$$2^x + \left(\frac{1}{2}\right)^x - 7 \geq 0$$

$$2^x \cdot 2^x = 2^{2x}$$

$$\boxed{2^x \cdot 2^{-x} = 1}$$

$$2^x + \frac{1}{2^x} - 7 \geq 0$$

$$\frac{2^{2x} + 1 - 7 \cdot 2^x}{2^x} \geq 0$$

$$\frac{2^{2x} - 7 \cdot 2^x + 1}{2^x} \geq 0$$

$$2^x > 0 \quad \forall x \in \mathbb{R}$$

$$2^x = y$$

$$2^{2x} - 7 \cdot 2^x + 1 \geq 0$$

$$\Delta = 49 - 4 > 0$$

$$y^2 - 7y + 1 \geq 0$$

$$y_{1/2} = \frac{7 \pm \sqrt{45}}{2}$$

$$y \leq \frac{7-\sqrt{45}}{2} \quad \vee \quad y \geq \frac{7+\sqrt{45}}{2}$$

$$2^x \leq \underbrace{\frac{7-\sqrt{45}}{2}}_{>0} \quad \vee \quad 2^x \geq \underbrace{\frac{7+\sqrt{45}}{2}}_{>0}$$

$$x \leq \log_2\left(\frac{7-\sqrt{45}}{2}\right) \quad \vee \quad x \geq \log_2\left(\frac{7+\sqrt{45}}{2}\right)$$

$$a^x \qquad a>0 \qquad a \neq 1$$

$$f(x) = \underbrace{(x^2+2x)}_{\text{base}}^{\overbrace{\sqrt{x^2-3x}}^{\text{esponente}}}$$

(f non costante)

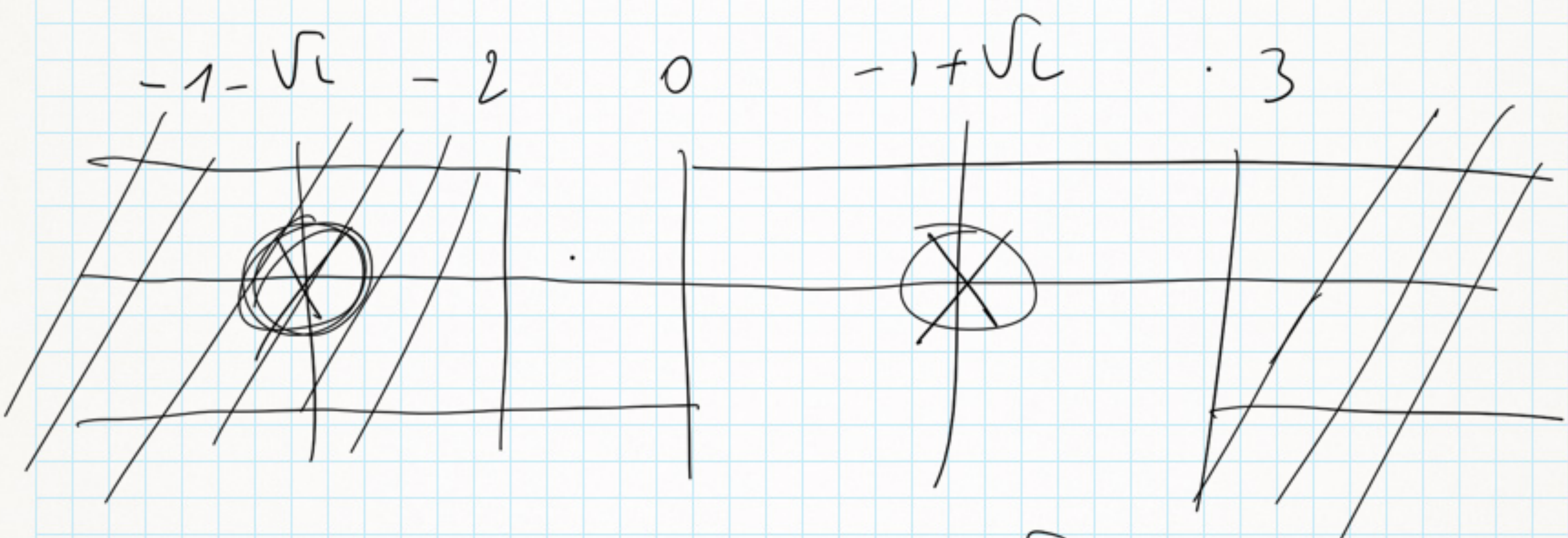
$$\begin{cases} x^2+2x>0 \\ x^2+2x \neq 1 \\ x^2-3x \geq 0 \\ x^2-3x \neq 0 \end{cases} \longrightarrow \text{FUNZIONE NON COSTANTE}$$

$$\begin{cases} x<-2 \ \vee \ x>0 \\ x \neq -1-\sqrt{2} \ \vee \ x \neq -1+\sqrt{2} \\ x<0 \vee x>3 \end{cases}$$

