

MATEMATICA - LEZIONE 9

giovedì 15 ottobre 2026

14:08

ESE 1

Risolvere $\sqrt{x+5} = 1-x$

$$\begin{cases} x+5 \geq 0 \\ 1-x \geq 0 \\ x+5 = (1-x)^2 \end{cases}$$

$$\begin{cases} 1-x \geq 0 \\ x+5 = (1-x)^2 \end{cases}$$

$$x \leq 1$$

$$x+5 = 1 - 2x + x^2$$

$$x^2 - 3x - 4 = 0$$

$$\Delta = 9 + 16 = 25$$

$$x_{1,2} = \frac{3 \pm 5}{2} = \begin{cases} 4 \\ -1 \end{cases}$$

$$\begin{cases} x \leq 1 \\ x \neq 4 \vee x = -1 \end{cases}$$

$x = -1$ è l'unica soluzione.

ESE 2

$$\sqrt{2x^2-1} = 2x^2-3$$

$$\begin{cases} 2x^2-1 \geq 0 \\ 2x^2-3 \geq 0 \\ 2x^2-1 = (2x^2-3)^2 \end{cases}$$

$$\begin{cases} 2x^2-3 \geq 0 & \textcircled{1} \\ 2x^2-1 = (2x^2-3)^2 & \textcircled{2} \end{cases}$$

$$\textcircled{1} \quad 2x^2-3 \geq 0 \iff x \leq -\sqrt{\frac{3}{2}} \vee x \geq \sqrt{\frac{3}{2}}$$

$$\textcircled{2} \quad 2x^2-1 = 4x^4 - 12x^2 + 9$$

$$4x^4 - 14x^2 + 10 = 0$$

$$2x^4 - 7x^2 + 5 = 0$$

$$y = x^2$$

$$2y^2 - 7y + 5 = 0$$

$$\Delta = 49 - 40 = 9$$

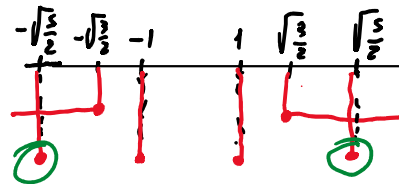
$$y_{1,2} = \frac{7 \pm 3}{4} = \begin{cases} \frac{10}{4} = \frac{5}{2} \\ 1 \end{cases}$$

$$y = \frac{5}{2} \quad \vee \quad y = 1$$

$$x^2 = \frac{5}{2} \quad \vee \quad x^2 = 1$$

$$x = \pm \sqrt{\frac{5}{2}} \quad \vee \quad x = \pm 1$$

$$\begin{cases} x \leq -\sqrt{\frac{3}{2}} & \vee & x \geq \sqrt{\frac{3}{2}} \\ x = \pm \sqrt{\frac{5}{2}} & \vee & x = \pm 1 \end{cases}$$



$$x = \pm \sqrt{\frac{5}{2}}$$

ESE 3

$$\sqrt{x^2 + x + 2} \geq 2x$$

$$\textcircled{1} \begin{cases} \cancel{2x} \geq 0 \\ x^2 + x + 2 \geq 0 \\ x^2 + x + 2 \geq 4x^2 \end{cases} \quad \vee \quad \begin{cases} \cancel{2x} < 0 \\ x^2 + x + 2 \geq 0 \end{cases} \textcircled{2}$$

oss $x^2 + x + 2 \geq 0$ $\forall x \in \mathbb{R}$.
 $\Delta = 1 - 8 = -7 < 0$

$$\textcircled{1} \begin{cases} x \geq 0 \\ x^2 + x + 2 \geq 4x^2 \end{cases} \quad \vee \quad x < 0 \quad \textcircled{2}$$

$$\rightarrow 3x^2 - x - 2 \leq 0$$

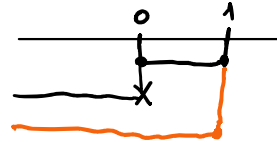
$$x_{1,2} = \frac{1 \pm \sqrt{1+24}}{6} = \frac{1}{6} \quad -\frac{4}{6} = -\frac{2}{3}$$

$$-\frac{2}{3} \leq x \leq 1$$

$$\begin{cases} x \geq 0 \\ -\frac{2}{3} \leq x \leq 1 \end{cases} \quad \text{V} \quad x < 0$$

$$0 \leq x \leq 1 \quad \text{V} \quad x < 0$$

Soluzione: $x \leq 1$.



