$$\sqrt[n]{f(x)} \begin{matrix} > \\ < \\ = \end{matrix} g(x) \qquad n \in \mathbb{N}$$

n dispari

$$\left[\sqrt[n]{f(x)}\right]^n \begin{matrix} < \\ > \\ = \end{matrix} [g(x)]^n$$

$$f(x) \begin{matrix} < \\ > \\ = \end{matrix} g(x)^n$$

n pari

$$\sqrt[n]{\underbrace{f(x)}_{}} = \underbrace{g(x)}_{} \qquad \geq 0$$

$$\begin{cases} f(x) \geq 0 \\ g(x) \geq 0 \\ \left[\sqrt[n]{f(x)}\right]^n = g(x)^n \end{cases}$$

$$\sqrt{x^2-3x} = x+7$$

$$\begin{cases} x^2-3x \geq 0 \\ x+7 \geq 0 \\ x^2-3x = (x+7)^2 \end{cases}$$

$x^2-3x=0$

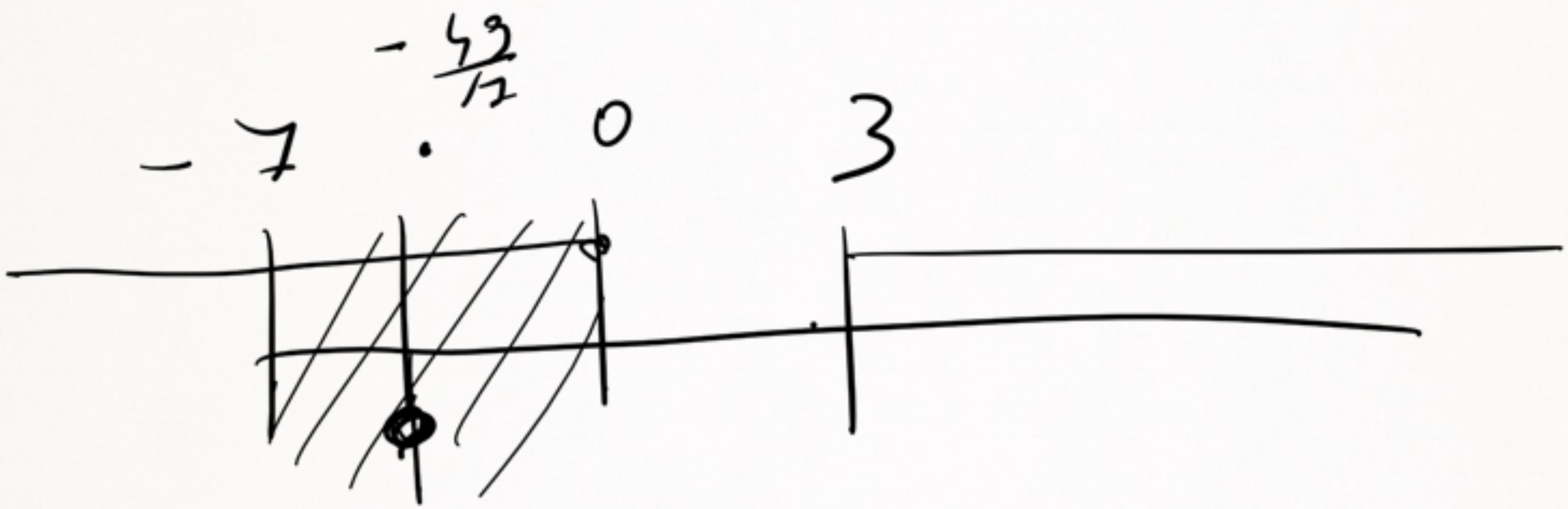
$x(x-3)=0$

$x=0 \lor x=3$

$$\begin{cases} x \leq 0 \lor x \geq 3 \\ x \geq -7 \\ \cancel{x^2}-3x = \cancel{x^2}+14x+49 \end{cases}$$

$$\begin{cases} x \leq 0 \vee x \geq 3 \\ x \geq -7 \\ -17x = 49 \end{cases}$$

$$x = -\frac{49}{17}$$



$$x = -\frac{49}{17}$$

$$\sqrt{x^2+1} = 3x$$

$$\begin{cases} x^2+1 \geq 0 & (\forall x \in \mathbb{R}) \\ 3x \geq 0 \\ x^2+1 = 9x^2 \end{cases}$$

