$$\int \frac{x}{\sqrt{1-x^2}} \cdot \arcsin x \, dx$$

Integriamo per parti

$g(x) = \arcsin x \qquad g'(x) = \dfrac{1}{\sqrt{1-x^2}}$

$f'(x) = \dfrac{x}{\sqrt{1-x^2}}$

$$\int \frac{x}{\sqrt{1-x^2}} \, dx = \int x \, (1-x^2)^{-\frac{1}{2}} \, dx =$$

$(1-x^2) = t \Rightarrow -2x\,dx = dt$

$$= -\frac{1}{2}\int -2x\,(1-x^2)^{-\frac{1}{2}}\,dx = -\frac{1}{2}\int t^{-\frac{1}{2}}\,dt =$$

$$= -\frac{1}{2}\,\frac{t^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + c = -\cancel{\frac{1}{2}}\,\frac{t^{\frac{1}{2}}}{\cancel{\frac{1}{2}}} + c$$

$$= -\,t^{\frac{1}{2}} + c \overset{t=1-x^2}{=\!=} -\sqrt{1-x^2} + c$$

$$\int \frac{x}{\sqrt{1-x^2}}\,\arcsin x\,dx$$

$g(x) = \arcsin x \qquad g'(x) = \frac{1}{\sqrt{1-x^2}}$

$f'(x) = \frac{x}{\sqrt{1-x^2}} \qquad f(x) = -\sqrt{1-x^2}$

$$\int \frac{x}{\sqrt{1-x^2}} \arcsin x \, dx = -\sqrt{1-x^2} \cdot \arcsin x \; +$$

$$+ \int \cancel{\sqrt{1-x^2}} \cdot \frac{1}{\cancel{\sqrt{1-x^2}}} \, dx = -\sqrt{1-x^2} \cdot \arcsin x$$

$$+ \int dx = -\sqrt{1-x^2} \cdot \arcsin x + x + c$$

$$\int_{\lg 2}^{\lg 3} \frac{e^x}{e^{2x} - 2e^x + 2} \, dx \quad (x)$$

$$e^x = t$$

$$e^x \, dx = dt$$

$$x = \lg 2 \qquad t = e^{\lg 2} = 2$$

$$x = \lg 3 \qquad t = e^{\lg 3} = 3$$

(4) $\displaystyle\int_2^3 \frac{1}{t^2-2t+2}\,dt$

$$\boxed{\frac{1}{t^2-2t+2} = \frac{A\,(2t-2)}{t^2-2t+2} + \frac{B}{t^2-2t+2}}$$

$$t^2-2t+2 = (t-1)^2+1$$

$$\int_2^3 \frac{1}{t^2-2t+1}\,dt = \int_2^3 \frac{1}{(t-1)^2+1}\,dt =$$

$$= \left[\operatorname{arctg}(t-1)\right]_2^3 = \operatorname{artg} 2 - \operatorname{artg} 1 = \operatorname{arctg} 2 - \frac{\pi}{4}$$

$$\int \frac{x+2}{x^2-x+1}\,dx$$

$$\frac{x+2}{\underbrace{x^2-x+1}_{P(x)}} = \frac{A\ P'(x)}{P(x)} + \frac{B}{P(x)} =$$

$$= \frac{A(2x-1)}{x^2-x+1} + \frac{B}{x^2-x+1} =$$

$$= \frac{2Ax - A + B}{x^2-x+1} \iff x+2 = 2Ax - A + B$$

$$\begin{cases} 2A = 1 \\ -A + B = 2 \end{cases} \qquad \begin{cases} A = \frac{1}{2} \\ B = 2 + A = 2 + \frac{1}{2} = \frac{5}{2} \end{cases}$$

$$\frac{x+2}{x^2 - x + 1} = \frac{1}{2} \frac{(2x-1)}{x^2 - x + 1} + \frac{5}{2} \frac{1}{x^2 - x + 1}$$

$$\int \frac{x+2}{x^2 - x + 1} dx = \underbrace{\frac{1}{2} \int \frac{2x-1}{x^2 - x + 1} dx} + \frac{5}{2} \int \frac{1}{x^2 - x + 1} dx$$