Nota Se assumiamo aiuto: $\sqrt{x^2+x+2} \geq -x$

$$\begin{cases} -2x \geq 0 \\ x^2+x+2 \geq 0 \\ x^2+x+2 \geq (-2x)^2 \end{cases} \quad \text{V} \quad \begin{cases} -2x < 0 \\ x^2+x+2 \geq 0 \end{cases}$$

$$\begin{cases} x \leq 0 \\ -\frac{2}{3} \leq x \leq 1 \end{cases} \quad \text{V} \quad x > 0$$

$$-\frac{2}{3} \leq x \leq 0 \quad \text{V} \quad x > 0$$

$$x \geq -\frac{2}{3}$$

ESE 4

$$2 + \sqrt{2x+1} \leq x$$

$$\sqrt{2x+1} \leq x-2$$

$$\begin{cases} 2x+1 \geq 0 \rightarrow x \geq -\frac{1}{2} \\ x-2 \geq 0 \rightarrow x \geq 2 \\ 2x+1 \leq (x-2)^2 \end{cases}$$

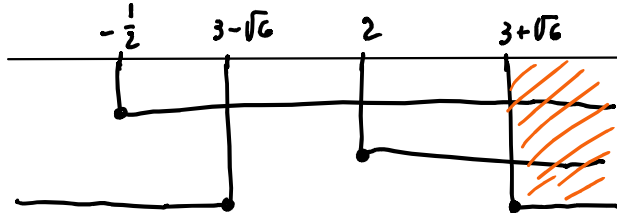
$$\rightarrow 2x+1 \leq x^2-4x+4$$

$$x^2-6x+3 \geq 0$$

$$x_{1,2} = 3 \pm \sqrt{6}$$

$$x \leq 3 - \sqrt{6} \quad \vee \quad x \geq 3 + \sqrt{6}$$

$$\begin{cases} x \geq -\frac{1}{2} \\ x \geq 2 \\ x \leq 3 - \sqrt{6} \quad \vee \quad x \geq 3 + \sqrt{6} \end{cases}$$



Soluzione finale: $x \geq 3 + \sqrt{6}$

ESE 5

$$\frac{\sqrt{2x+4}}{3x+2} < 1$$

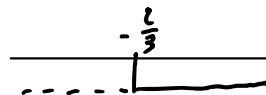
$$\frac{\sqrt{2x+4}}{3x+2} - 1 < 0$$

$$\frac{\sqrt{2x+4} - (3x+2)}{3x+2} < 0$$

$$\begin{aligned} 2x+4 &\geq 0 \\ x &\geq -2 \end{aligned}$$

Studiamo il segno di numeratore e denominatore.

$$D: 3x+2 > 0 \Leftrightarrow x > -\frac{2}{3}$$



$$N: \sqrt{2x+4} - (3x+2) > 0$$

$$\sqrt{2x+4} > 3x+2$$

$$\textcircled{1} \begin{cases} 3x+2 \geq 0 \\ 2x+4 \geq 0 \\ 2x+4 > (3x+2)^2 \end{cases} \quad \vee \quad \begin{cases} 3x+2 < 0 \\ 2x+4 \geq 0 \end{cases}$$

$$\textcircled{1} \begin{cases} 3x+7 \geq 0 \rightarrow x \geq -\frac{7}{3} \\ 2x+4 > 9x^2+12x+4 \end{cases}$$

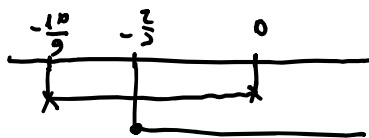
$$9x^2+10x < 0$$

$$x(9x+10) < 0$$

$$x_1=0, x_2=-\frac{10}{9}$$

$$-\frac{10}{9} < x < 0$$

$$\begin{cases} x \geq -\frac{7}{3} \\ -\frac{10}{9} < x < 0 \end{cases}$$



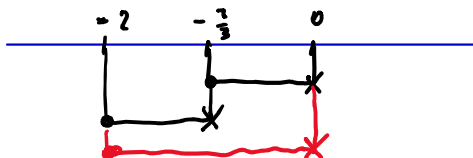
$$-\frac{7}{3} \leq x < 0$$

$$\textcircled{2} \begin{cases} 3x+2 < 0 \\ 2x+4 \geq 0 \end{cases} \Leftrightarrow \begin{cases} x < -\frac{2}{3} \\ x \geq -2 \end{cases} \Leftrightarrow -2 \leq x < -\frac{2}{3}$$

