

MIRELLA CAPPELLETTI MONTANO

DIPARTIMENTO DI MATEMATICA

III PIANO, UFFICIO 12

MERCOLEDÌ 16-18

mirella.cappelletti@montanocunibe.it

dm.cunibe.it

personale

$$x^2 - 5x + 6 = 0$$

$$ax^2 + bx + c = 0$$

$$\Delta = b^2 - 4ac$$

$$x_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a}$$

$$x^2 - 5x + 6 > 0$$

VALORI ESTERNI

$$x < 2 \vee x > 3$$

$$]-\infty, 2[\cup]3, +\infty[$$

$$\Delta = 25 - 24 = 1 > 0$$

$$x_{1,2} = \frac{5 \pm \sqrt{1}}{2}$$

$$= \frac{5 \pm 1}{2} \quad \begin{array}{l} 3 \\ 2 \end{array}$$

$$x^2 - x + 1 = 0$$

NON ADMITTE SOLUT.

$$\Delta = (-1)^2 - 4 \cdot 1 = 1 - 4 = -3 < 0$$

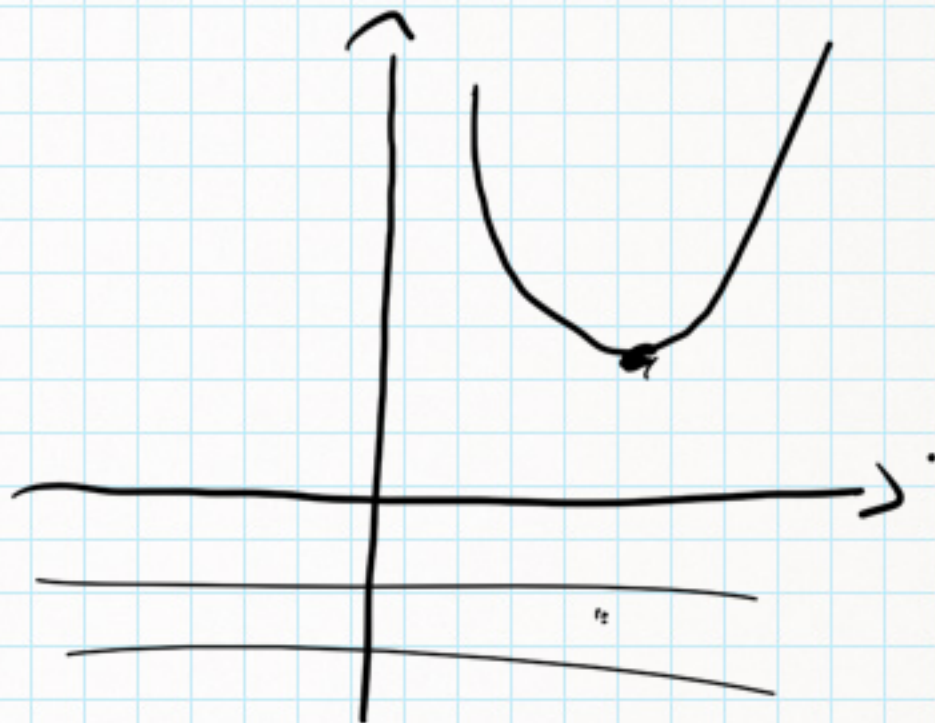
$$x^2 - x + 1 < 0$$

NO SOLUTIONS

$$y = x^2 - x + 1$$

$$V \left(-\frac{b}{2a}, \frac{-\Delta}{4a} \right)$$

> 0



$$x^2 - 2x + 1 \leq 0$$

$$\boxed{\Delta = (-2)^2 - 4 = 4 - 4 = 0}$$

$$(x-1)^2 \leq 0$$

$$x = 1$$

$$[f(x)]^{2n} \geq 0$$

$$(x-1)^2 > 0$$

$$\forall x \neq 1$$

$$x^3 - 3x^2 + 2x = 0 \quad ||$$

$$x(x^2 - 3x + 2) = 0 \quad (\Leftrightarrow) \quad x = 0 \quad \checkmark$$

$$x^2 - 3x + 2 = 0$$

↓

$$x = 1 \quad \checkmark \quad x = 2$$

$$x^3 - 4x - 3 = 0$$

$$\pm 1, \quad \pm 3$$

$$\textcircled{-1}$$

$$(-1)^3 - 4(-1) - 3 =$$

$$= -1 + 4 - 3 = 0$$

$$\begin{array}{c|ccc|c} & 1 & 0 & -4 & -3 \\ -1 & & -1 & 1 & 3 \\ \hline & 1 & -1 & -3 & // \end{array}$$

