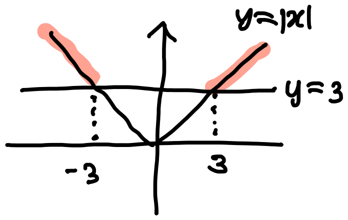
MATEMATICA – LEZIONE 14 – 24/10/24 14.00 - 16.00 Aula 4

Esercizio per casa:

II appello 2022/23

a) $\dfrac{|1-4x|-3}{4-x} < 0$

b) $\dfrac{|1-4e^{2x}|-3}{4-e^{2x}} < 0$



Svolgimento: a) Studio del segno

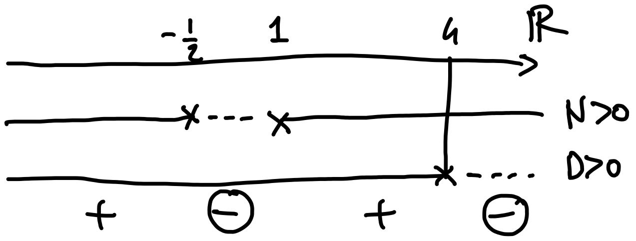
$N > 0$: $|1-4x|-3 > 0 \iff |1-4x| > 3$

$\iff 1-4x < -3 \ \vee \ 1-4x > 3$

$\iff 4 < 4x \ \vee \ -2 > 4x$

$\iff x > 1 \ \vee \ x < -\frac{1}{2}$

$D > 0$: $4-x > 0 \iff x < 4$



$-\frac{1}{2} < x < 1 \ \vee \ x > 4$

b) $\dfrac{|1-4e^{2x}|-3}{4-e^{2x}} < 0$ sostituzione $t = e^{2x}$

$\dfrac{|1-4t|-3}{4-t} < 0 \implies -\frac{1}{2} < t < 1 \ \vee \ t > 4$

$\implies -\frac{1}{2} < e^{2x} < 1 \ \vee \ e^{2x} > 4$

$\implies \begin{cases} e^{2x} > -\frac{1}{2} & \forall x \in \mathbb{R} \\ e^{2x} < 1 \end{cases} \ \vee \ e^{2x} > 4$

$\implies e^{2x} < e^0 \ \vee \ 2x > \ln 4$

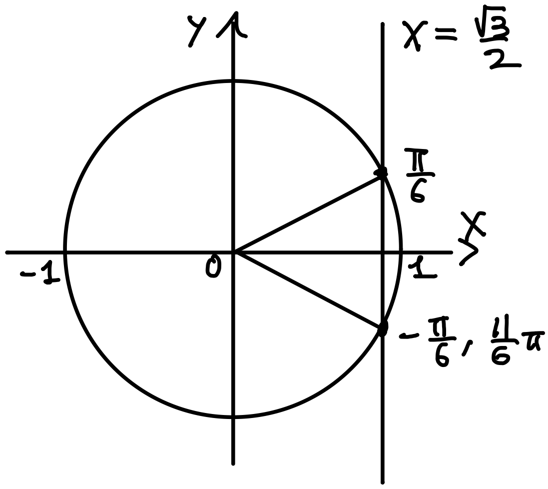
$\implies 2x < 0 \ \vee \ x > \frac{1}{2} \ln 4 = \ln 4^{\frac{1}{2}} = \ln \sqrt{4} = \ln 2$