$$\begin{cases} \forall x \in \mathbb{R} \\ x \geq 0 \\ 8x^2 = 1 \end{cases} \Rightarrow \quad x = \pm\sqrt{\frac{1}{8}}$$

l'unica sol. accettabile è $\quad x = \sqrt{\frac{1}{8}}$

$$\sqrt[n]{f(x)} > g(x)$$

$$\begin{cases} f(x) \geq 0 \\ g(x) < 0 \end{cases} \vee \begin{cases} g(x) \geq 0 \\ \cancel{f(x) \geq 0} \\ f(x) > \underbrace{g^n(x)}_{\geq 0} \end{cases}$$

RICAPITOLANDO

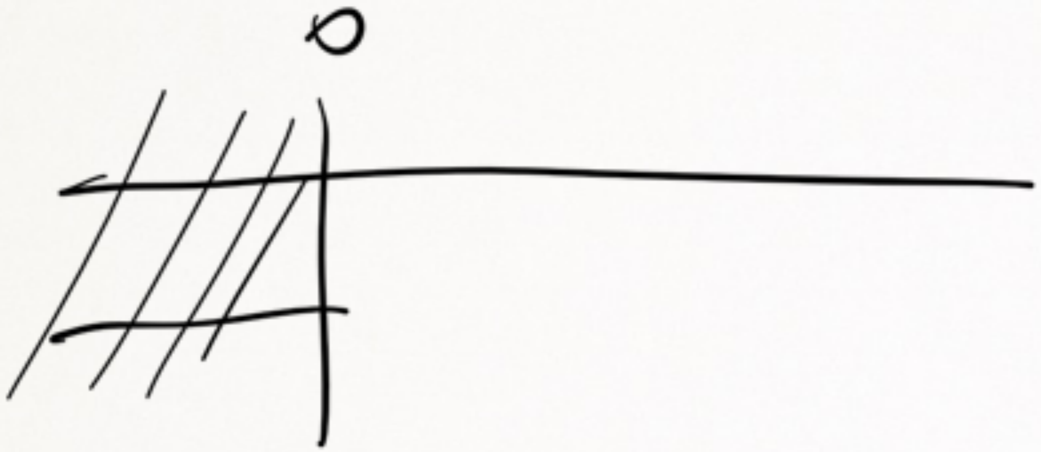
$$\begin{cases} f(x) \geq 0 \\ g(x) < 0 \end{cases} \vee \begin{cases} g(x) \geq 0 \\ f(x) > g^n(x) \end{cases}$$

$$\sqrt{x^2+1} > 3x$$

$$\begin{cases} x^2+1 \geq 0 & (\forall x \in \mathbb{R}) \\ 3x < 0 \end{cases}$$

oppure $\begin{cases} 3x \geq 0 \\ x^2+1 > 9x^2 \end{cases}$

$$\underline{\underline{\begin{cases} \forall x \in \mathbb{R} \\ x < 0 \end{cases}}} \lor \begin{cases} x \geq 0 \\ 8x^2 - 1 < 0 \end{cases}$$