$$x^3 - 4x - 3 = 0$$

$$x^3 - 4x - 3 = (x+1)(x^2 - x - 3) = 0$$

$$\Leftrightarrow x+1=0 \quad \vee \quad x^2 - x - 3 = 0$$

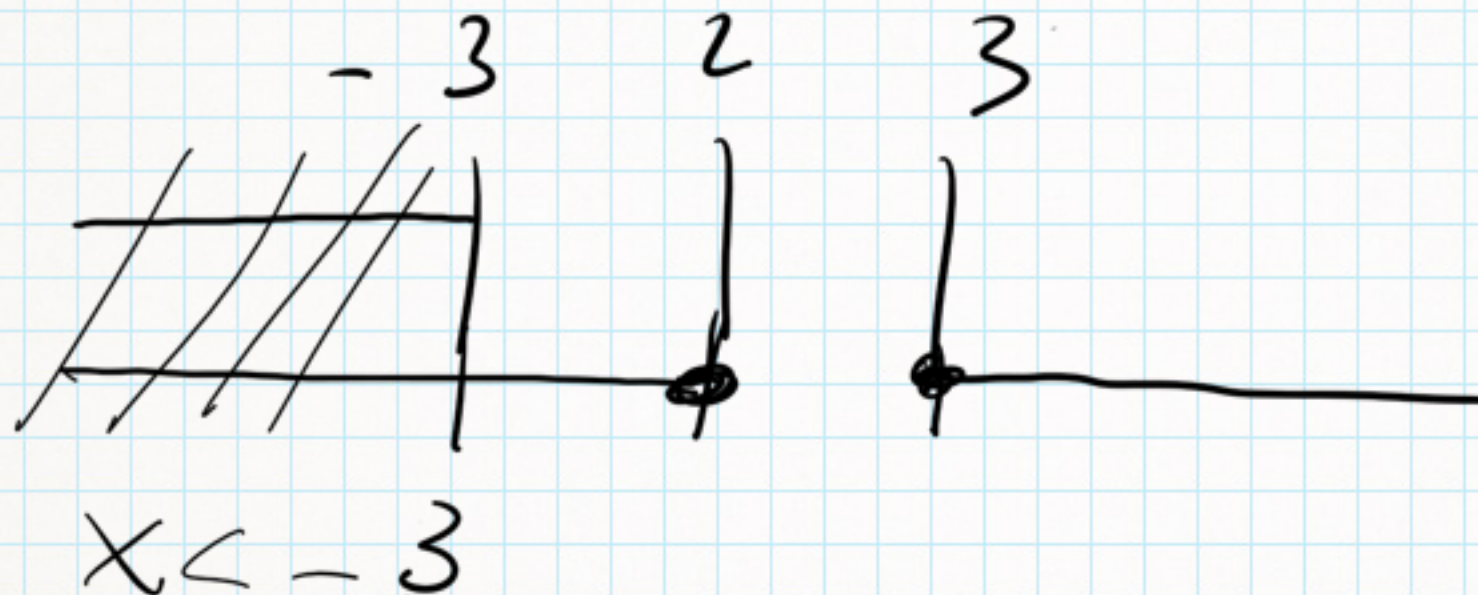
$$x_{1,2} = \frac{1 \pm \sqrt{13}}{2}$$

SISTEMI DI DISEQUAZIONI

$$\begin{cases} x + 3 < 0 \\ x^2 - 5x + 6 \geq 0 \end{cases}$$

INTERSEZIONE
TRA INSERITE
DELLE SOLUZIONI

$$\begin{cases} x < -3 \\ x \leq 2 \vee x \geq 3 \end{cases}$$



$$x^3 - 3x^2 + 2x = 0$$

$$x = 0$$

$$x = 1$$

$$x = 2$$

$$x^3 - 3x^2 + 2x < 0$$

$$x(x^2 - 3x + 2) \leq 0$$

↑

↑

$$\begin{cases} x < 0 \\ x^2 - 3x + 2 > 0 \end{cases}$$

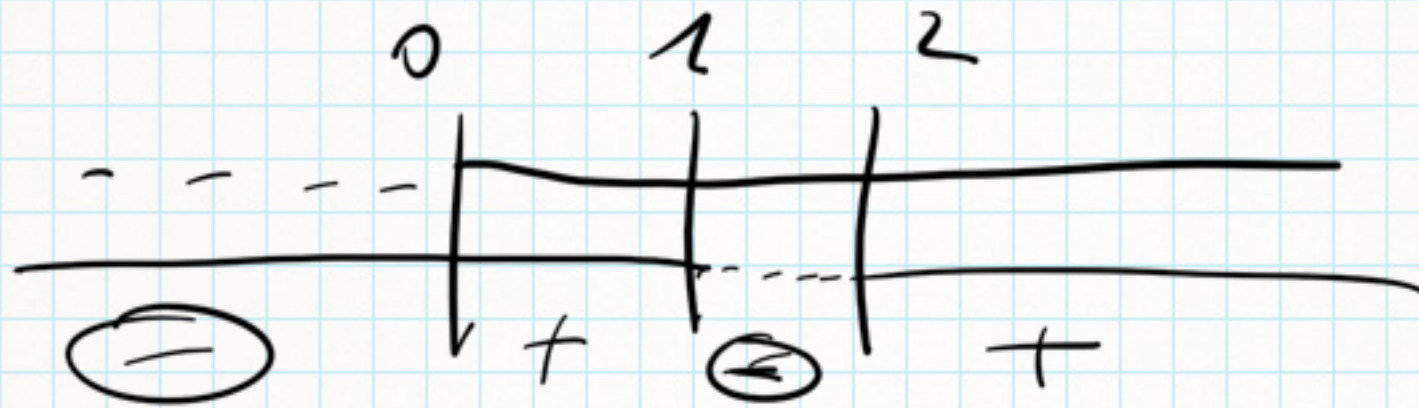
$$\vee \begin{cases} x > 0 \\ x^2 - 3x + 2 < 0 \end{cases}$$

$$x > 0$$

$$x(x^2 - 3x + 2)$$

$$x^2 - 3x + 2 > 0$$

$$x < 1 \vee x > 2$$



$$x < 0 \vee 1 < x < 2$$

$$x^3 - 4x - 3 > 0$$

(e case)

