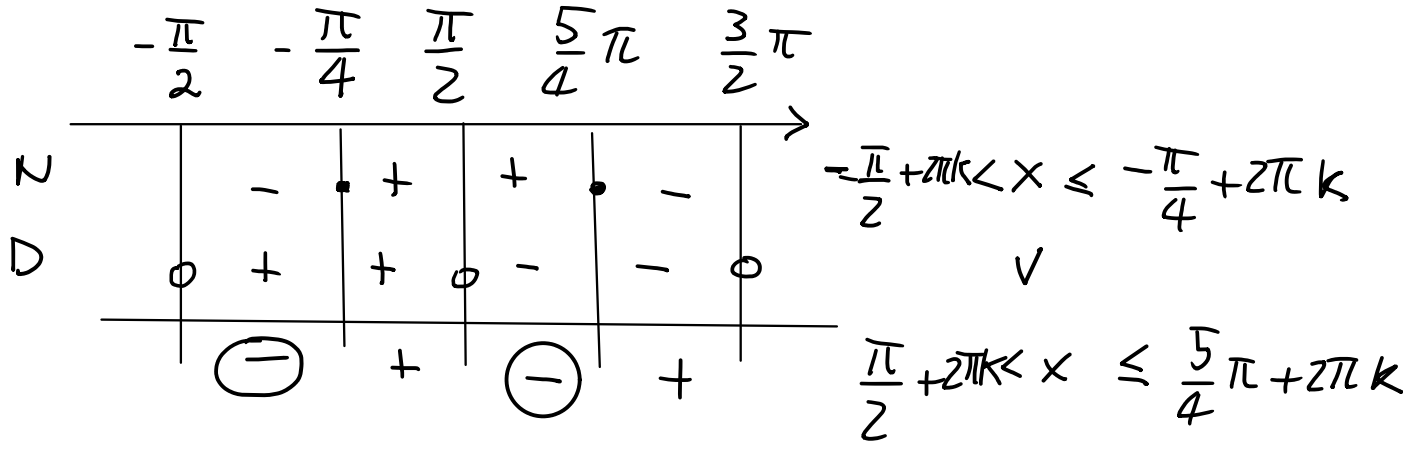
- $2\sin x + \sqrt{2} \ge 0 \quad \Leftrightarrow \quad \sin x \ge -\frac{\sqrt{2}}{2} = -\frac{\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}} = -\frac{1}{\sqrt{2}}$

· $2\sin x + \sqrt{2} \geq 0 \iff \sin x \geq -\frac{\sqrt{2}}{2} = -\frac{\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}} = -\frac{1}{\sqrt{2}}$

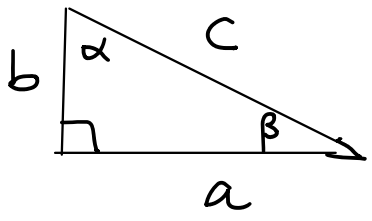


$-\frac{\pi}{4} + 2\pi k \leq x \leq \frac{5}{4}\pi + 2\pi k$

· $\cos x > 0 \iff -\frac{\pi}{2} + 2\pi k < x < \frac{\pi}{2} + 2\pi k.$



Teoremi sui triangoli

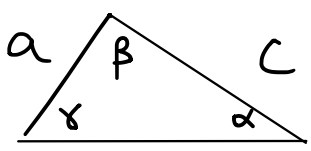


$b = c\cos\alpha = c\sin\beta$

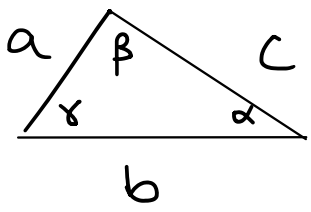
$a = c\cos\beta = c\sin\alpha$

$\frac{b}{a} = \frac{c\sin\beta}{c\cos\beta} = \tan\beta \; ; \; \frac{a}{b} = \tan\alpha$

Teorema dei seni



$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$



$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

Teorema di Carnot (del coseno)

$$a^2 = b^2 + c^2 + 2bc\cos\alpha$$

se $\alpha = \frac{\pi}{2} \Rightarrow a^2 = b^2 + c^2$