$$= \frac{1}{2} \lg (x^2 - x + 1) + \frac{5}{2} \int \frac{1}{x^2 - x + 1} dx$$

$$\int \frac{1}{\underbrace{x^2 - x + 1}} \, dx \ \circledast$$

$$ax^2 + bx + c$$

$$\Delta = b^2 - 4ac$$

$$x^2 - x + 1 = \left(x + \frac{b}{2a}\right)^2 - \frac{\Delta}{4a} =$$

$$= \left(x - \frac{1}{2}\right)^2 + \frac{3}{4}$$

$$\circledast \int \frac{1}{x^2 - x + 1} \, dx = \int \frac{1}{\left(x - \frac{1}{2}\right)^2 + \frac{3}{4}} \, dx =$$

$$= \int \frac{1}{\frac{3}{4}\left[\frac{\left(x-\frac{1}{2}\right)^2}{\frac{3}{4}} + 1\right]}\,dx =$$

$$= \frac{4}{3}\int \frac{1}{\left(\frac{x-\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)^2 + 1}\,dx = \qquad \boxed{\frac{4}{3} = \frac{2}{\sqrt{3}} \cdot \frac{2}{\sqrt{3}}}$$

$$= \frac{4}{3}\int \frac{1}{\left(\frac{2x-1}{\cancel{2}} \cdot \frac{\cancel{2}}{\sqrt{3}}\right)^2 + 1}\,dx = \frac{4}{3}\int \frac{1}{\left(\frac{2x-1}{\sqrt{3}}\right)^2 + 1}\,dx$$

$$= \frac{2}{\sqrt{3}}\int \frac{\frac{2}{\sqrt{3}}}{\left(\frac{2x-1}{\sqrt{3}}\right)^2 + 1}\,dx = \frac{2}{\sqrt{3}}\,\mathrm{arctg}\left(\frac{2x-1}{\sqrt{3}}\right) + c$$

$$\int_2^3 \frac{x+\sqrt{x-1}}{x-5}\,dx$$

$$\sqrt{x-1} = t \Rightarrow x-1 = t^2 \Rightarrow$$

$$\Rightarrow \underline{x = t^2+1}$$

$$dx = 2t\,dt$$

$$x = 2 \quad t = \sqrt{2-1} = 1$$

$$x = 3 \quad t = \sqrt{3-1} = \sqrt{2}$$

$$\int_2^3 \frac{x+\sqrt{x-1}}{x-3}\,dx = \int_1^{\sqrt{2}} \frac{t^2+1+t}{t^2+1-5}\cdot 2t\,dt$$

$$= 2\int_1^{\sqrt{2}} \frac{t^3+t^2+t}{t^2-4}\,dt$$

$$\begin{array}{lll|l}
t^3 & +t^2 & +t & t^2-4 \\ \cline{4-4}
t^3 & & -4t & t+1 \\ \cline{1-3}
// & t^2 & +5t & \\
 & t^2 & -4 & \\ \cline{2-3}
 & // & 5t+4 &
\end{array}$$

$$\frac{t^3+t^2+t}{t^2-4} = t+1 + \frac{5t+4}{t^2-4}$$

$$\int \frac{t^3+t^2+t}{t^2-4}\,dt = \int \left(t+1 + \frac{5t+4}{t^2-4}\right) dt$$

$$= \int t\,dt + \int dt + \int \frac{5t+4}{t^2-4}\,dt =$$

$$= \frac{t^2}{2} + t + \int \frac{5t-4}{t^2-4}\,dt$$

$$t^2 - 4 = (t-2)(t+2)$$

$$\frac{5t+4}{t^2-4} = \frac{A}{t-2} + \frac{B}{t+2} =$$

$$= \frac{At + 2A + Bt - 2B}{(t-2)(t+2)} = \frac{t(A+B) + 2A - 2B}{t^2-4}$$

$$\iff \quad 5t+4 = t(A+B) + 2A - 2B$$

$$\begin{cases} A + B = 5 \\ 2A - 2B = 4 \end{cases} \qquad \begin{cases} A + B = 5 \\ A - B = 2 \end{cases} \qquad \begin{cases} A = 5 - B \\ 5 - 2B = 2 \end{cases}$$

$$\begin{cases} A = 5 - B \\ -2B = -3 \end{cases} \qquad \begin{cases} A = 5 - \frac{3}{2} = \frac{7}{2} \\ B = \frac{3}{2} \end{cases}$$