$$x^2+2x-1=0$$

$$x_{1,2}=\frac{-2\pm\sqrt{8}}{2}=\frac{-2\pm2\sqrt{2}}{2}=$$

$$=\frac{\cancel{2}(-1\pm\sqrt{2})}{\cancel{2}}=-1\pm\sqrt{2}.$$

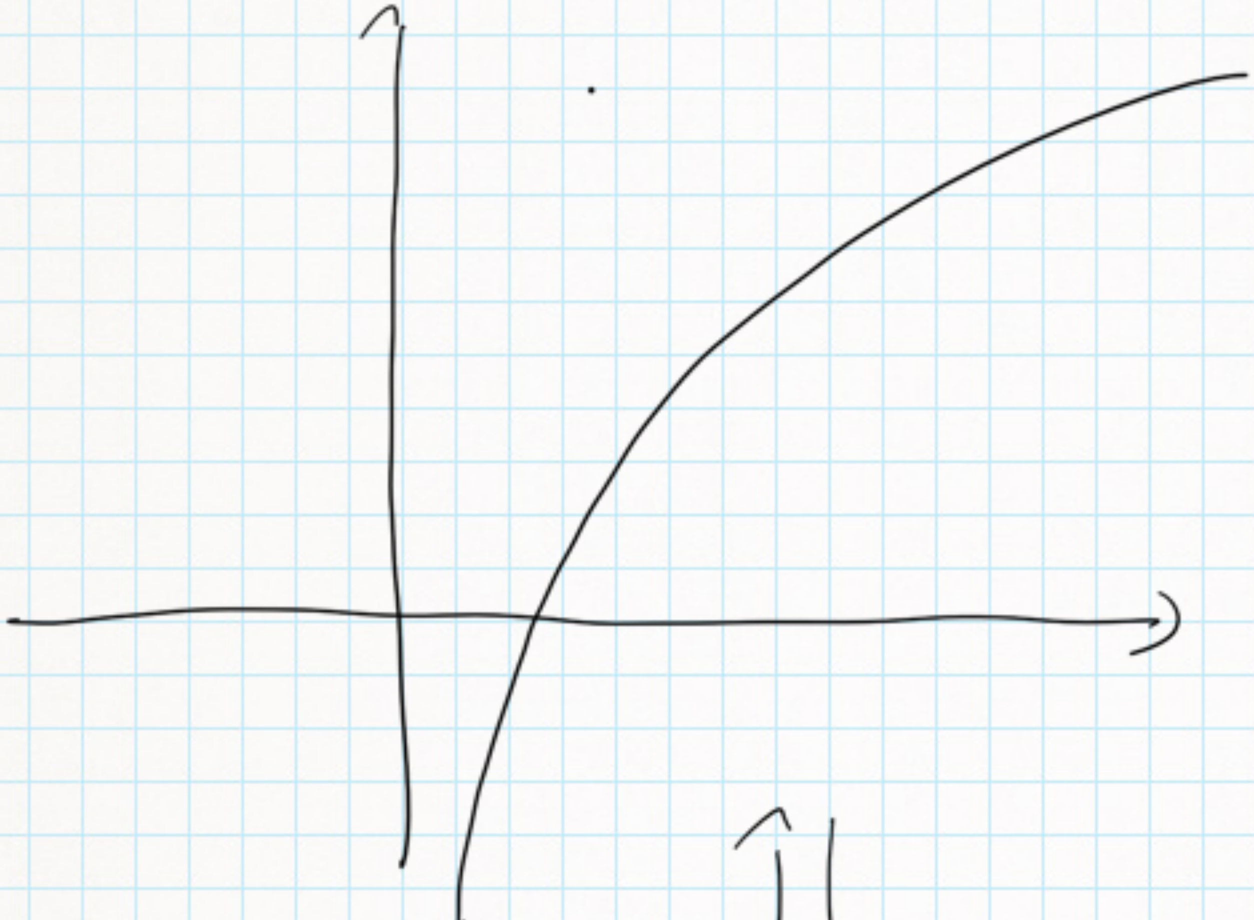


Sol: $x<-2,\ x\neq-1-\sqrt{2}$

oppure $x>3$

$f(x) = \lg_a x$

$a > 1$



$\boxed{x > 0}$

$0 < a < 1$

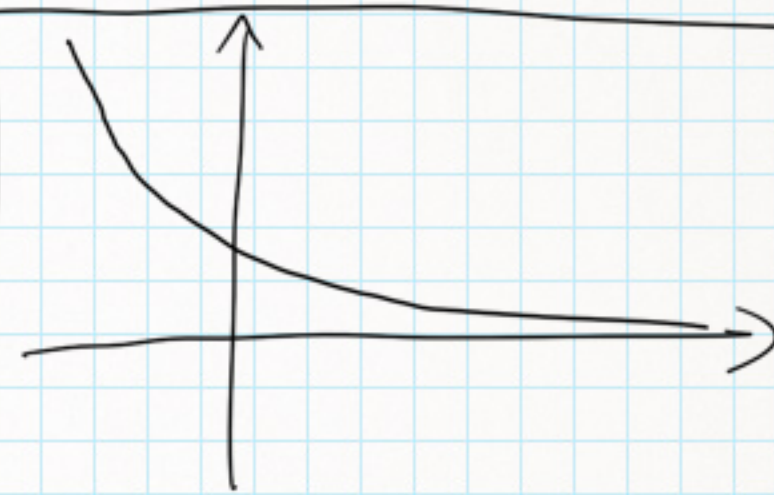
$\boxed{x > 0}$

- $\log_{\frac{1}{2}} x = 3$

$$\frac{1}{2}^{\log_{\frac{1}{2}} x} = \left(\frac{1}{2}\right)^3$$

$$x = \frac{1}{2^3} = \frac{1}{8}$$

$\boxed{x>0}$



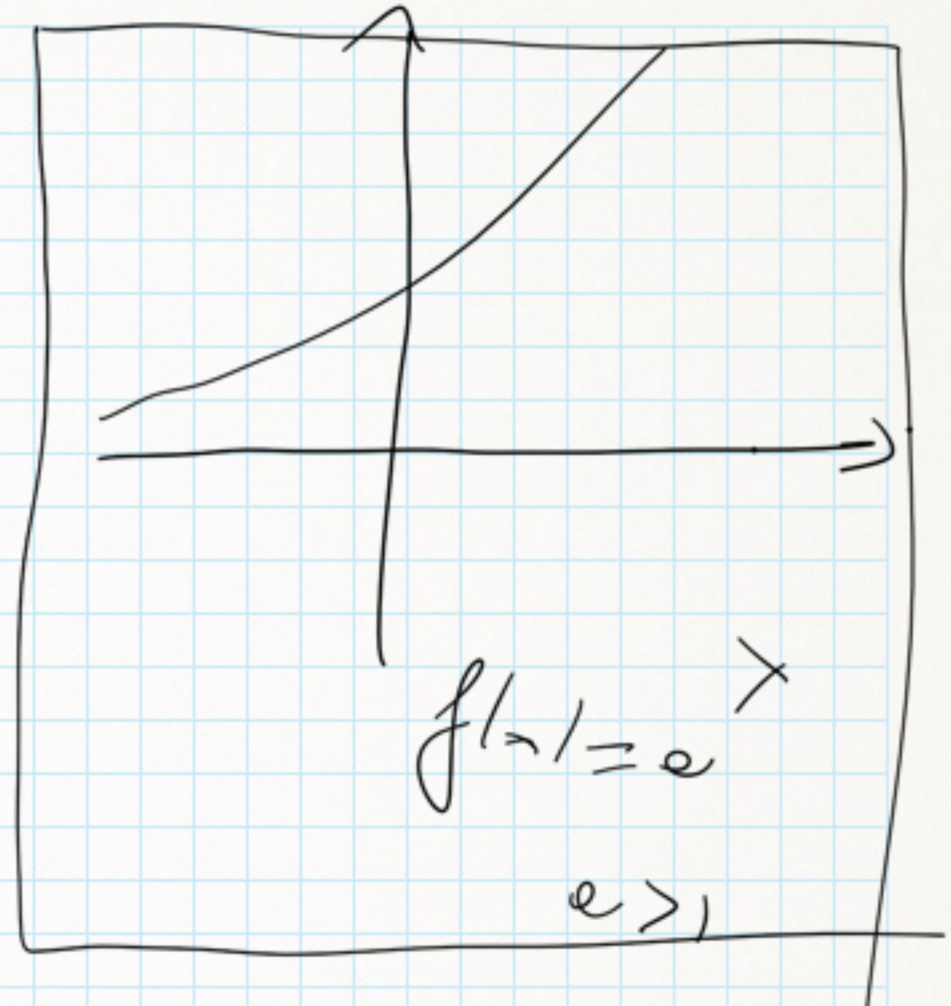
$f(x) = a^x \quad 0 < a < 1$

- $\log_{\frac{1}{2}} x > 3$

$$\frac{1}{2}^{\log_{\frac{1}{2}} x} < \left(\frac{1}{2}\right)^3$$

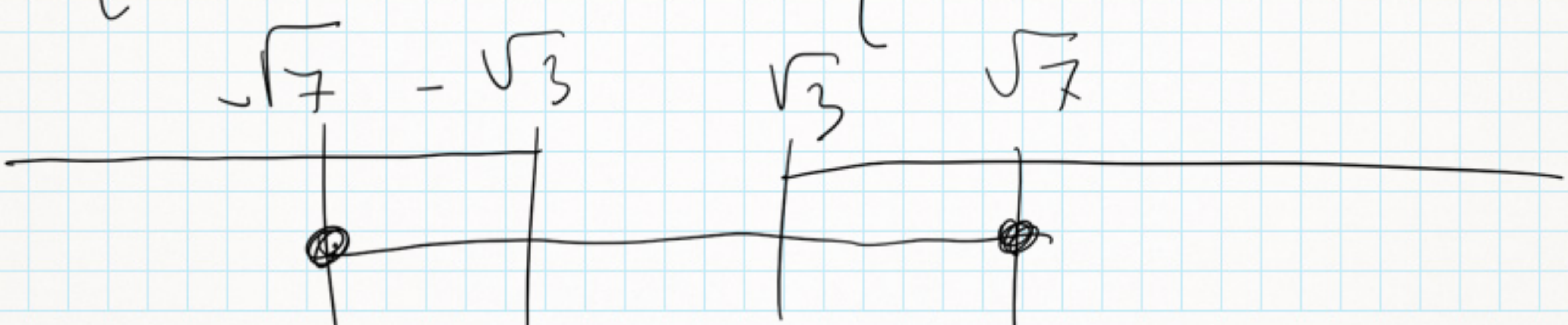
$$\boxed{0 < x} < \frac{1}{8}$$

$$\log_2 (x^2-3) \le 2$$



$$\begin{cases} x^2 - 3 > 0 \\ x^2 - 3 \le 2^2 = 4 \end{cases}$$

$$\begin{cases} x^2 - 3 > 0 \\ x^2 - 7 \le 0 \end{cases} \qquad \begin{cases} x < -\sqrt{3} \vee x > \sqrt{3} \\ -\sqrt{7} \le x \le \sqrt{7} \end{cases}$$