$$\frac{x^2 - 3x + 2}{x - 7} \leq 0$$

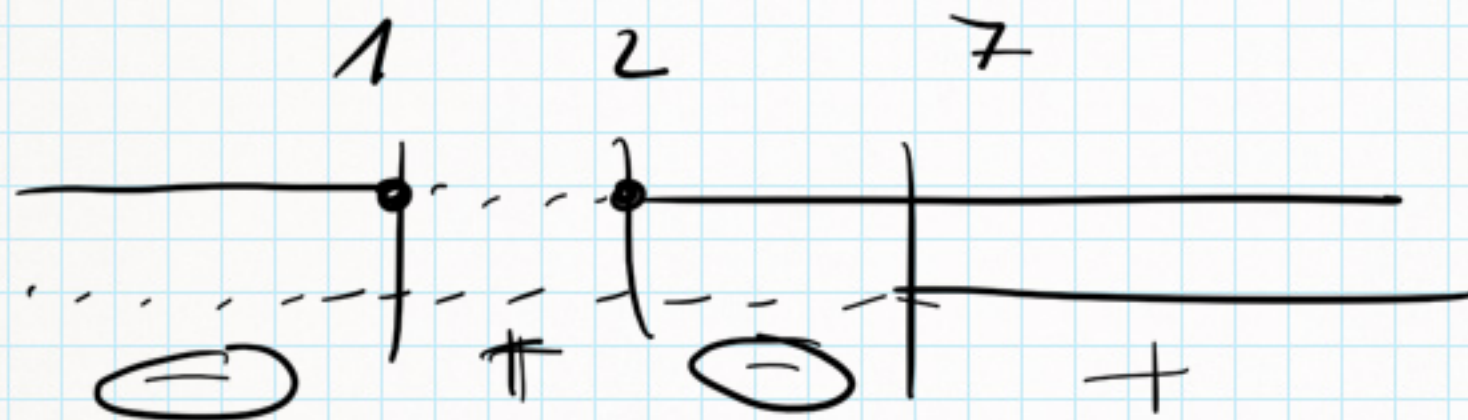
$$x \neq 7$$

$$x^2 - 3x + 2 \geq 0$$

$$x \leq 1 \vee x \geq 2$$

$$x - 7 > 0$$

$$x > 7$$



$$x \leq 1 \vee 2 \leq x < 7$$

$$\begin{cases} \frac{5-x}{x+2} > 0 & \textcircled{1} \\ x^2 - x - 2 \geq 0 & \textcircled{2} \end{cases}$$