$\implies x < 0 \ \vee \ x > \ln 2$

Equazioni e disequazioni goniometriche

1) $\cos x = \frac{\sqrt{3}}{2}$

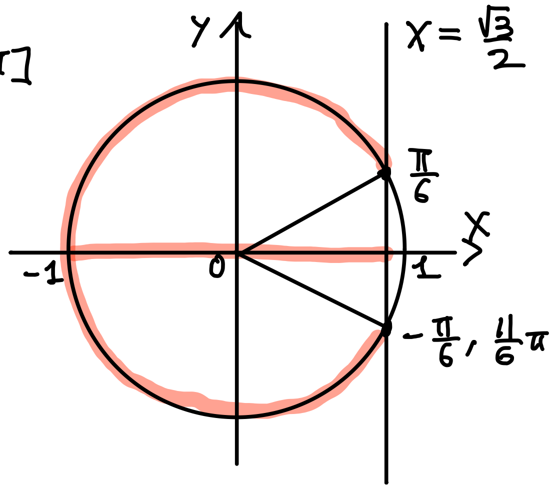
$x = \frac{\pi}{6} + 2k\pi \ \vee \ x = -\frac{\pi}{6} + 2k\pi, \ k \in \mathbb{Z}$



2) $\cos x \leq \frac{\sqrt{3}}{2}$

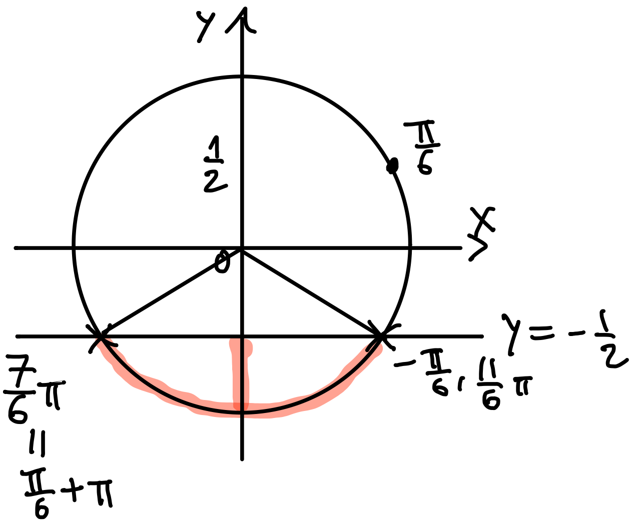
$\frac{\pi}{6} \leq x \leq \frac{11}{6}\pi$ soluzioni in $[0, 2\pi]$

$\frac{\pi}{6} + 2k\pi \leq x \leq \frac{11}{6}\pi + 2k\pi, \ k \in \mathbb{Z}$



3) $3^{\sin x} - \frac{1}{\sqrt{3}} < 0 \iff 3^{\sin x} < \frac{1}{\sqrt{3}} = 3^{-\frac{1}{2}}$

$\iff \sin x < -\frac{1}{2}$



$\frac{7}{6}\pi < x < \frac{11}{6}\pi$ soluzioni $[0, 2\pi]$

$\frac{7}{6}\pi + 2k\pi < x < \frac{11}{6}\pi + 2k\pi, \ k \in \mathbb{Z}$

soluzioni in $\mathbb{R}$

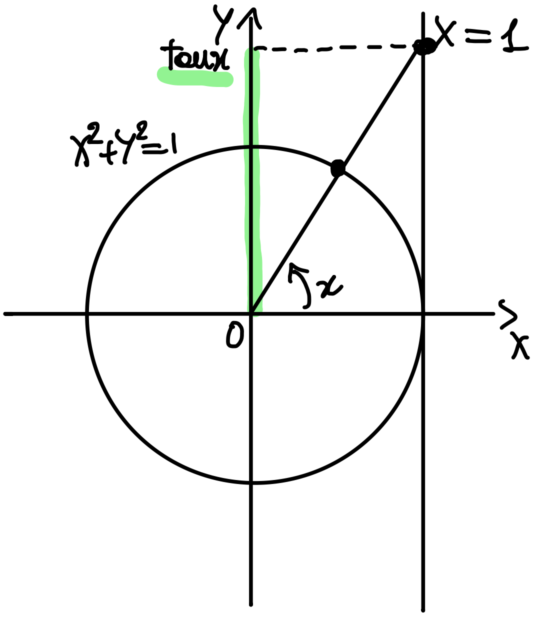
4) $\tan x \geq -1$

$\forall x \in \mathbb{R}$ tale che $\cos x \neq 0$: $\tan x := \frac{\sin x}{\cos x}$