INSIEME DELLE SOLUZIONI

$$\left[-\sqrt{7}, -\sqrt{3}\right[\cup \left]\sqrt{3}, \sqrt{7}\right]$$

$$-\sqrt{7} \leq x < -\sqrt{3} \quad \vee \quad \sqrt{3} < x \leq \sqrt{7}$$

- $\lg^2 x - 3 \lg x \geq 0$

$\ln^2 x - 3 \ln x \geq 0$

STESSA COSA

$\boxed{x > 0}$

$y = \lg x \qquad y^2 - 3y \geq 0$

$y^2 - 3y = 0$ $\qquad$ $y = 0 \vee y = 3$

$y(y-3) = 0$

$y \leq 0 \quad \vee \quad y \geq 3$

$x > 0$

$\lg x \leq 0 \quad \vee \quad \lg x \geq 3$

$x \leq e^0 = 1 \quad \vee \quad x \geq e^3$

$$\begin{cases} x \leq 1 \vee x \geq e^3 \\ x > 0 \end{cases}$$

$0 < x \leq 1 \vee x \geq e^3$

$$f(x) = \log_{\frac{1}{3}}\left(\frac{3x-1}{x+1}\right)$$

DOMINIO

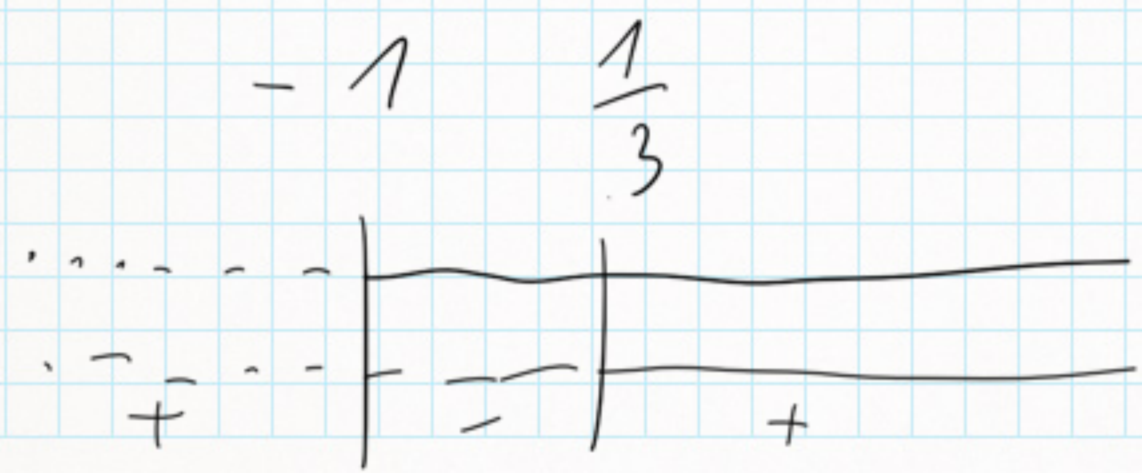
POSITIVITÀ $(f(x) > 0)$

DOMINIO