Soluzione finale, delle disequazioni al numeratore:

$$-\frac{7}{3} \leq x < 0 \quad \vee \quad -2 \leq x < -\frac{2}{3}$$

$$\boxed{-2 \leq x < 0}$$

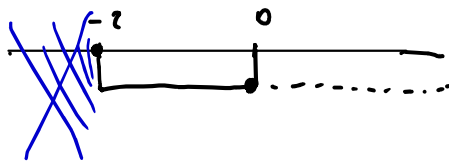


Conclusione:

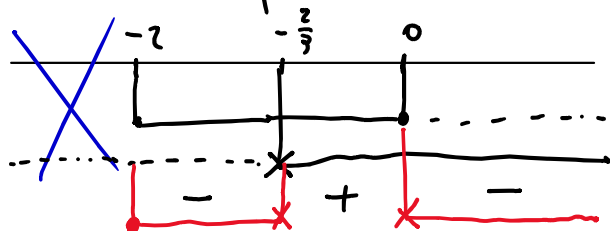
$$N: N > 0 \Leftrightarrow -2 \leq x < 0$$

$$\sqrt{2x+4} - (3x+7)$$

$$\text{c.e. } x \geq -2$$



Segno della frazione



Soluzione: $-2 \leq x < -\frac{2}{3} \quad \vee \quad x > 0.$

Logaritmi

$$\bullet \log_4 2 = \log_4 \sqrt{4} = \log_4 4^{\frac{1}{2}} = \frac{1}{2}$$

$$\bullet \log_{10} \sqrt[11]{10} = \log_{10} (10^{\frac{1}{11}}) = \frac{1}{11}$$

$$\begin{aligned} \bullet \log_{27} \sqrt{3} &= \log_{27} 3^{\frac{1}{2}} = \log_{27} \left((27^{\frac{1}{3}})^{\frac{1}{2}} \right) \\ &= \log_{27} (27^{\frac{1}{6}}) = \frac{1}{6} \end{aligned}$$

$$\log_{27} \sqrt{3} = \frac{\log_3 \sqrt{3}}{\log_3 27} = \frac{\frac{1}{2}}{3} = \frac{1}{6}$$

$$\bullet \log_a 1 = 0 \quad (\log_a 1 = 0)$$

$$\begin{aligned} \bullet \log_{\sqrt{2}} 64 &= \log_{\sqrt{2}} 2^6 = \log_{\sqrt{2}} ((\sqrt{2})^2)^6 = \log_{\sqrt{2}} (\sqrt{2}^{12}) = 12 \\ &\quad \searrow \frac{\log_2 2^6}{\log_2 \sqrt{2}} = \frac{6}{\frac{1}{2}} = 6 \cdot 2 = 12 \end{aligned}$$

ESE 6

$$\bullet 3^{x+1} = \frac{1}{\sqrt[5]{3}}$$

$$\log_3 3^{x+1} = \log_3 \left(\frac{1}{\sqrt[5]{3}} \right)$$

$$x+1 = -\frac{1}{5}$$

$$x = -1 - \frac{1}{5}$$

$$x = -\frac{6}{5}$$

ESE 7

$$\left(\frac{1}{2}\right)^x < \sqrt{2}$$

$$\log_{\frac{1}{2}} \left(\frac{1}{2}\right)^x > \log_{\frac{1}{2}} \sqrt{2}$$

$$x > -\frac{1}{2}$$

$0 < \frac{1}{2} < 1$, se passiamo
ai log, bisogna invertire
la disuguaglianza

$$\left(\log_{\frac{1}{2}} \sqrt{2} = \frac{\log_2 \sqrt{2}}{\log_2 \frac{1}{2}} = \frac{\frac{1}{2}}{-1} = -\frac{1}{2} \right)$$