$$\boxed{x < 0}$$

$$\left]-\infty, 0\right[\cup \left[0, \frac{1}{\sqrt{8}}\right[$$

$$= \left]-\infty, \frac{1}{\sqrt{8}}\right[$$

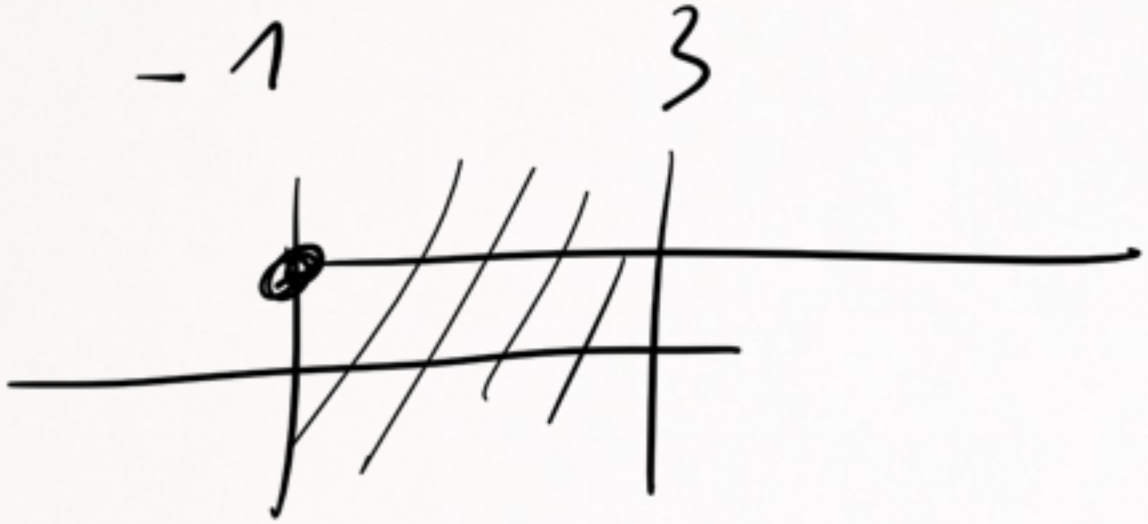
$$\begin{cases} x \geq 0 \\ -\dfrac{1}{\sqrt{8}} < x < \dfrac{1}{\sqrt{8}} \end{cases}$$

$$0 \leq x < \frac{1}{\sqrt{8}}$$

$$\sqrt{x+1} > x-3$$

$$\begin{cases} x+1 \geq 0 \\ x-3 < 0 \end{cases} \vee \begin{cases} x-3 \geq 0 \\ x+1 > (x-3)^2 \end{cases}$$

$$\begin{cases} x \geq -1 \\ x < 3 \end{cases} \vee \begin{cases} x \geq 3 \\ x+1 > x^2-6x+9 \end{cases}$$

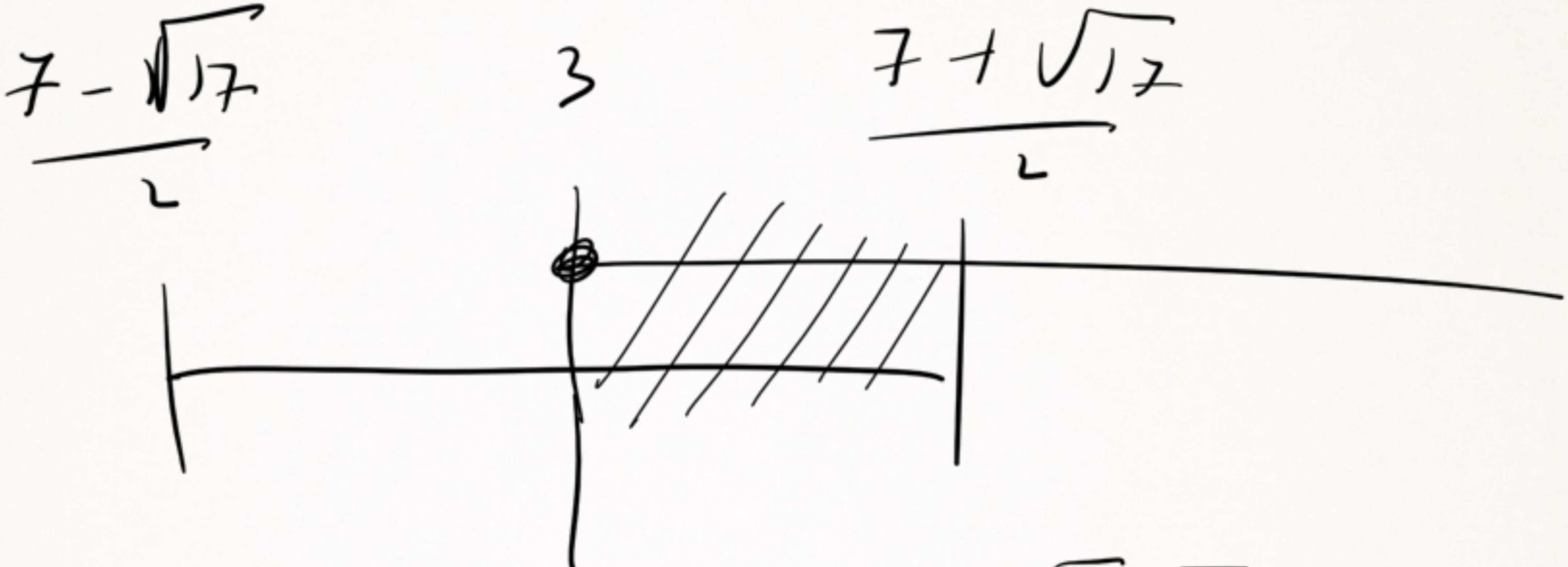


$[-1, 3[$

$$\begin{cases} x \geq 3 \\ x^2 - 7x + 8 \leq 0 \\ x^2 - 7x + 8 = 0 \end{cases}$$

