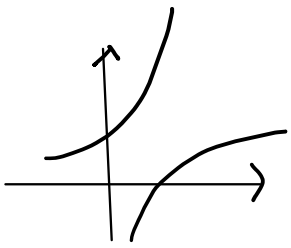
Esercitazione 05/11/2026

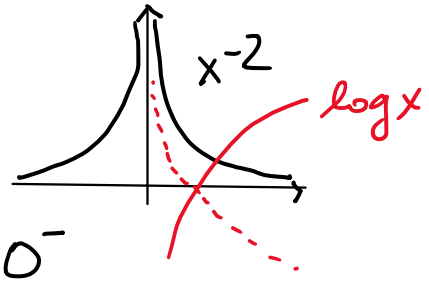
giovedì 5 novembre 2026 14:02

- $\lim_{x\to+\infty} \frac{3^x}{\log_2 x} = +\infty$



- $\lim_{x\to+\infty} \frac{\log x}{\sqrt{x}} = 0^+$ la potenza vince sul log.

- $\lim_{x\to 0^+} \frac{\log x}{x^{-2}} = 0^-$



- $\lim_{x\to 0^+} \frac{\log x}{x^{-2}} = \lim_{x\to 0^+} x^2 \log x = 0^-$

- $\lim_{x\to+\infty} (e^x - x^3 - \arctan x) = \lim_{x\to+\infty} e^x \left(1 - \frac{x^3}{e^x} - \frac{\arctan x}{e^x}\right)$

$e^x \to \infty$, $\frac{x^3}{e^x} \to 0$, $\frac{\arctan x}{e^x} \to 0$

$= +\infty$

- $\lim_{x\to\infty} (\log x - x^3) = -\infty$

$\log x - x^3 = -x^3\left(1 - \frac{\log x}{x^3}\right)$

- $\lim_{x\to+\infty} \frac{x^3 + \log_2 x}{x^2 + 5x + e^{-x}} = \lim_{x\to+\infty} \frac{x^3\left(1 + \frac{\log x}{x^3}\right)}{x^2\left(1 + \frac{5}{x} + \frac{e^{-x}}{x^2}\right)} = +\infty$

- $\lim_{x\to-\infty} \frac{3^{-x} + x^3}{\log|x| + e^x} = \lim_{x\to-\infty} \frac{3^{-x}(1 + x^3 \cdot 3^x)}{\log|x|\left(1 + \frac{e^x}{\log|x|}\right)} = +\infty$

- $\lim_{x\to 0^+} \frac{x^2 + x^7}{\sqrt{x} + x^3} = \lim_{x\to 0^+} \frac{x^2(1 + x^5)}{\sqrt{x}(1 + x^{5/2})}$

$x \to 0^+ \quad \sqrt{x} + x^5 \qquad x \to 0^+ \quad \sqrt{x}(1 + x^{5/2})$

$$= \lim_{x \to 0^+} x^{3/2} \cdot \frac{(1+x^5)}{(1+x^{5/2})} = 0^+$$

$\frac{(1+x^5)}{(1+x^{5/2})} \to 1$

- <u>limiti notevoli</u>:

$$\lim_{x \to 0} \frac{\sin x}{x} = 1 \qquad \lim_{x \to 0} \frac{a^x - 1}{x} = \log a$$

$$\lim_{x \to 0} \frac{1 - \cos x}{x^2} = \frac{1}{2} \qquad \lim_{x \to 0} \frac{\tan x}{x} = 1$$

$$\lim_{x \to 0} \frac{\arctan x}{x} = 1 \qquad \lim_{x \to 0} \frac{\log_a(1+x)}{x} = \frac{1}{\log a}$$

$$\lim_{x \to 0} \frac{\arcsin x}{x} = 1 \qquad \lim_{x \to 0} \frac{(x+1)^\alpha - 1}{x} \to \alpha$$

$$\lim_{x \to 0^+} \frac{x^3 + \sin x}{\sqrt{x} + x^5} = \lim_{x \to 0^+} \frac{x^3 + x\left(\frac{\sin x}{x}\right)^{\to 1}}{\sqrt{x} + x^5}$$