$$\frac{5t+4}{t^2-4} = \frac{7}{2} \cdot \frac{1}{t-2} + \frac{3}{2} \cdot \frac{1}{t+2}$$

$$\int \frac{5t+4}{t^2-4}\,dt = \frac{7}{2}\int \frac{1}{t-2}\,dt + \frac{3}{2}\int \frac{1}{t+2}\,dt$$

$$= \frac{7}{2}\lg|t-2| + \frac{3}{2}\lg|t+2| + C$$

$$\int_1^{\sqrt{2}} \frac{t^3+t^2+t}{t^2-4}\,dt = \Big[\frac{t^2}{2} + t + \frac{7}{2}\lg|t-2| +$$

$$+\frac{3}{2}\lg|t+2|\Big]_1^{\sqrt{2}} = \cancel{1} + \sqrt{2} + \frac{7}{2}\underbrace{\lg|\sqrt{2}-2|} +$$

$$+\frac{3}{2}\lg(\sqrt{2}+2) - \frac{1}{2} \cancel{-1} - \cancel{\frac{7}{2}\lg|-1|}$$

$$-\frac{3}{2}\lg 3 = \sqrt{2} + \frac{7}{2}\lg(2-\sqrt{2}) +$$

$$+\frac{3}{2}\lg(2+\sqrt{2}) - \frac{1}{2} - \frac{3}{2}\lg 3$$

$$\int \frac{1}{4\sin x + 3\cos x}\,dx$$

USIAMO LE FORMULE PARAMETRICHE

$$t = \operatorname{tg}\frac{x}{2} \qquad \sin x = \frac{2t}{1+t^2}$$

$$\frac{x}{2} = \operatorname{arctg} t$$

$$x = 2\operatorname{arctg} t \qquad \cos x = \frac{1-t^2}{1+t^2}$$

$$dx = \frac{2}{t^2+1}\,dt$$

$$\int \frac{1}{4\sin x + 3\cos x}\,dx = \int \frac{1}{4\,\frac{2t}{1+t^2} + 3\,\frac{1-t^2}{1+t^2}} \cdot \frac{2}{t^2+1}\,dt$$

$$= \int \frac{1}{\frac{8t}{1+t^2} + \frac{3-3t^2}{1+t^2}} \cdot \frac{2}{t^2+1}\,dt$$

$$= \int \frac{1}{\frac{-3t^2+8t+3}{t^2+1}} \cdot \frac{2}{t^2+1}\,dt =$$

$$= \int \frac{\cancel{t^2+1}}{-3t^2+8t+3} \cdot \frac{2}{\cancel{t^2+1}}\,dt = -2\int \frac{1}{3t^2-8t-3}\,dt.$$

DEVO RISOLVERE

$$\int \frac{1}{3t^2-8t-3}\,dt$$

$$\Delta = 64+36 = 100 > 0$$

$$3t^2-8t-3=0$$

$$t_{1,2} = \frac{8 \pm \sqrt{100}}{6} = \frac{8 \pm 10}{6} = \begin{cases} -\frac{1}{3} \\ 3 \end{cases}$$

$$3t^2-8t-3 = 3(t-3)\left(t+\frac{1}{3}\right) =$$

$$= (t-3)(3t+1)$$

$$\frac{1}{3t^2-8t-3} = \frac{A}{t-3} + \frac{B}{3t+1} =$$

$$= \frac{3At + A + Bt - 3B}{3t^2-8t-3} = \frac{t(3A+B) + A - 3B}{3t^2-8t-3}$$

$$\iff t(3A+B) + A - 3B = 1$$

$$\begin{cases} 3A + B = 0 \\ A - 3B = 1 \end{cases} \qquad \begin{cases} B = -3A \\ A + 9A = 1 \end{cases}$$

$$\begin{cases} B = -3A \\ 10A = 1 \end{cases} \qquad \begin{cases} B = -\frac{3}{10} \\ A = \frac{1}{10} \end{cases}$$

$$\int \frac{1}{3t^2-5t-3}\,dt = \frac{1}{10}\int \frac{1}{t-3}\,dt - \frac{3}{10}\int \frac{1}{3t+1}\,dt$$