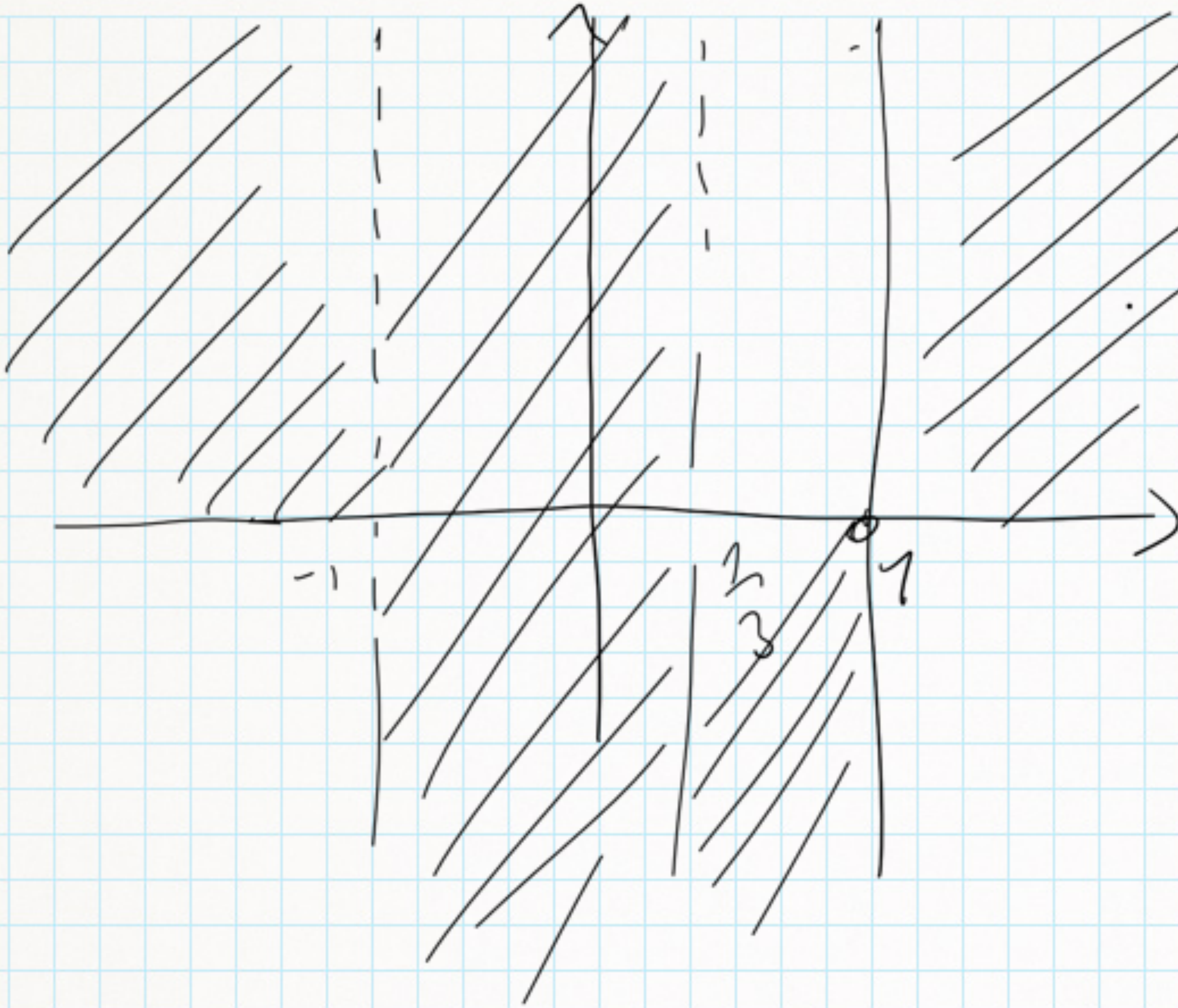
$$\frac{3x-1}{x+1} > 0$$

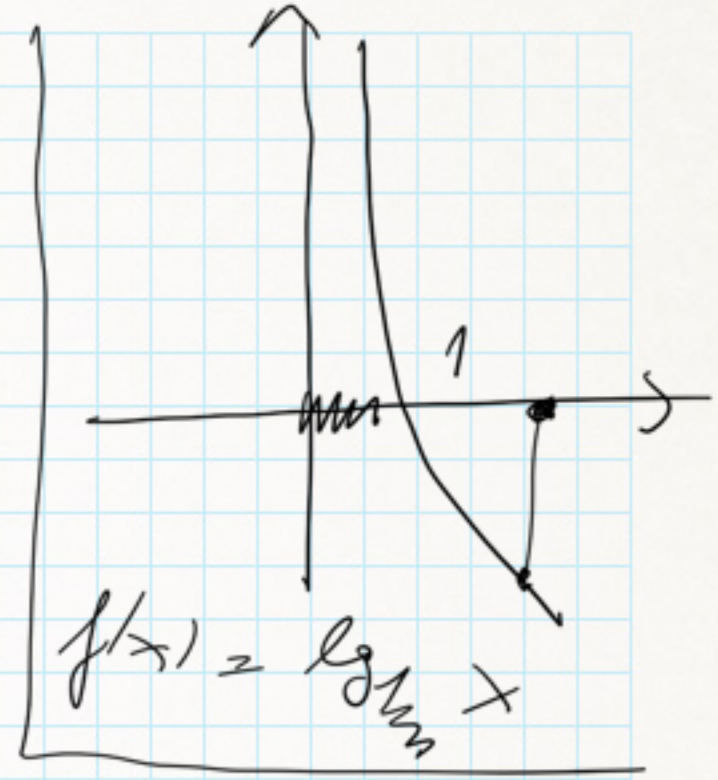
$3x-1>0 \qquad x>\frac{1}{3}$

$x+1>0 \qquad x>-1$



$$\boxed{x<-1 \vee x>\frac{1}{3}}$$





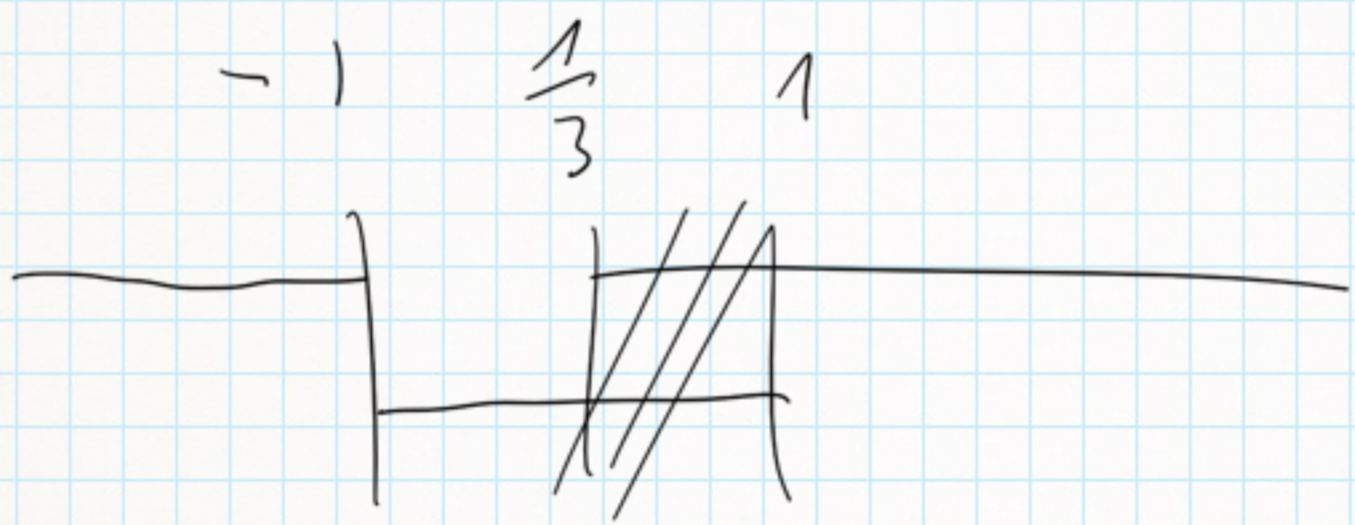
$f(x) > 0$ $\qquad \log_{\frac{1}{3}}\left(\frac{3x-1}{x+1}\right) > 0$

$$\Leftrightarrow \quad 0 < \frac{3x-1}{x+1} < 1 \qquad \begin{cases} \dfrac{3x-1}{x+1} > 0 \\ \dfrac{3x-1}{x+1} < 1 \end{cases}$$

$$\begin{cases} x < -1 \vee x > \frac{1}{3} \\ \frac{3x-1-x-1}{x+1} < 0 \end{cases} \qquad \begin{cases} x < -1 \vee x > \frac{1}{3} \\ \frac{2x-2}{x+1} < 0 \end{cases}$$

$$\begin{cases} x < -1 \vee x > \frac{1}{3} \\ \frac{x-1}{x+1} < 0 \end{cases} \qquad \begin{cases} x < -1 \vee x > \frac{1}{3} \\ -1 < x < 1 \end{cases}$$