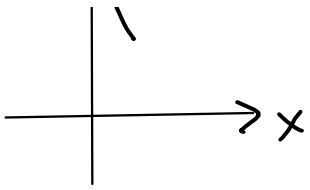
$$= \lim_{x \to 0^+} \frac{x\left(\left(\frac{\sin x}{x}\right)^{\to 1} + x^2\right)}{x^{1/2}(1 + x^{9/2})} = 0^+$$

$$\lim_{x \to 0^+} \frac{\sin x}{x^2} = \lim_{x \to 0^+} \left(\frac{\sin x}{x}\right) \cdot \frac{1}{x} = +\infty$$

$\left(\frac{\sin x}{x}\right) \to 1$

$\lim_{x \to 0^+} f(x) = +\infty$ $\{x = 0\}$ è un asintoto verticale.

verticale.



N.B. Il grafico di una funzione non interseca mai un asintoto verticale.

- $\lim_{x\to 0} \frac{1-\cos x}{x} = \lim_{x\to 0} \frac{1-\cos x}{x^2} \cdot x = 0$ ($\frac{1-\cos x}{x^2} \to \frac{1}{2}$)

$\{x=0\}$ non è un asintoto.

- $\lim_{x\to 0^+} \frac{\sin(x^3)}{x^5} = \lim_{x\to 0^+} \frac{\sin(x^3)}{x^3} \cdot \frac{1}{x^2} = +\infty$ ($\frac{\sin(x^3)}{x^3} \to 1$)

- $\lim_{x\to 2} \frac{\sin(x-2)}{x-2} = 1$

- $\lim_{x\to 0^+} \frac{\sin x \cdot \tan x}{x^3} = \lim_{x\to 0^+} \frac{\sin x}{x} \cdot \frac{\tan x}{x} \cdot \frac{1}{x} = +\infty.$
($\frac{\sin x}{x} \to 1$, $\frac{\tan x}{x} \to 1$, $\frac{1}{x} \to +\infty$)

- $\lim_{x\to 0} \frac{\log(1+x^3)}{x} = \lim_{x\to 0} \frac{\log(1+x^3)}{x^3} \cdot \frac{x^3}{x} = 0$
($\frac{\log(1+x^3)}{x^3} \to 1$)

$\frac{\log(1+x^3)}{x^3} \to 1$: sostituisci $\log(1+x^3)$ con x^3

$\lim \frac{\sin 5x}{\quad} = \lim 5 \cdot \frac{\sin 5x}{\quad} \cdot \frac{x}{\quad} = 5$

$$\lim_{x\to 0} \frac{\sin 5x}{\log(1+x)} = \lim_{x\to 0} 5 \cdot \frac{\sin 5x}{5x} \cdot \frac{x}{\log(1+x)} = 5$$

($\frac{\sin 5x}{5x} \to 1$, $\frac{x}{\log(1+x)} \to 1$)

$$\lim_{x\to 0} \frac{\sin(x^3)\log(1+2x)}{\tan(x^2)} = \lim_{x\to 0} \frac{x^3 \cdot 2x}{x^2} = \lim_{x\to 0} 2x^2 = 0$$

$$\frac{\sin(x^3)\log(1+2x)}{\tan(x^2)} = \frac{x^3 \cdot \frac{\sin(x^3)}{x^3} \cdot 2x \cdot \frac{\log(1+2x)}{2x}}{x^2 \cdot \frac{\tan(x^2)}{x^2}}$$

$$\lim_{x\to 0} \frac{(e^x-1)\cdot \sin 2x}{1-\cos x} = \lim_{x\to 0} \frac{x \cdot 2x}{\frac{1}{2}x^2} =$$