$$= \frac{1}{10}\lg|t-3| - \frac{1}{10}\int \frac{3}{3t+1}\,dt$$

$$= \frac{1}{10}\lg|t-3| - \frac{1}{10}\lg|3t+1| + c$$

$$= \frac{1}{10}\left[\lg|t-3| - \lg|3t+1|\right] + c$$

$t = \operatorname{tg}\frac{x}{2}$

$$= \frac{1}{10}\lg\left|\frac{t-3}{3t+1}\right| + c = \frac{1}{10}\lg\left|\frac{\operatorname{tg}\frac{x}{2}-3}{3\operatorname{tg}\frac{x}{2}+1}\right| + c$$

$$\int \frac{x-5}{x^2-6x+9}\,dx = \int \frac{x-5}{(x-3)^2}\,dx$$

$$\frac{x-5}{(x-3)^2} = \frac{A}{x-3} + \frac{B}{(x-3)^2} =$$

$$= \frac{A(x-3)+B}{(x-3)^2} = \frac{Ax-3A+B}{(x-3)^2}$$

$$\Leftrightarrow \quad x-5 = Ax-3A+B$$

$$\begin{cases} A = 1 \\ -3A+B = -5 \end{cases} \qquad \begin{cases} A = 1 \\ -3+B = -5 \end{cases} \qquad \begin{cases} A = 1 \\ B = -2 \end{cases}$$

$$\frac{x-5}{(x-3)^2} = \frac{1}{x-3} - \frac{2}{(x-3)^2}$$

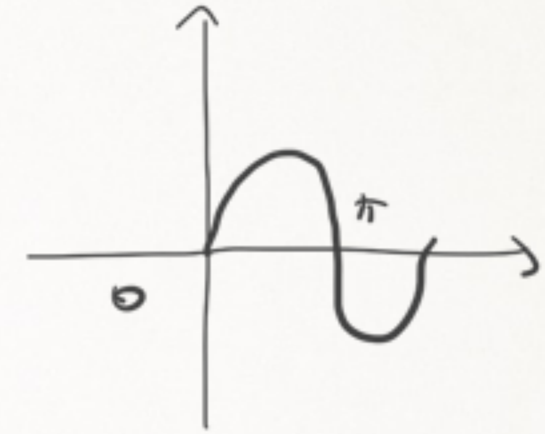
$$\int \frac{x-5}{(x-3)^2}\,dx = \int \frac{1}{x-3}\,dx - 2\int \frac{1}{(x-3)^2}\,dx$$

$$= \lg|x-3| - 2\int (x-3)^{-2}\,dx =$$

$$= \lg|x-3| - 2\,\frac{(x-3)^{-2+1}}{\underbrace{-2+1}_{=-1}} + C =$$

$$= \lg|x-3| + \frac{2}{x-3} + c$$

$$\int_0^{\frac{1}{2}} \frac{x^3}{\sqrt{\underbrace{1-x^2}}}\, dx$$



$x = \cos t \qquad t \in [0, \pi]$

$\sin t \geq 0$ in $[0, \pi]$

$1 - x^2 = 1 - \cos^2 x = \sin^2 t$

$\sqrt{1-x^2} = \sqrt{\sin^2 t} = |\sin t| = \sin t$

$dx = -\sin t \, dt$

$x = 0 \quad . \quad \cos t = 0 \quad t = \frac{\pi}{2}$

$x = \frac{1}{2} \quad \cos t = \frac{1}{2} \quad t = \frac{\pi}{3}$

$$\int_{\frac{\pi}{2}}^{\frac{\pi}{3}} \frac{\cos^3 t}{\cancel{\sin t}} (-\cancel{\sin t})\, dt =$$

$$= -\int_{\frac{\pi}{2}}^{\frac{\pi}{3}} \cos^3 t\, dt = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cos^3 t\, dt.$$

$$= \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cos t \cdot \cos^2 t\, dt =$$

$$= \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cos t (1 - \sin^2 t)\, dt$$

$\sin t = s$

$\cos t\, dt = ds$

$\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$\sin \frac{\pi}{2} = 1$

$$= \int_{\frac{\sqrt{3}}{2}}^{1} (1 - s^2)\, ds = \left[s - \frac{s^3}{3} \right]_{\frac{\sqrt{3}}{2}}^{1}$$

$$\int \sin^3 x \, dx = \int \sin x \cdot \sin^2 x \, dx$$

$$= \int \sin x \,(1 - \cos^2 x)\, dx$$