$$\begin{cases} -2 < x < 5 \end{cases}$$

$$\textcircled{1} \quad \frac{5-x}{x+2} > 0 \quad x \neq -2$$

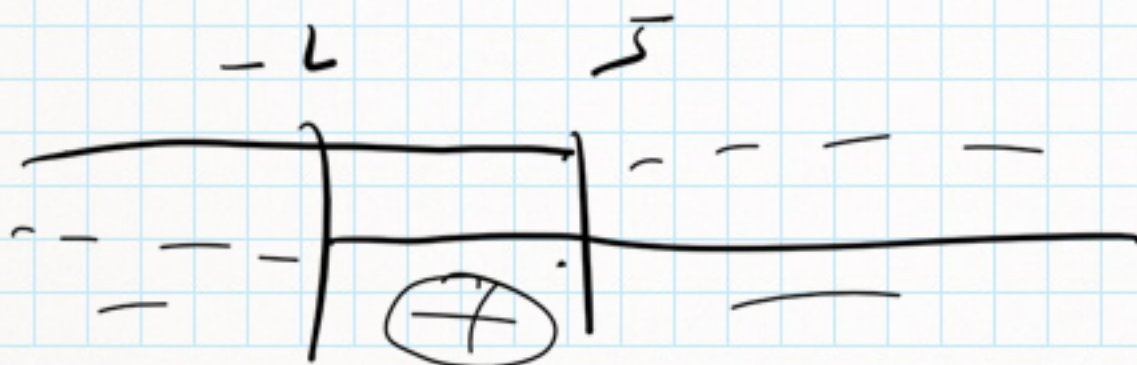
$$5-x > 0,$$

$$-x > -5$$

$$x < 5$$

$$x+2 > 0$$

$$x > -2$$



$$x^2 - x - 2 > 0$$

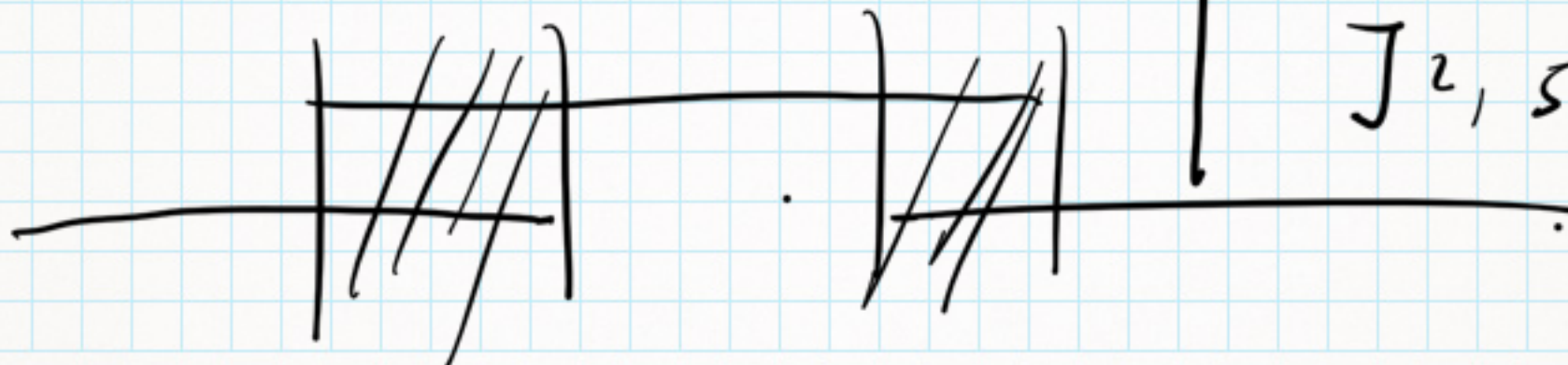
$$x < -1 \vee x > 2$$

$$x^2 - x - 2 = 0$$

$$x_{1,2} = \frac{1 \pm \sqrt{1+8}}{1} = \frac{1 \pm 3}{1}$$

$$\begin{cases} -2 < x < 5 \\ x < -1 \vee x > 2 \end{cases}$$

$$\begin{cases} -2 < x < -1 \\ \checkmark \\ 2 < x < 5 \\]-2, -1[\vee \\]2, 5[\end{cases}$$



VALORE ASSOLUTO

$$x \in \mathbb{R}$$

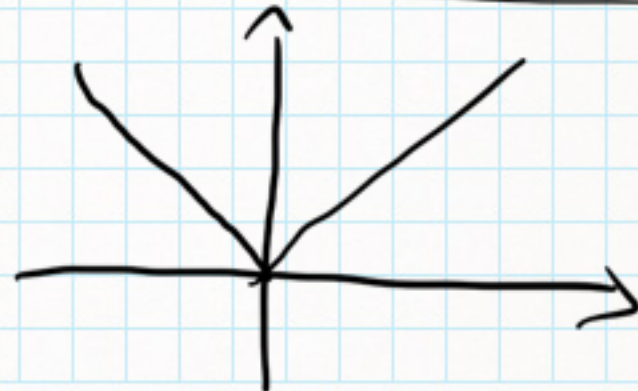
$$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