$x_{1,2} = \frac{7 \pm \sqrt{17}}{2}$

$$\begin{cases} x \geq 3 \\ \frac{7-\sqrt{17}}{2} < x < \frac{7+\sqrt{17}}{2} \end{cases}$$



$$\left[3, \frac{7+\sqrt{17}}{2}\right[$$

$$[-1, 3[\ \cup \ \left[3, \frac{7+\sqrt{17}}{2}\right[$$

$$= \left[-1; \frac{7+\sqrt{17}}{2}\right[$$

$$\sqrt[n]{f(x)} < g(x)$$

$$\xcancel{\begin{cases} f(x) \geq 0 \\ g(x) < 0 \end{cases}}$$

$$\boxed{\begin{cases} f(x) \geq 0 \\ g(x) \geq 0 \\ f(x) < g^n(x) \end{cases}}$$

$$\sqrt{x^2-2x} \le x+3$$

$$\begin{cases} x^2-2x \ge 0 \\ x+3 \ge 0 \\ x^2-2x \le (x+3)^2 \end{cases}$$

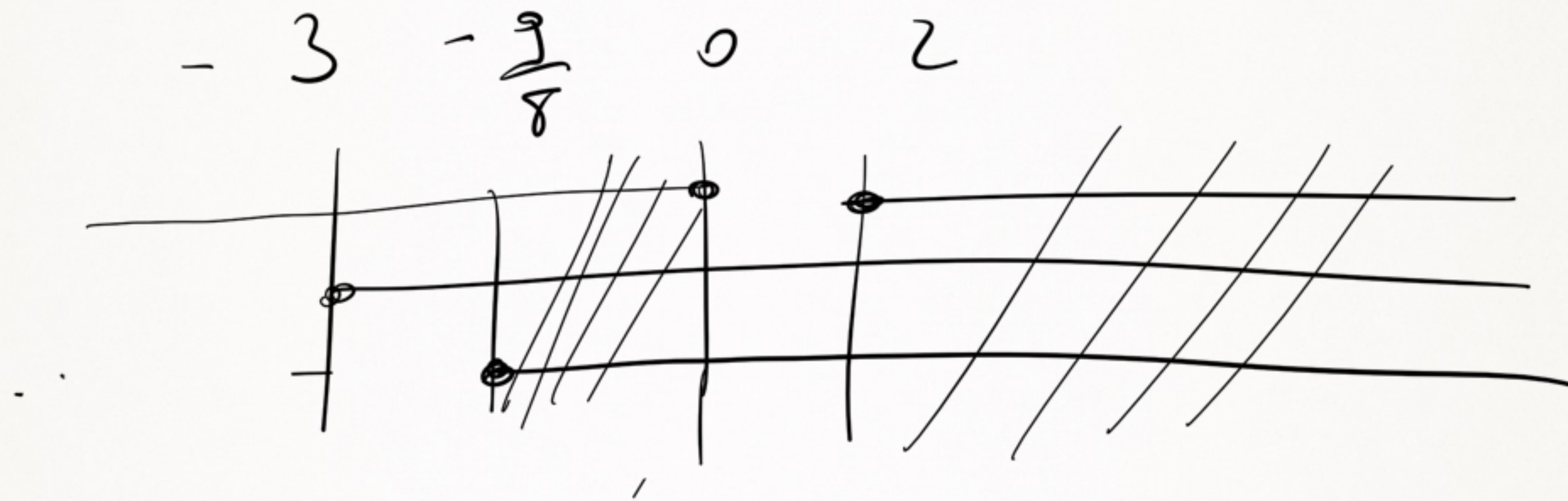
$$x^2-2x=0$$

$$x(x-2)=0$$

$$x=0 \vee x=2$$

$$\begin{cases} x \le 0 \vee x \ge 2 \\ x \ge -3 \\ \cancel{x^2}-2x \le \cancel{x^2}+6x+9 \end{cases}$$