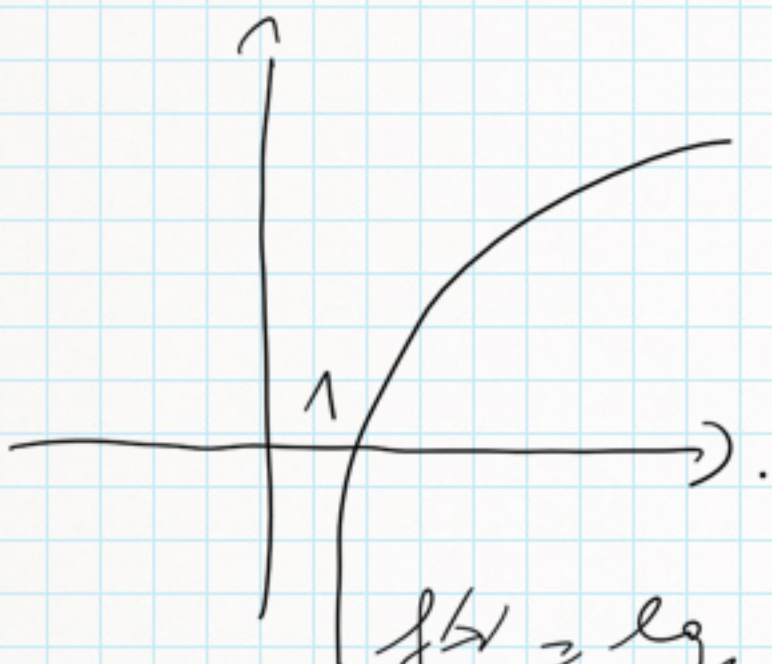
$$\frac{1}{3} < x < 1$$



$$f(x) = \log_3\left(\frac{3x-1}{x+1}\right)$$

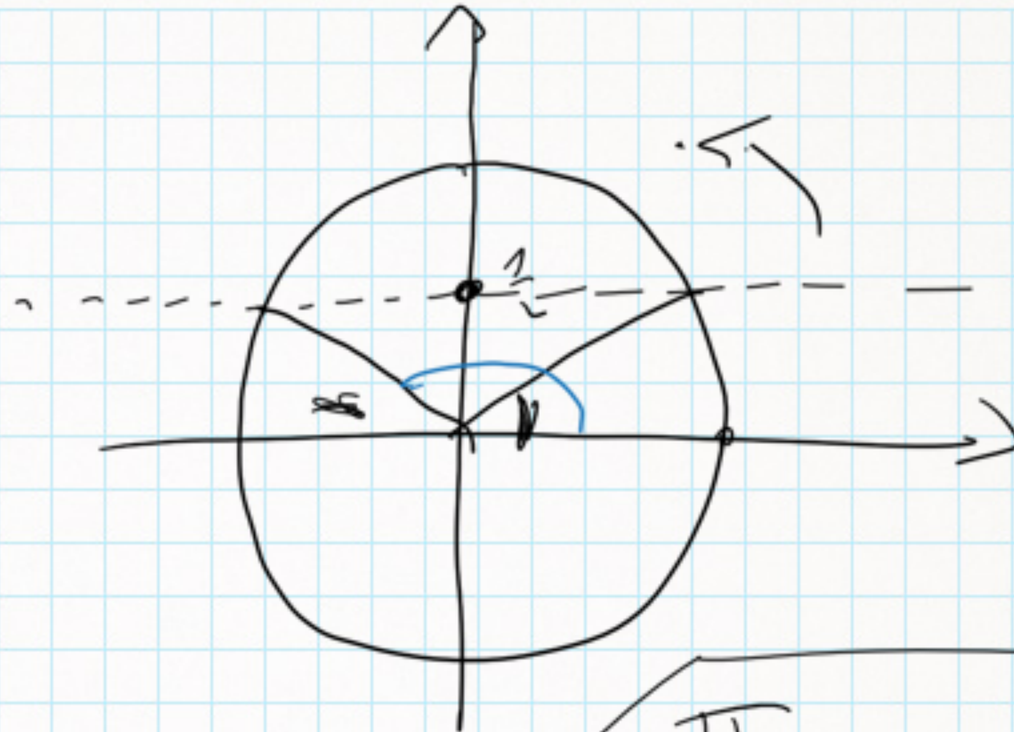
DOMINIO $\longrightarrow$ $\dfrac{3x-1}{x+1} > 0$

POSITIVITA' $\longrightarrow$ $\dfrac{3x-1}{x+1} > 1$



$f(x) = \log_3 x$

$$\sin x = \frac{1}{2}$$



$[0, 2\pi]$

$$x = \frac{\pi}{6}$$

$$x = \pi - \frac{\pi}{6} = \frac{5}{6}\pi$$

$$\frac{\pi}{6}$$

$$\sin \frac{\pi}{6} = \frac{1}{2}$$

$$\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

$$[\sin x = \sin(\pi - x)]$$

$$\mathbb{R}: \quad x = \frac{\pi}{6} + 2k\pi \quad \vee \quad x = \frac{5}{6}\pi + 2k\pi$$