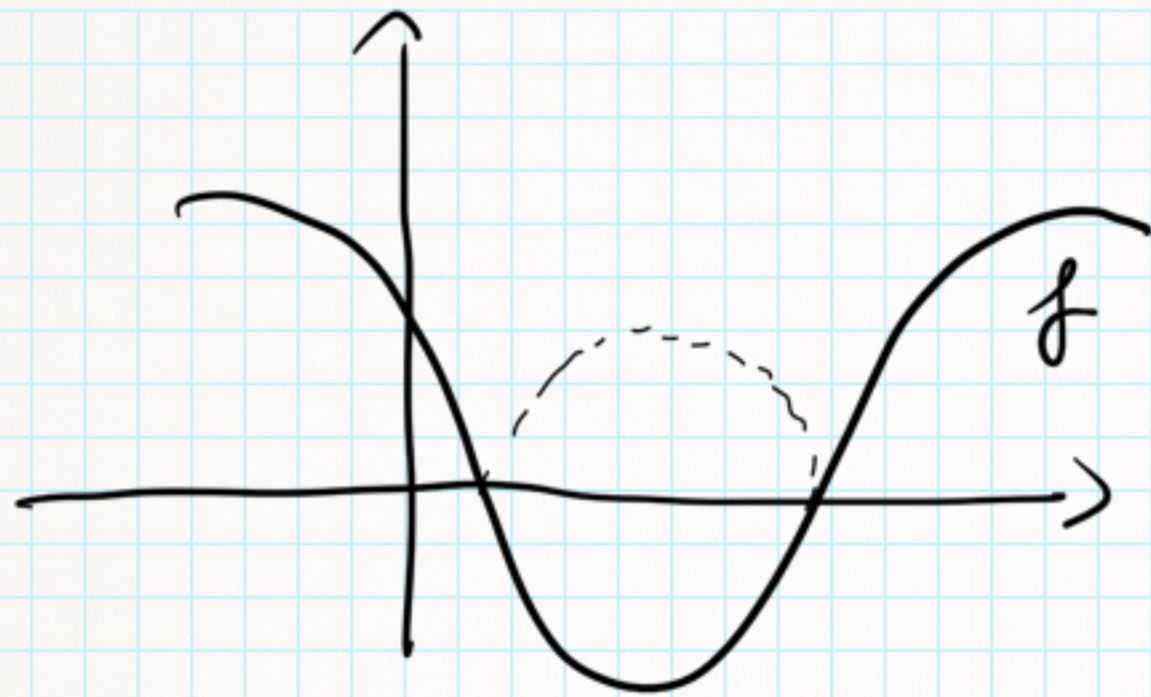
$$|3| = 3$$

$$|-\sqrt{5}| = \sqrt{5}$$

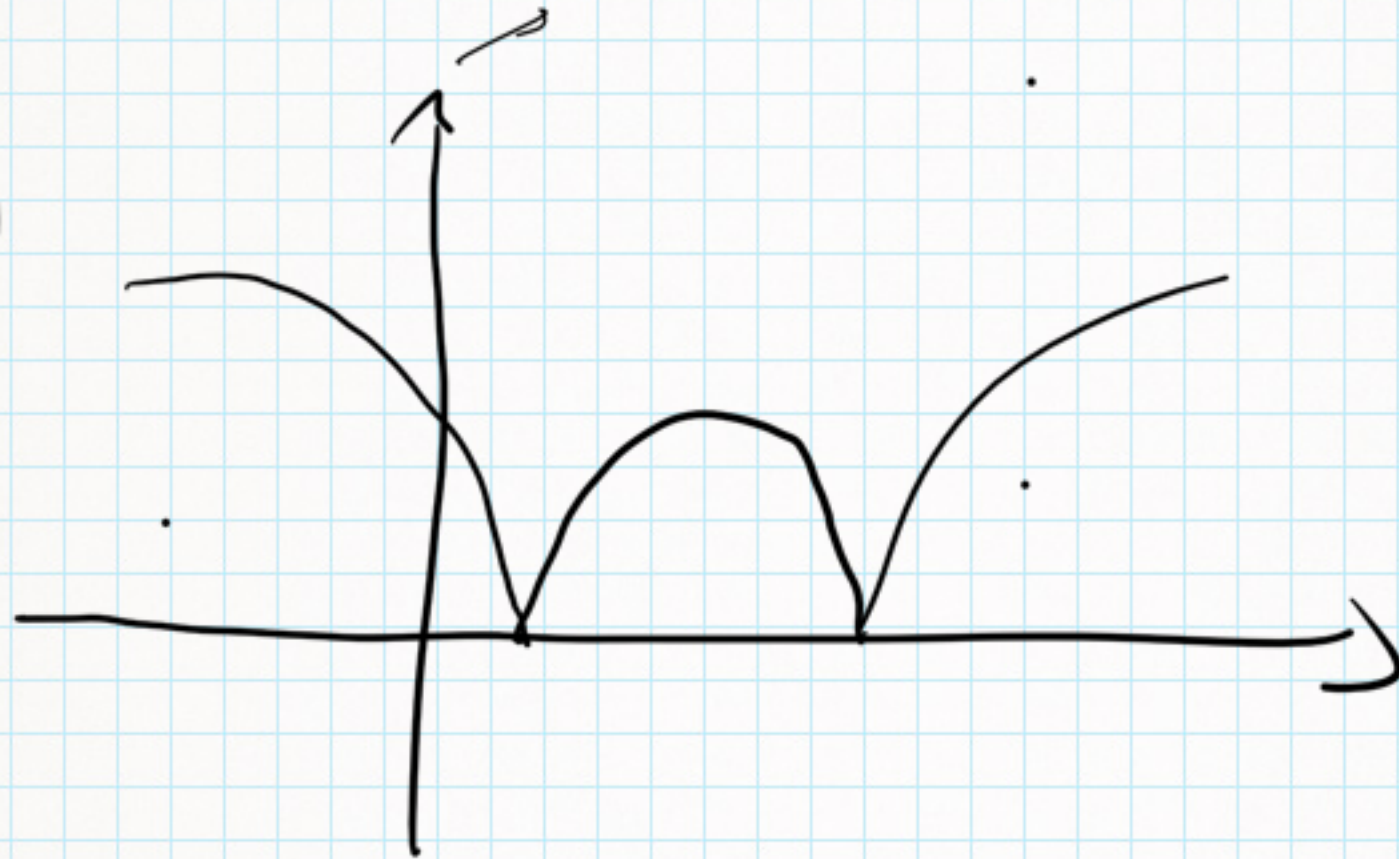
$$|x| \geq 0 \quad \forall x \in \mathbb{R} \quad \text{e} \quad \underline{|x| = 0 \Leftrightarrow x = 0}$$