$$\begin{cases} x \le 0 \lor x \ge 2 \\ x \ge -3 \\ -8x \le 9 \qquad\qquad x \ge -\dfrac{9}{8} \end{cases}$$



$$-\frac{9}{8} \le x \le 0 \quad \lor \quad x \ge 2$$

ESPONENZIALE E LOGARITMO

$a > 0, \quad a \neq 1$ BASE

$r \in \mathbb{R}$

$r = p, \alpha_1 \alpha_2 \alpha_3 \dots \alpha_n \dots$

$r > 0, \quad \boxed{a > 1}$

$$a^r = \sup\{ a^{p,\alpha_1 \dots \alpha_n} \mid n \in \mathbb{N}, n \geq 1 \}$$

$$(\sqrt{2})^{\pi}$$

Si $a < 0 < 1$ $\quad (\Rightarrow \frac{1}{a} > 1)$

$$a^r = \frac{1}{\left(\frac{1}{a}\right)^r}$$

Si $r < 0$ $\quad a^r = \dfrac{1}{a^{-r}}$

$(-r > 0)$

$1^r = 1 \qquad 0^r = 0 \quad r > 0$

Propietà $\qquad a^r > 0 \quad \forall r \in \mathbb{R}$

$a, b > 0, \quad a, b \neq 1, \quad r, s \in \mathbb{R}$

$$a^r \cdot a^s = a^{r+s} \qquad (*)$$

$$(ab)^r = a^r \cdot b^r$$

$$(a^r)^s = a^{r \cdot s}$$

$$r < s \Longrightarrow \begin{cases} a^r < a^s & \text{se } a > 1 \\ a^r > a^s & 0 < a < 1 \end{cases}$$

$$a \neq 1 \qquad a^r = a^s \Longrightarrow r = s \qquad (**)$$

Logaritmi

$a > 0$, $a \neq 1$ base

Sia $x > 0$

Si definisce LOGARITMO IN BASE a di x l'unico $y \in \mathbb{R}$ t.c.

$$a^y = x$$

e si denota con $\log_a x$

$\log_a x$

$$y = \lg_e x \iff e^{y} = x$$

$$\boxed{x > 0}$$

NOTAZIONI

e NUMERO DI NEPERO

$\lg_e x$ — $\lg x$, $\ln x$

$\lg_{10} x$ — $\operatorname{Lg} x$

$\log_{10} x$ — $\operatorname{Log} x$

PROPRIETÀ $\quad x, y > 0 \quad a, b > 0$

$a, b \neq 1$

(1) $\log_a 1 = 0 \qquad (a^0 = 1)$

(2) $\log_a a = 1 \qquad (a^1 = a)$

(3) $a^{\log_a x} = x \qquad (\log_a a^x = x)$

(4) $\log_a (x \cdot y) = \log_a x + \log_a y$

Γ PER LA (3) $\boxed{a^{\log_a (x \cdot y)}} = xy =$

$= \underbrace{a^{\log_a x} \, a^{\log_a y}}$ $\overset{*}{=}$ $\boxed{a^{\log_a x + \log_a y}}$

$\Rightarrow$

$$\lg_e(x \cdot y) \qquad \lg_e x + \lg_e y$$

$$0 \qquad = \qquad 0$$

per $\forall x$ $\qquad \lg_e(x \cdot y) = \lg_e x + \lg_e y$

$\boxed{\text{OSS}}$ $\quad x > 0 \quad$ la proprietà si può

$y > 0 \quad$ applicare

Potrei avere $x, y \neq 0$ tc

$x \cdot y > 0 \qquad \lg_e(x \cdot y)$ ha senso

ma $\lg_e x$ e $\lg_e y$ potrebbero non aver senso

$x \cdot y > 0 \qquad x \cdot y = |x \cdot y| = |x| \cdot |y|$

$$\lg_a(x \cdot y) = \lg_a(|x \cdot y|) =$$

$$= \lg_a(|x| \cdot |y|) = \lg_a|x| + \lg_a|y|$$

RICAPITOLANDO

se $x \cdot y > 0$

$$\lg_a(xy) = \lg_a|x| + \lg_a|y|$$

$x > 0, \quad y > 0$

- $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$
- $\forall r \in \mathbb{R} \quad \log_a (x^r) = r \log_a x$
- $\log_a x = \frac{\log_b x}{\log_b a}$
- $\log_a b = \frac{1}{\log_b a}$