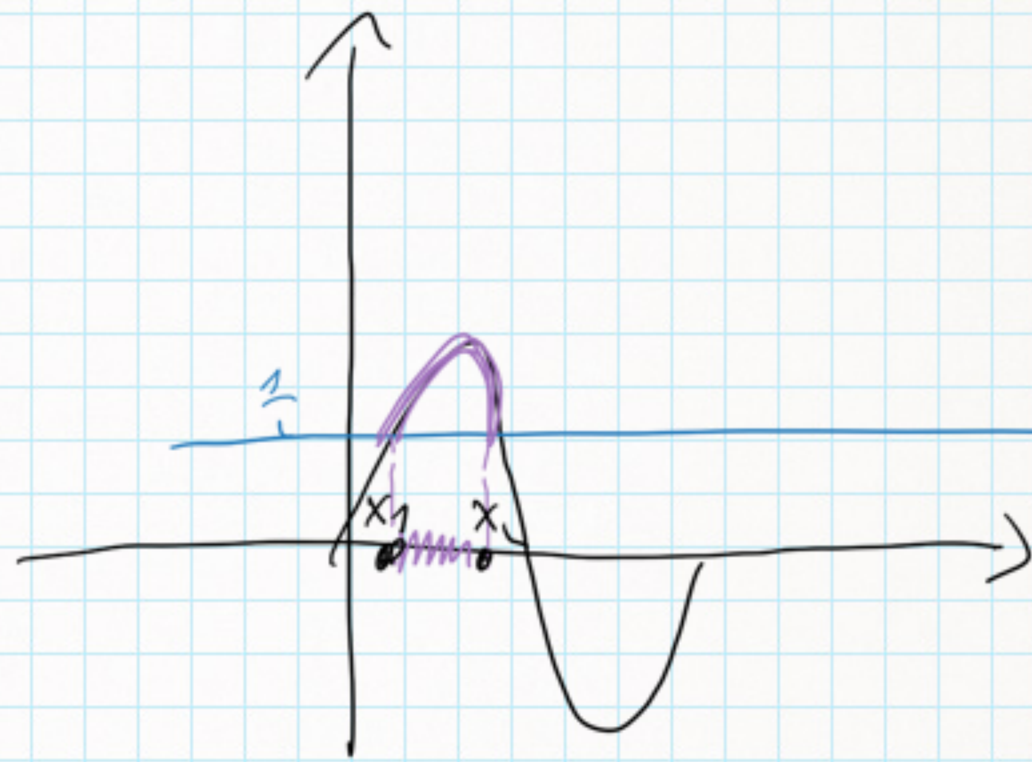
$$k \in \mathbb{Z}$$

$\sin x > \frac{1}{2}$

$[0, 2\pi]$

$f(x) = \sin x$

$g(x) = \frac{1}{2}$



$x_1 = \frac{\pi}{6}$

$\sin x = \frac{1}{2}$

$x_2 = \frac{5}{6}\pi$

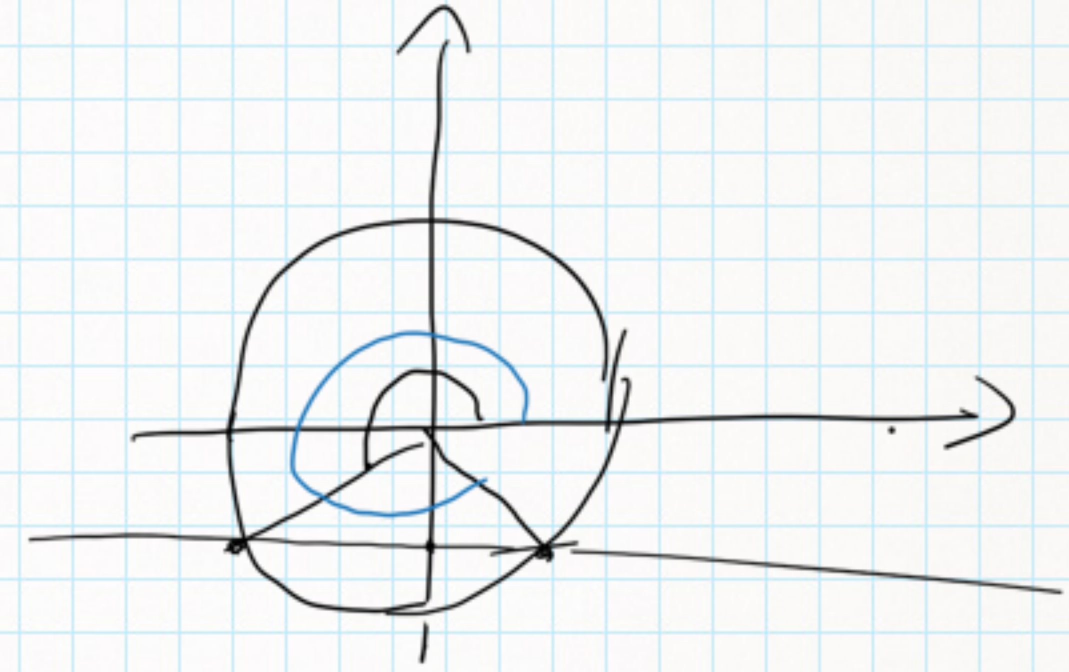
$\frac{\pi}{6} < x < \frac{5}{6}\pi$

R: $2k\pi + \frac{\pi}{6} < x < \frac{5}{6}\pi + 2k\pi, \quad k \in \mathbb{Z}$

$\sin x \leq -\frac{\sqrt{2}}{2}$

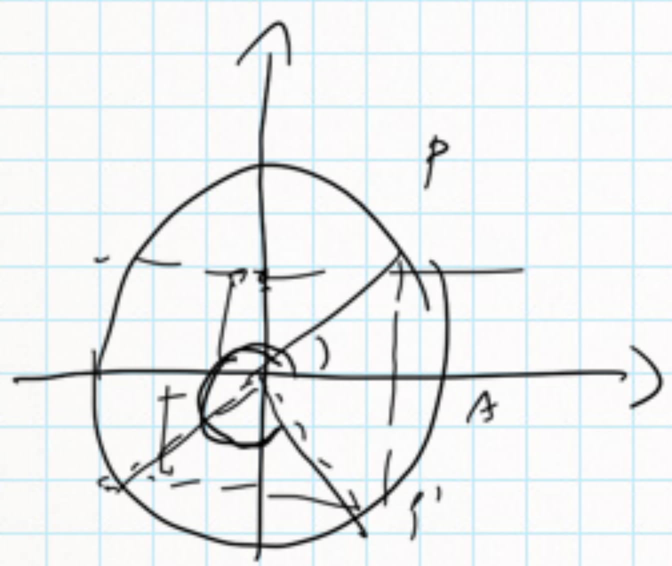
$[0, 2\pi]$

$\sin x = -\frac{\sqrt{2}}{2}$



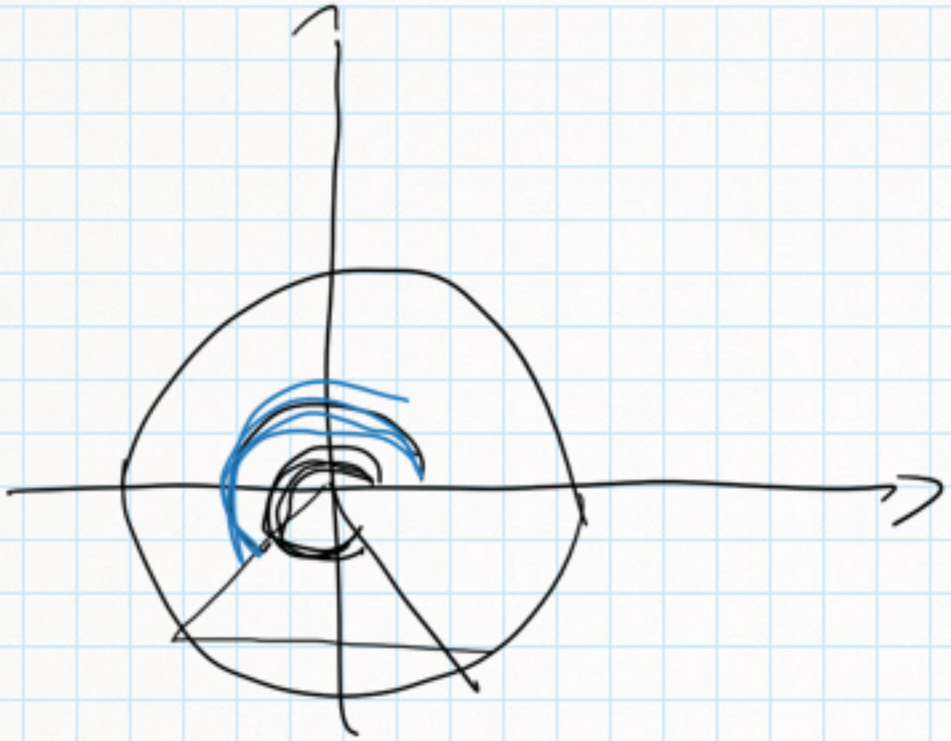
$x = \frac{5}{4}\pi \vee \frac{7}{4}\pi$

$\frac{\pi}{4}$



$\sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$

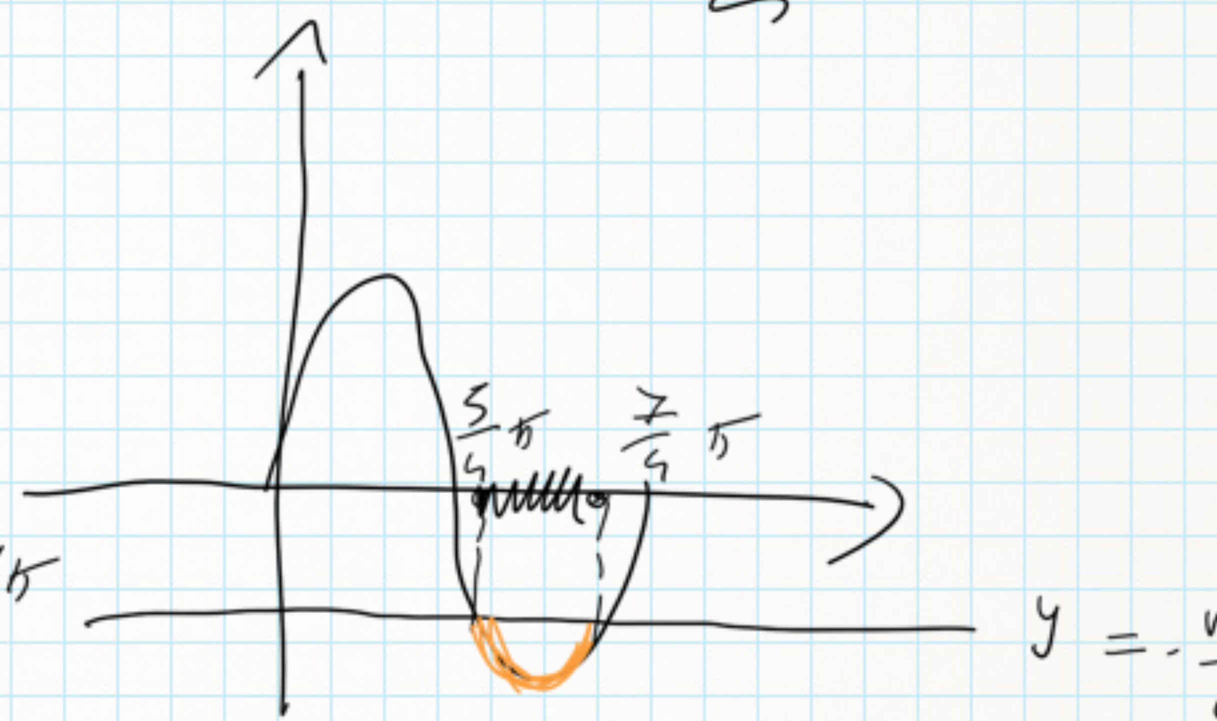
$\cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$