$$\begin{array}{l} | | : \mathbb{R} \rightarrow \mathbb{R} \\ f \\ x \mapsto |x| \end{array}$$





$|f|$



$$|f(x)| = c$$

$$\boxed{c \geq 0}$$

$$- c = 0$$

$$|f(x)| = 0 \Leftrightarrow f(x) = 0$$

$$\left| \frac{x-3}{x-1} \right| = 0 \Leftrightarrow$$

$$\frac{x-3}{x-1} = 0$$

$$\Leftrightarrow x = 3$$

$$\left| \frac{x-3}{x-1} \right| \geq 1 \Leftrightarrow$$

$$\frac{x-3}{x-1} \geq 1 \quad \vee$$

$$\frac{x-3}{x-1} \leq -1$$

$$\frac{x-3}{x-1} = 1 \Leftrightarrow \cancel{x}-3 = \cancel{x}-1 \Leftrightarrow -3 = -1$$

↓
Solo

EQUATION

NO DIS-EQUATION

$$\frac{x-3}{x-1} = -1 \Leftrightarrow x-3 = -x+1$$

$$\Leftrightarrow 2x = 4 \Leftrightarrow \boxed{x = 2}$$

$$|f(x)| = e^{\text{>0}} \Leftrightarrow f(x) = e \vee f(x) = -e$$

$$|x-3| + x^2 = 1$$

$$|x-3| = \underbrace{1-x^2}$$

$$\begin{cases} x-3 \geq 0 \\ x-3+x^2=1 \end{cases}$$

✓

$$\begin{cases} x-3 < 0 \\ -(x-3)+x^2=1 \end{cases}$$

$$\begin{cases} x \geq 3 \\ x^2+x-4 \geq 0 \end{cases}$$

$$\begin{cases} x < 3 \\ x^2-x+2 = 0 \end{cases}$$

no sol.

$\Delta < 0$

$$x^2 - 4 = 0$$

$$x_{1,2} = \frac{-1 \pm \sqrt{17}}{2}$$

$$\left\{ \begin{array}{l} x \geq 3 \end{array} \right.$$

$$\left\{ \begin{array}{l} x_1 = \frac{-1 - \sqrt{17}}{2} \end{array} \right.$$

✓

$$\left\{ \begin{array}{l} x = \frac{-1 + \sqrt{17}}{2} \end{array} \right.$$

So LU is not, NOW ACCEPTABLE

$$|f(x)| > c$$

$$c \in \mathbb{R}$$

$$c < 0$$

$$\forall x \in \mathbb{R}$$

(, $\forall x$ nel
dominio di f)