Appure:

$$\left(\frac{1}{2}\right)^x < \sqrt{2}$$

$$2 > 1.$$

$$\log_2 \left(\frac{1}{2}\right)^x < \log_2 \sqrt{2}$$

$$x \log_2 \frac{1}{2} < \frac{1}{2}$$

$$-x < \frac{1}{2}$$

$$x > -\frac{1}{2}.$$

ESE 8

$$4^x - 5 \cdot 2^x + 4 > 0$$

$$y = 2^x \quad \left(y^2 = (2^x)^2 = 2^{2x} = (2^2)^x = 4^x \right)$$

$$y^2 - 5y + 4 > 0$$

$$y_{1,2} = \frac{5 \pm \sqrt{25-16}}{2} = \begin{matrix} 4 \\ 1 \end{matrix}$$

$$y < 1 \quad \vee \quad y > 4$$

$$2^x < 1 \quad \vee \quad 2^x > 4$$

$$x < \log_2 1 \quad \vee \quad x > \log_2 4$$

$$x < 0 \quad \vee \quad x > 2.$$

ESE 9

$$\cdot \log_3^2 x + \log_3 x \geq 0.$$

$$\text{c.e.: } x > 0.$$

$$y = \log_3 x$$

$$y^2 + y \geq 0$$

$$y(y+1) \geq 0$$

$$y \leq -1 \quad \vee \quad y \geq 0$$

$$\log_3 x \leq -1 \quad \vee \quad \log_3 x \geq 0$$

$$3^{\log_3 x} \leq 3^{-1} \quad \vee \quad 3^{\log_3 x} \geq 3^0$$

$$x \leq \frac{1}{3} \quad \vee \quad x \geq 1$$

$$\begin{cases} x > 0 \\ x \leq \frac{1}{3} \quad \vee \quad x \geq 1 \end{cases}$$

$$0 < x \leq \frac{1}{3} \quad \vee \quad x \geq 1.$$

Regole da ricordare

$$\cdot a^x = b > 0$$

$$\log_a a^x = \log_a b$$

$$x = \log_a b$$

$$\log_a x = b \quad \text{c.e. } x > 0$$

$$a^{\log_a x} = a^b$$

$$x = a^b$$

ESE 10

$$\log_{\frac{1}{3}} x \geq 4$$

$$\text{c.e. } x > 0.$$

$$\left(\frac{1}{3}\right)^{\log_{\frac{1}{3}} x} \leq \left(\frac{1}{3}\right)^4$$

$$x \leq \frac{1}{81}$$

Soluzione:

$$0 < x \leq \frac{1}{81}$$

ESE 11

a) Risolvere $\frac{|x-4| - 3}{1-2x} \geq 0$

b) Risolvere $\frac{|\ln x - 4| - 3}{1-2\ln x} \geq 0.$

a) N: $|x-4| - 3 \geq 0$

$$|x-4| \geq 3$$

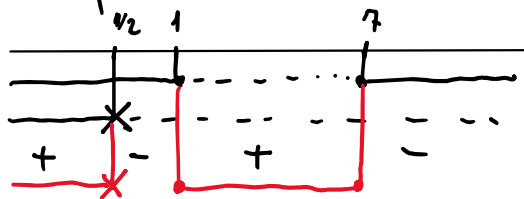
$$x-4 \geq 3 \quad \vee \quad x-4 \leq -3.$$

$$x \geq 7 \quad \vee \quad x \leq 1$$

Segno del numeratore:



Segno della frazione:



$$x < \frac{1}{2} \quad \vee \quad 1 \leq x \leq 7$$

b) $\frac{|\ln x - 4| - 3}{1-2\ln x} \geq 0. \quad \text{c.e. } \underline{x > 0}$