$\mathbb{R}$ $\quad x = \frac{5}{4}\pi + 2k\pi \lor x = \frac{7}{4}\pi + 2k\pi$

$\sin x \le -\frac{\sqrt{2}}{2}$

$\frac{5}{4}\pi + 2k\pi \le x \le \frac{7}{4}\pi + 2k\pi$



$$2\sin^2 x - \sin x \geq 0 \qquad \sin x = y$$

$$2y^2 - y \geq 0 \qquad -1 \leq \sin x \leq 1$$

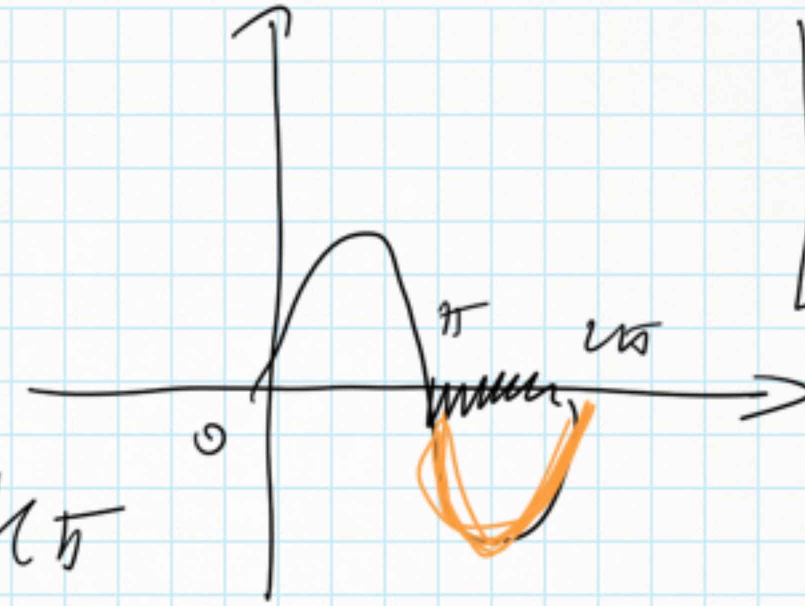
$$y(2y-1) \geq 0$$

$$y \leq 0 \quad \vee \quad y \geq \frac{1}{2}$$

$$\sin x \leq 0 \quad \vee \quad \underbrace{\sin x \geq \frac{1}{2}}$$

↓ FATTA PRECEDENTEMENTE

$\sin x \le 0$



$\pi + 2k\pi \le x \le 2\pi + 2k\pi$

$\cos x = \cos(2\pi - x)$

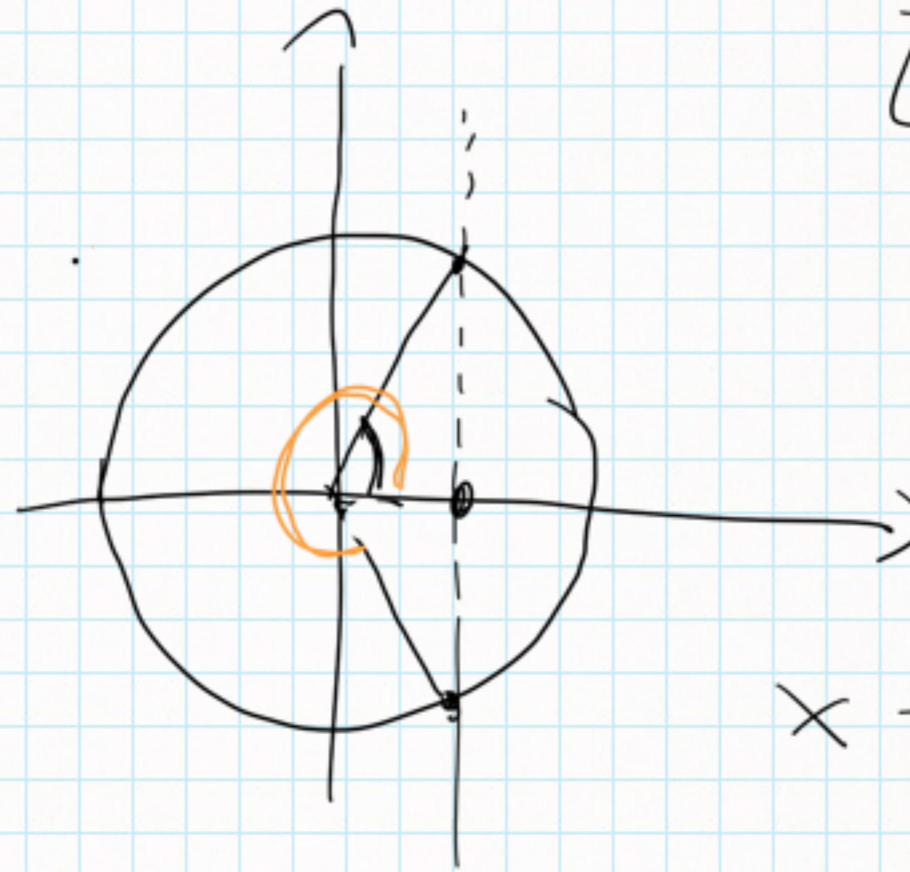
$[0, 2\pi]$

$\cos x = \frac{1}{2}$

$\frac{\pi}{3}$

$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$\cos\frac{\pi}{3} = \frac{1}{2}$