$\frac{e^x-1}{x} \to 1$, $x\to 0$ $\qquad \frac{1-\cos x}{x^2} \to \frac{1}{2}$

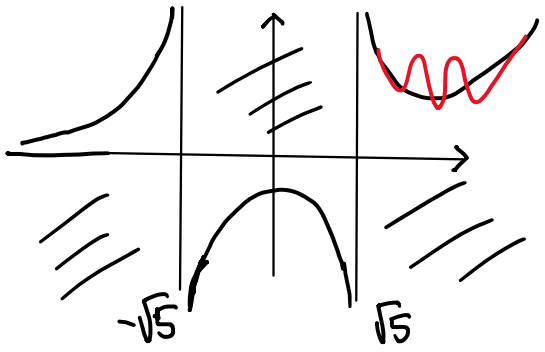
$$\lim_{x\to 0} \frac{(e^x-1)\sin 2x}{1-\cos x} = \lim_{x\to 0} \frac{\frac{e^x-1}{x} \cdot x \cdot \frac{\sin 2x}{2x} \cdot 2x}{\frac{1-\cos x}{x^2} \cdot x^2}$$

($\frac{e^x-1}{x} \to 1$, $\frac{\sin 2x}{2x} \to 1$, $\frac{1-\cos x}{x^2} \to \frac{1}{2}$)

$$= \lim_{x\to 0} \frac{1 \cdot x \cdot 1 \cdot (2x)}{\frac{1}{2} \cdot x^2} = \lim_{x\to 0} \frac{2x \cdot x}{\frac{1}{2}x^2} = 4$$

$f(x) = \frac{e^x}{x^2-5}$ Dominio, segno, asintoti, limiti.

$D(f) = \{x^2 - 5 \neq 0\} = \mathbb{R} \setminus \{\pm\sqrt{5}\}$



· $f(x) > 0 \iff x < -\sqrt{5} \vee x > \sqrt{5}$

· $e^x > 0 \quad \forall x \in \mathbb{R}$.

· $x^2 - 5 > 0$, $x < -\sqrt{5} \vee x > \sqrt{5}$.

· asintoti.

$$\lim_{x \to \sqrt{5}^{\pm}} \frac{e^x}{x^2-5} = \pm\infty \; ; \lim_{x \to -\sqrt{5}^{\pm}} \frac{e^x}{x^2-5} = \mp\infty$$

$$\lim_{x \to +\infty} \frac{e^x}{x^2-5} = +\infty \; ; \lim_{x \to -\infty} \frac{e^x}{x^2-5} = 0^+$$

$\{x = \sqrt{5}\}$, $\{x = -\sqrt{5}\}$ sono asintoti verticali.

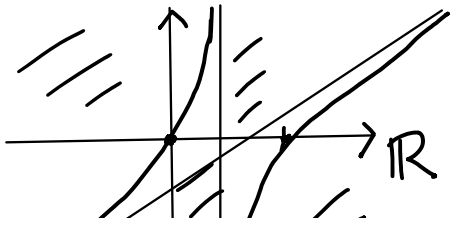
$\{y = 0\}$ è un asintoto orizzontale.

· $\nexists$ asintoti obliqui perché $\frac{f(x)}{x} \xrightarrow[x \to +\infty]{} +\infty$.

$f(x) = \frac{x^2 - 3x}{x-1}$; f ha un asintoto obliquo.

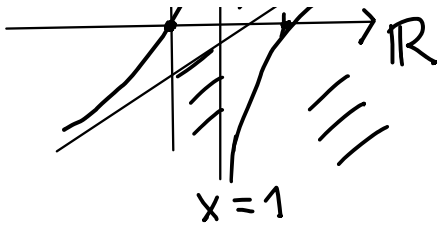
Dominio, segno, asintoti.

$D(f) = \mathbb{R} \setminus \{1\}$. ; $f(x) > 0$



$$\frac{x^2 - 3x}{x-1} > 0$$

$x(x-3)$



$$\frac{x(x-3)}{x-1} > 0$$

	0		1		3	
x	−	+		+		+
$x-3$	−	−		−		+
$x-1$	−	−		+		+
	−	+		−		+

$f(0) = f(3) = 0$

asintoti:

- $\lim_{x\to 1^+} \frac{x^2-3x}{x-1} = \frac{-2}{0^+} = -\infty$; $\lim_{x\to 1^-} \frac{x^2-3x}{x-1} = +\infty$

$\{x=1\}$ è un asintoto verticale.