$[\forall x \in \mathbb{R}$ tale che $x \neq \frac{\pi}{2} + k\pi, \ k \in \mathbb{Z}]$

$$\tan(x+\pi) = \frac{\sin(x+\pi)}{\cos(x+\pi)} = \frac{-\sin x}{-\cos x} = \frac{\sin x}{\cos x} = \tan x$$

$\Rightarrow$ tan è π-periodica



x	$\tan x$
0	0
$\frac{\pi}{6}$	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
$\frac{\pi}{4}$	1
$\frac{\pi}{3}$	$\sqrt{3}$
$\frac{\pi}{2}$	$\nexists$

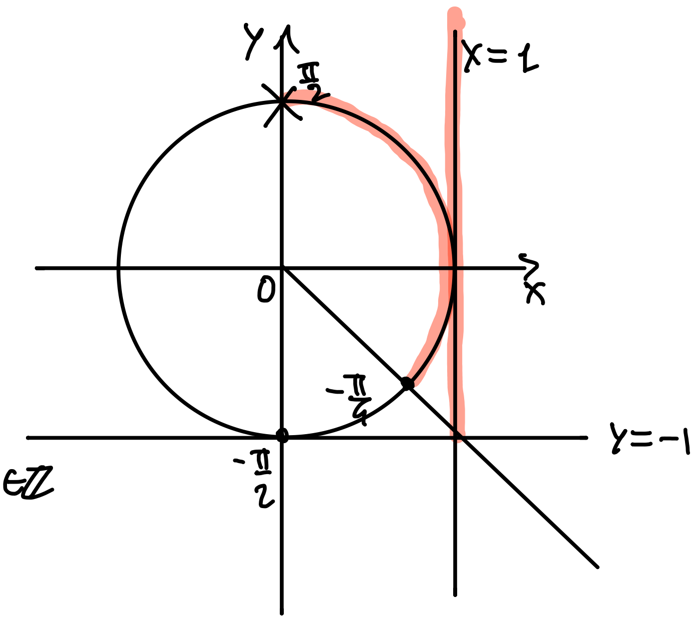
$$\tan(-x) = \frac{\sin(-x)}{\cos(-x)} = \frac{-\sin x}{\cos x} = -\tan x$$

tan è una funzione dispari

$\tan x \geq -1$

$-\frac{\pi}{4} \leq x < \frac{\pi}{2}$
soluzione in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$-\frac{\pi}{4} + k\pi \leq x < \frac{\pi}{2} + k\pi, \quad k \in \mathbb{Z}$
soluzioni in $\mathbb{R}$

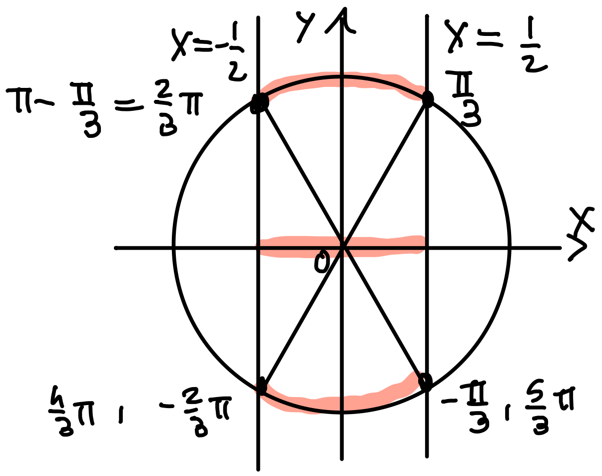


5) $3\cos^2 x - \sin^2 x \leq 0 \quad \Longleftrightarrow \quad 3\cos^2 x - (1-\cos^2 x) \leq 0$