$$\log_2 8 = 3$$

$$\log_2 8 = \log_2 \left(2^3\right) = 3 \underbrace{\log_2 2}_{=1} = 3 \cdot 1 = 3$$

$$\log_3\left(\frac{1}{9}\right) = \log_3 (3)^{-2} = -2 \log_3 3 = -2.$$

- $\log_{25} 5 + \log_{25} 125 =$

$$= \log_{25}(5 \cdot 125) = \log_{25} 625 =$$

$$= \log_{25} 25^2 = 2$$

$$\log_e \sqrt{x^2-1} - \frac{1}{2} \log_e (x-1)$$

$$\begin{cases} \sqrt{x^2-1} > 0 \\ x^2-1 \geq 0 \\ x-1 > 0 \end{cases} \qquad \begin{cases} x^2-1 > 0 \\ x-1 > 0 \end{cases}$$

$$\begin{cases} x < -1 \vee x > 1 \\ x > 1 \end{cases} \quad \Bigg| \quad \boxed{x > 1}$$

$$\lg_e \sqrt{x^2-1} - \frac{1}{2}\lg(x-1) =$$

$$= \lg_e (x^2-1)^{\frac{1}{2}} - \frac{1}{2}\lg(x-1) =$$

$$= \frac{1}{2}\lg_e (x^2-1) - \frac{1}{2}\lg(x-1) =$$

$$= \frac{1}{2}\left[\lg_e (x^2-1) - \lg(x-1)\right] =$$

$$= \frac{1}{2}\lg_e \left(\frac{x^2-1}{x-1}\right) =$$

$$= \frac{1}{2}\lg_e \left(\frac{(x+1)\cancel{(x-1)}}{\cancel{x-1}}\right) = \frac{1}{2}\lg(x+1)$$

Proprietà

$$0<x<y \Rightarrow \begin{cases} \log_a x < \log_a y & a>1 \\ \log_a x > \log_a y & 0<a<1 \end{cases}$$

$$x<y \Rightarrow \begin{cases} a^x < a^y & a>1 \\ a^x > a^y & 0<a<1 \end{cases}$$

- $2^x = \frac{1}{4} = 2^{-2} \Rightarrow x = -2$
- $2^x = 73 \Rightarrow \log_2 2^x = \log_2 73$

$$\boxed{x = \log_2 73}$$

$$2^x < 73$$

$$\log_2 2^x < \log_2 73$$

$$x < \log_2 73$$

$$\left(\frac{1}{3}\right)^x \geq 9 \Rightarrow \log_{\frac{1}{3}} \left(\frac{1}{3}\right)^x \leq \log_{\frac{1}{3}} 9$$

$$x \leq \lg_{\frac{1}{3}} 9 = \lg_{\frac{1}{3}} \left(\frac{1}{3}\right)^{-2} = -2$$

$$\left(\frac{1}{4}\right)^x + \left(\frac{1}{2}\right)^x - 2 \geq 0$$

$$\left(\frac{1}{2}\right)^{2x} + \left(\frac{1}{2}\right)^x - 2 \geq 0$$

$$\frac{1}{2}^x = y \qquad y^2 - y - 2 \geq 0$$

$$y^2 + y - 2 = 0 \qquad y = -2 \quad \vee\, y = 1$$

$$y \le -2 \quad \vee \quad y \ge 1$$

$$\cancel{\left(\tfrac{1}{2}\right)^x \le -2} \quad \vee \quad \left(\tfrac{1}{2}\right)^x \ge 1$$

$$\left(\tfrac{1}{2}\right)^x > 0$$

$$\left(\tfrac{1}{2}\right)^x \ge 1$$

$$\log_{\frac{1}{2}} \left(\tfrac{1}{2}\right)^x \le \log_{\frac{1}{2}} 1$$

1

$$x \le \log_{\frac{1}{2}} 1 = 0$$

$$x \le 0$$