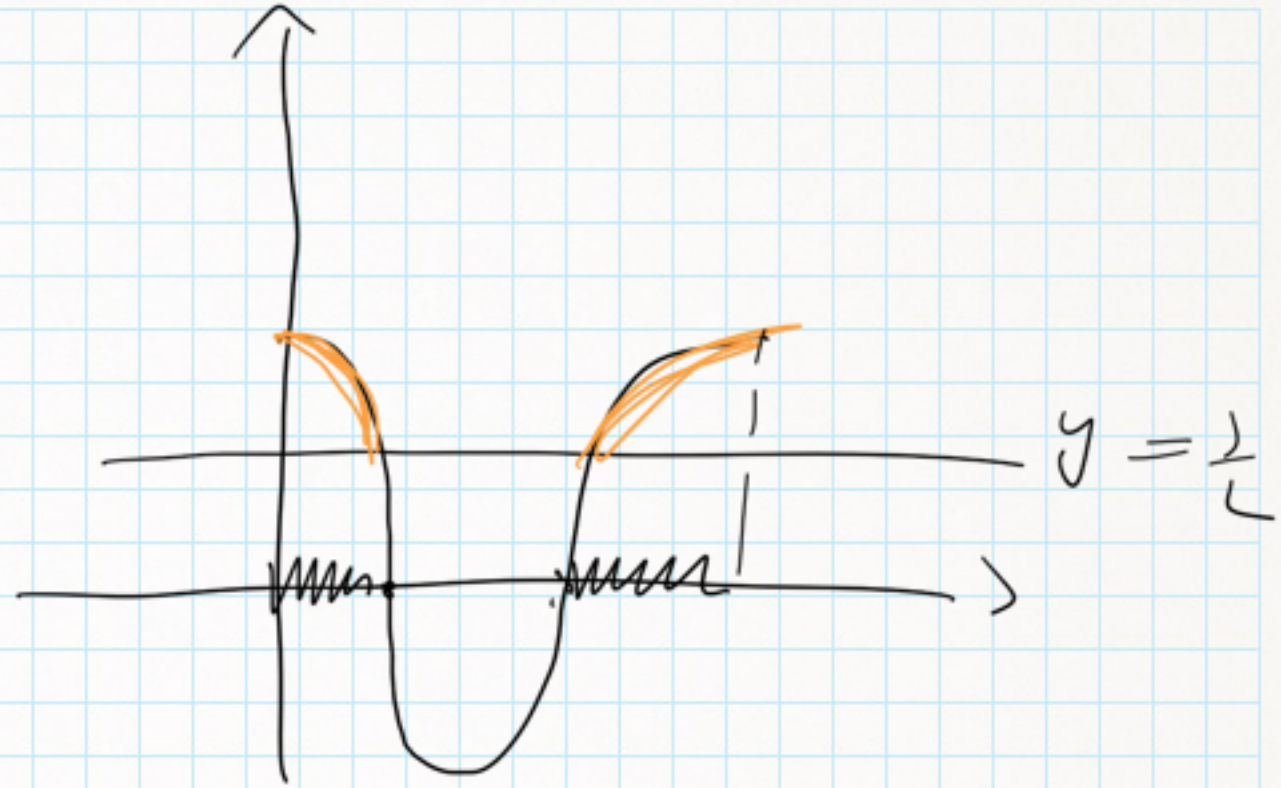


$x = \frac{\pi}{3} + 2k\pi$

$x = 2\pi - \frac{\pi}{3} + 2k\pi = \frac{5}{3}\pi + 2k\pi$

$\cos x > \frac{1}{2}$

$[0, 2\pi]$

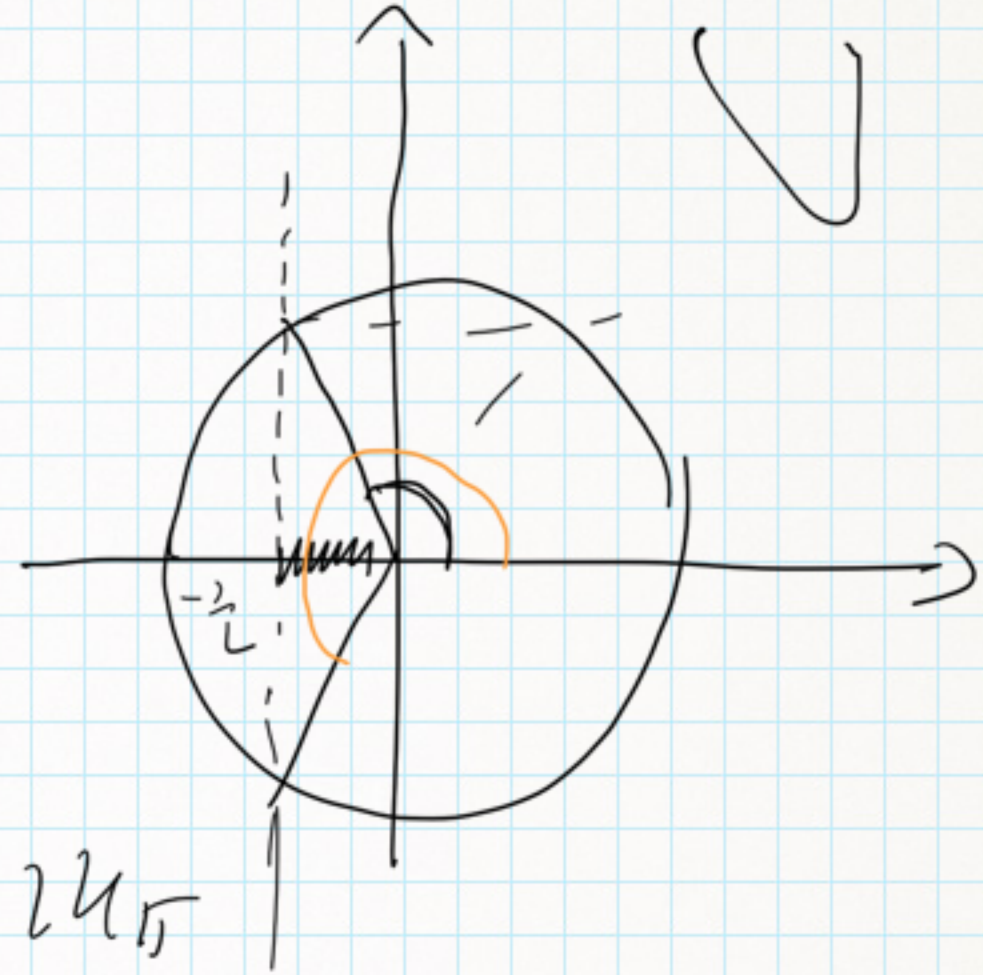


$0 \leq x < \frac{\pi}{3} \quad \vee \quad \frac{5}{3}\pi < x \leq 2\pi$

(aggiungere la periodicità)

$\cos x \leq -\frac{1}{2}$

$\cos x = -\frac{1}{2}$

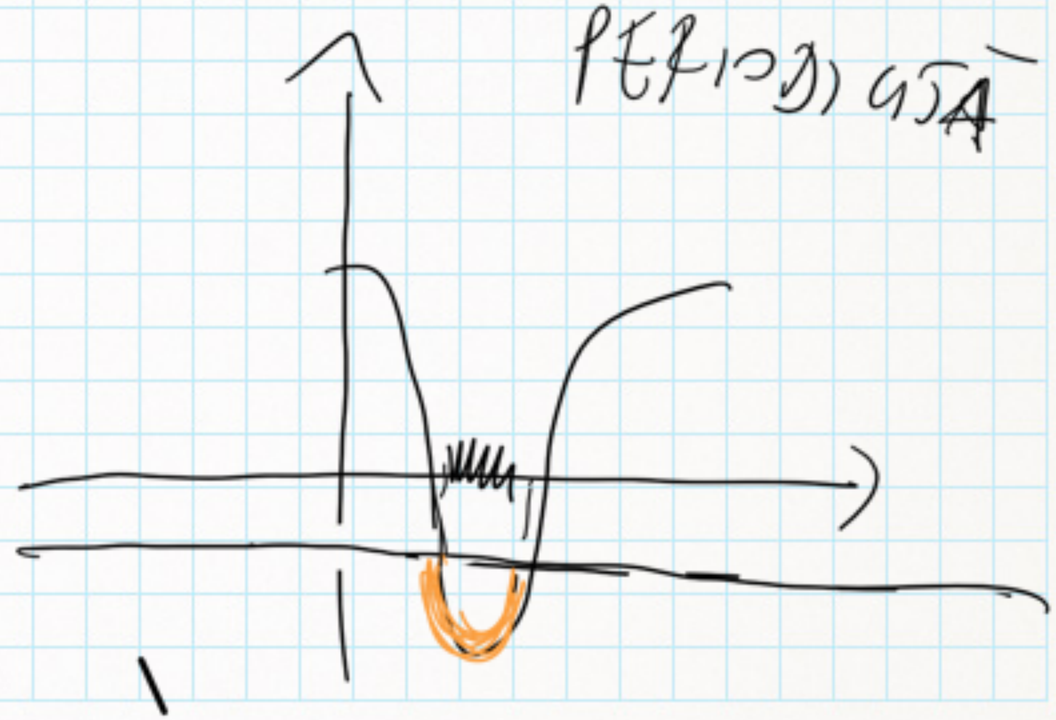


$x = \pi - \frac{\pi}{3} = \frac{2}{3}\pi$

$+2k\pi$

$x = \pi + \frac{\pi}{3} = \frac{4}{3}\pi$

AGGIUNGERE PERIODICITÀ



$\cos x \leq -\frac{1}{2}$

$\frac{2}{3}\pi \leq x \leq \frac{5}{3}\pi$