$\forall x \in \mathbb{R}: \cos^2 x + \sin^2 x = 1$

$\Longleftrightarrow \quad 3\cos^2 x - 1 + \cos^2 x \leq 0 \quad \Longleftrightarrow \quad 4\cos^2 x - 1 \leq 0$

$t = \cos x \qquad 4t^2 - 1 \leq 0 \quad \Longleftrightarrow \quad -\frac{1}{2} \leq t \leq \frac{1}{2} \quad \Longleftrightarrow \quad -\frac{1}{2} \leq \cos x \leq \frac{1}{2}$



$\frac{\pi}{3} \le x \le \frac{2}{3}\pi \ \vee \ \frac{4}{3}\pi \le x \le \frac{5}{3}\pi$ soluzioni in $[0,2\pi]$

$\frac{\pi}{3}+2k\pi \le x \le \frac{2}{3}\pi+2k\pi \ \vee$

$\frac{4}{3}+2k\pi \le x \le \frac{5}{3}\pi+2k\pi, \ k\in\mathbb{Z}$

$\iff \frac{\pi}{3}+k\pi \le x \le \frac{2}{3}\pi+k\pi, \ k\in\mathbb{Z}$

soluzioni in $\mathbb{R}$

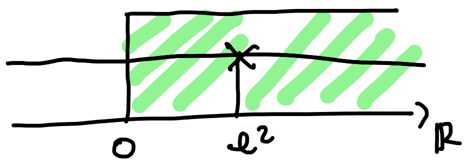
Esercizi sui domini

1) $f(x) = \dfrac{\ln^2 x}{2-\ln x}$

C.E. $\begin{cases} 2-\ln x \neq 0 \\ x>0 \end{cases}$

$2-\ln x \neq 0 \iff \ln x \neq 2 \iff x \neq e^2$

C.E. $\begin{cases} x \neq e^2 \\ x>0 \end{cases}$



$0<x<e^2 \ \vee \ x>e^2 \iff x\in(0,e^2)\cup(e^2,+\infty)$

2) $f(x) = \log_2\left(\sqrt{x^2-5}\right)$

C.E. $\begin{cases} \sqrt{x^2-5}>0 \\ x^2-5\ge 0 \end{cases} \iff \begin{cases} x^2-5>0 \\ x^2-5\ge 0 \end{cases}$

$\iff x^2-5>0$

$\iff x<-\sqrt{5} \ \vee \ x>\sqrt{5}$

$\iff x\in(-\infty,-\sqrt{5})\cup(\sqrt{5},+\infty)$

3) $f(x) = \dfrac{1}{8^{x^2-5x+6}-1}$

C.E. $8^{x^2-5x+6}-1 \neq 0 \iff 8^{x^2-5x+6} \neq 1 = 8^0$

$\iff x^2-5x+6 \neq 0$

$\iff (x-2)(x-3) \neq 0$

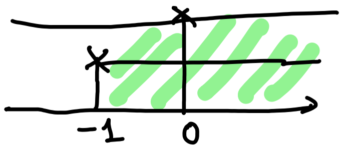
$\iff x \neq 2$ e $x \neq 3$

$\iff x \in \mathbb{R} \setminus \{2,3\} = (-\infty,2) \cup (2,3) \cup (3,+\infty)$

4) $f(x) = \dfrac{1}{\ln(x+1)}$

C.E. $\begin{cases} x+1>0 \\ \ln(x+1) \neq 0 \end{cases} \Rightarrow \begin{cases} x>-1 \\ \ln(x+1) \neq \ln 1 \end{cases}$