- $\lim_{x\to\infty} \frac{x^2-3x}{x-1} = +\infty$; $\lim_{x\to-\infty} \frac{x^2-3x}{x-1} = \lim_{x\to-\infty} \frac{x^2}{x}$

$\left[\frac{x^2-3x}{x-1} = \frac{x^2(1-3/x)}{x(1-1/x)} \sim \frac{x^2}{x} = x\right]$

$= \lim_{x\to-\infty} x$

$= -\infty$

- $\lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{x^2-3x}{x-1} \cdot \frac{1}{x} = \lim_{x\to+\infty} \frac{x^2-3x}{x^2-x} = 1$

- $\lim_{x\to\infty} \left(f(x) - 1\cdot x\right) = \lim_{x\to\infty} \left(\frac{x^2-3x}{x-1} - x\right) = \lim_{x\to\infty} \frac{x^2-3x-x(x-1)}{x-1}$

$= \lim_{x\to\infty} \frac{-3x+x}{x-1} = -2$

$y = mx+q$

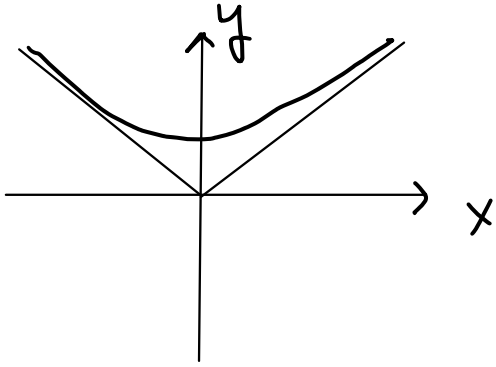
$m = \lim_{x\to\infty} \frac{f(x)}{x}$, $q = \lim_{x\to+\infty} \left(f(x) - mx\right)$

se questi 2 limiti esistono e sono finiti (entrambi) allora la retta $\{y = mx+q : x\in\mathbb{R}\}$

(entrambi), allora la retta $\{y = mx+q : x \in \mathbb{R}\}$ è un asintoto obliquo

$y = x-2$ è un asintoto obliquo a $+\infty$.
e anche a $-\infty$

- Gli asintoti obliqui sono al più 2 (possono essere destriti)



$y^2 - x^2 = 1$

$y = \sqrt{1+x^2}$

$y = x$ è asintoto obliquo a $+\infty$
$y = -x$ è asintoto obliquo a $-\infty$.

$$\lim_{x\to+\infty} \frac{\sqrt{1+x^2}}{x} = \lim_{x\to+\infty} \frac{\sqrt{x^2\left(1+\frac{1}{x^2}\right)}}{x}$$

$$= \lim_{x\to\infty} \frac{\cancel{x}\sqrt{1+\frac{1}{x^2}}}{\cancel{x}} = 1$$

$$\lim_{x\to+\infty} \sqrt{1+x^2} - x = \lim_{x\to\infty} \sqrt{x^2\left(1+\frac{1}{x^2}\right)} - x$$

$$= \lim_{x\to\infty} \left(x\sqrt{1+\frac{1}{x^2}} - x\right) =$$

$\delta = \frac{1}{x^2}$

$\frac{(1+\delta)^{\frac{1}{2}} - 1}{\delta} \to 1/2$

$= \lim_{x\to\infty} x \left(\sqrt{1+\left(\frac{1}{x^2}\right)} - 1 \right)$

$\left(\frac{1}{x^2}\right) \to 0$

$= \lim_{x\to\infty} x \cdot \frac{1}{x^2} \cdot \frac{1}{2}$

$= \lim_{x\to\infty} \frac{1}{x} \cdot \frac{1}{2} = 0$

f pari $y = mx + q$ è un asintoto obliquo $a +\infty$

$\Rightarrow$ $y = -mx + q$ è un asintoto obliquo $a -\infty$.

$f(x) = \frac{\log x}{|x|+7}$; $D(f)$ = dominio, segno, zeri, asintoti, limiti.