Sostituzione: $y = \ln x \quad (\ln = \log_e)$

$$\frac{|y-4| - 3}{1-2y} \geq 0$$

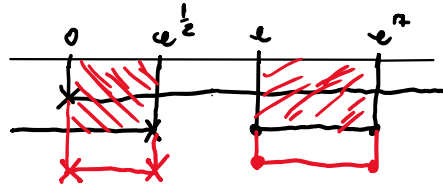
$$y < \frac{1}{2} \quad \vee \quad 1 \leq y \leq 7.$$

$$e^{\ln x} < e^{\frac{1}{2}}$$

$$\ln x < \frac{1}{2} \quad \vee \quad 1 \leq \ln x \leq 7.$$

$$x < e^{\frac{1}{2}} \quad \vee \quad e^1 \leq x \leq e^7$$

$$\begin{cases} x > 0 \\ x < e^{\frac{1}{2}} \vee e \leq x \leq e^7 \end{cases}$$



Soluzione:

$$0 < x < e^{\frac{1}{2}} \quad \vee \quad e \leq x \leq e^7.$$

ESE 12

$$\frac{3 - |1 - 4 \cdot 2^x|}{4 - 2^x} < 0$$

$$y = 2^x$$

$$\frac{3 - |1 - 4y|}{4 - y} < 0$$

Numeratore:

$$3 - |1 - 4y| \geq 0$$

$$3 \geq |1 - 4y|$$

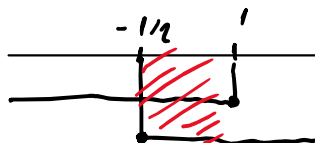
$$|1 - 4y| \leq 3$$

$$-3 \leq 1 - 4y \leq 3$$

$$\begin{cases} -3 \leq 1 - 4y \\ 1 - 4y \leq 3 \end{cases} \Rightarrow \begin{cases} 4y \leq 4 \\ 4y \geq -2 \end{cases}$$

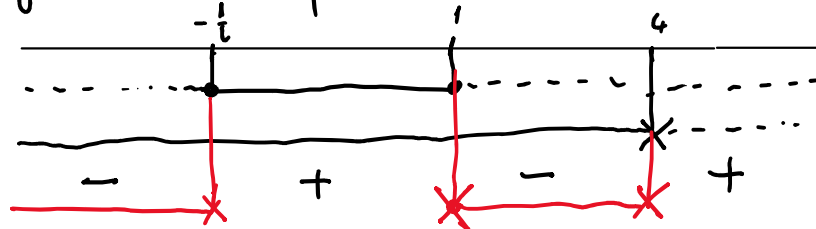
$$\begin{cases} y \leq 1 \\ y \geq -\frac{1}{2} \end{cases}$$

$$-\frac{1}{2} \leq y \leq 1.$$



Segno di N : 

Segno della frazione:



$$y < -\frac{1}{2} \quad \vee \quad 1 < y < 4$$

$$y = 2^x$$

$$2^x < -\frac{1}{2} \quad \vee \quad 1 < 2^x < 4$$

(impossibile) $\log_2 1 < x < \log_2 4$

$$0 < x < 2$$

Soluzione finale: $0 < x < 2$.

ESE 13

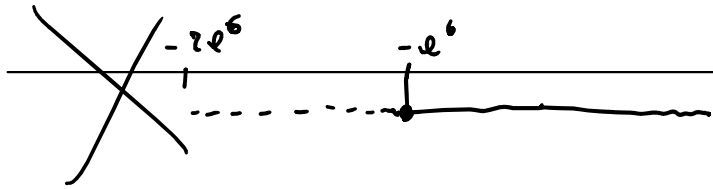
$$\frac{\frac{1}{6} \ln(x + 2e^6) - 1}{|2x - 3| - 1} \geq 0$$

c.e.:

$$x + 2e^6 > 0$$

$$x > -2e^6$$

$$\begin{aligned} \cdot N: \quad \frac{1}{6} \ln(x + 2e^6) - 1 &\geq 0 \\ \ln(x + 2e^6) - 6 &\geq 0 \\ \ln(x + 2e^6) &\geq 6 \\ x + 2e^6 &\geq e^6 \\ x &\geq -e^6 \end{aligned}$$



. D: $D > 0 \Leftrightarrow |2x - 3| > 1$

$\Leftrightarrow 2x - 3 > 1 \vee 2x - 3 < -1$

$\Leftrightarrow 2x > 4 \vee 2x < 2$

$\Leftrightarrow x > 2 \vee x < 1$

. Segno della frazione