$\Rightarrow \begin{cases} x>-1 \\ x \neq 0 \end{cases}$



$\Rightarrow -1<x<0 \vee x>0$

$\Rightarrow x \in (-1,0) \cup (0,+\infty)$

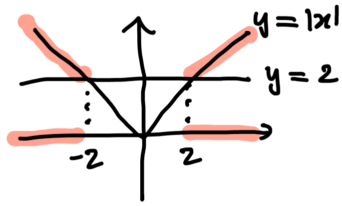
5) $f(x) = \sqrt{2^x - 4^x}$

C.E. $2^x - 4^x \geq 0 \iff 2^x \geq 4^x \iff 1 \geq \dfrac{4^x}{2^x} = 2^x$

$\iff 2^x \leq 1 = 2^0 \iff x \leq 0$

6) $f(x) = \sqrt{|x^2-3x+4|-2}$

C.E. $|x^2-3x+4|-2 \geq 0 \iff |x^2-3x+4| \geq 2$



$\iff x^2-3x+4 \leq -2 \vee x^2-3x+4 \geq 2$

$\iff x^2-3x+6 \leq 0 \vee x^2-3x+2 \geq 0$

$\Delta = (-3)^2 - 4\cdot 6 < 0$

$\Delta = (-3)^2 - 4\cdot 2 = 1$

$\dfrac{3 \pm 1}{2} \begin{cases} 1 \\ 2 \end{cases}$

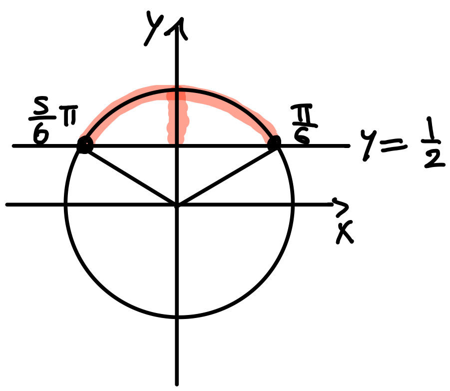
$\iff \nexists x \in \mathbb{R} \vee x \leq 1 \cup x \geq 2$

$\iff x \leq 1 \vee x \geq 2 \iff x \in (-\infty,1] \cup [2,+\infty)$

7) $f(x) = e^{x^2} + \sqrt{2\sin x - 1}$

C.E. $2\sin x - 1 \geq 0 \iff \sin x \geq \frac{1}{2}$

$\frac{\pi}{6} + 2k\pi \leq x \leq \frac{5}{6}\pi + 2k\pi \quad , \; k \in \mathbb{Z}$



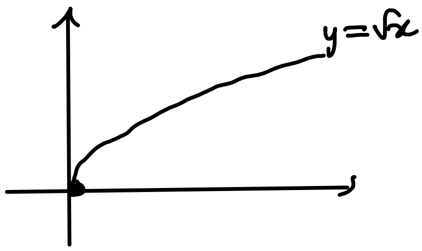
8) $f(x) = \dfrac{1}{e^x - 2}$

C.E. $e^x - 2 \neq 0 \implies e^x \neq 2 \implies x \neq \ln 2$

$\implies x \in \mathbb{R} \setminus \{\ln 2\} = (-\infty, \ln 2) \cup (\ln 2, +\infty)$

9) $f(x) = \dfrac{1}{\sqrt{x^4 + 9}}$

C.E. $x^4 + 9 > 0 \quad \forall x \in \mathbb{R}$



10) $f(x) = \sqrt[6]{x^4 - 1}$

C.E. $x^4 - 1 \geq 0$

I metodo : $x^4 - 1 = (x^2-1)(x^2+1) = (x-1)(x+1)(x^2+1) \geq 0$

$\iff (x-1)(x+1) \geq 0$

$\iff x \leq -1 \lor x \geq 1$

II metodo : $t = x^2 \qquad t^2 - 1 \geq 0 \implies t \leq -1 \lor t \geq 1$

$\implies x^2 \leq -1 \lor x^2 \geq 1$

impossibile

$\implies x \leq -1 \lor x \geq 1$

III metodo : metodo grafico