$$c = 0$$

$$|f(x)| > 0$$

$$\forall x \in \mathbb{R} \text{ t.c.}$$

$$f(x) \neq 0$$

$$c > 0$$

$$|f(x)| > c \iff$$

$$f(x) < -c \quad \vee \quad f(x) > c$$

$$|f(x)| > c$$

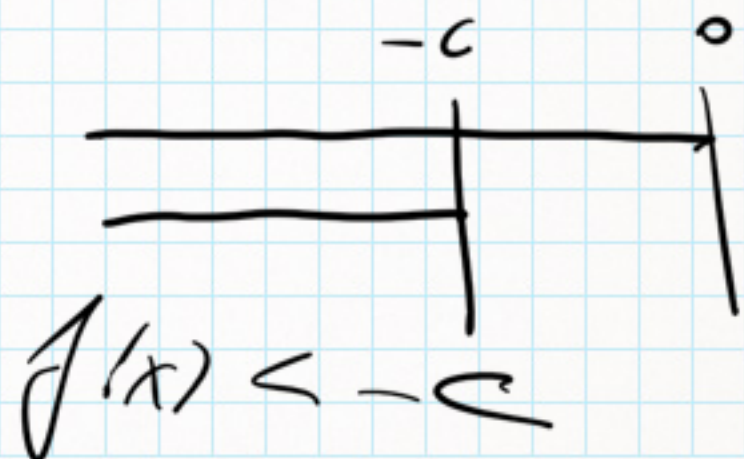
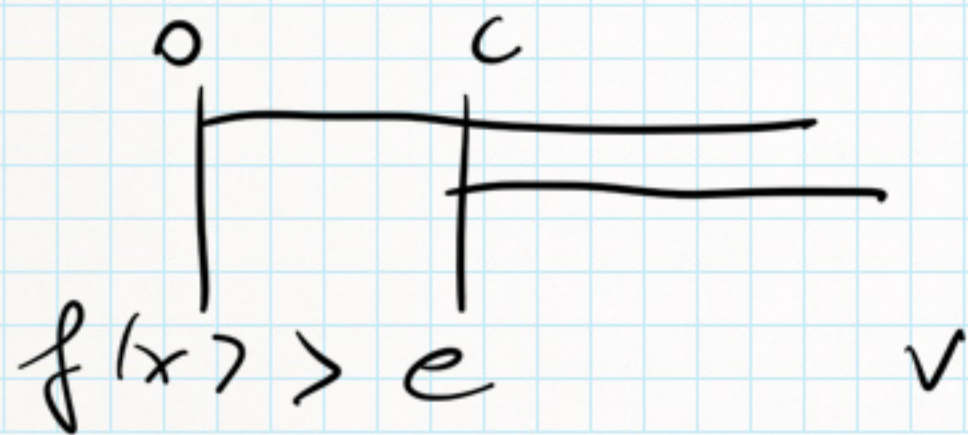
$c > 0$

$$\begin{cases} f(x) \geq 0 \\ f(x) > c \end{cases} \quad \checkmark$$

$$\begin{cases} f(x) < 0 \\ -f(x) > c \end{cases}$$

$$\begin{cases} f(x) \geq 0 \\ f(x) > c \end{cases} \quad \checkmark$$

$$\begin{cases} f(x) < 0 \\ f(x) < -c \end{cases}$$



$$|f(x)| > c \Leftrightarrow$$

$$f(x) < -c \vee f(x) > c$$

$$\left| \frac{x-1}{3x-1} \right| \geq 2$$

$(\Rightarrow)$

$$\frac{x-1}{3x-1} \leq -2 \vee \frac{x-1}{3x-1} \geq 2$$

$$\frac{x-1}{3x-1} + 2 \leq 0 \vee \frac{x-1}{3x-1} - 2 \geq 0$$

$$\frac{x-1+6x-2}{3x-1} \leq 0 \vee \frac{x-1-6x+2}{3x-1} \geq 0$$

$$\frac{7x-3}{3x-1} \leq 0$$

✓

$$\frac{-5x+1}{3x-1} \geq 0$$

$$\frac{7x-3}{3x-1} \leq 0$$

✓

$$\frac{5x-1}{3x-1} \leq 0$$

$$\frac{1}{3} < x \leq \frac{3}{7}$$

✓

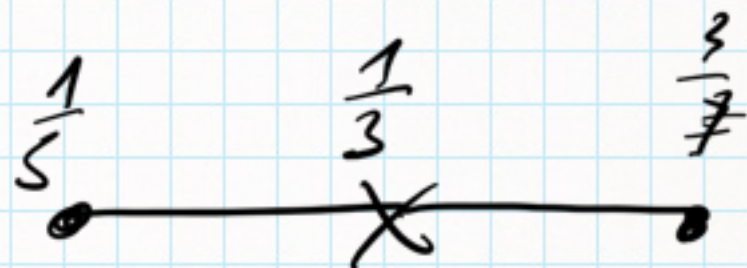
$$\frac{1}{5} \leq x < \frac{1}{3}$$

$$\left] \frac{1}{3}, \frac{3}{7} \right]$$

∪

$$\left[\frac{1}{5}, \frac{1}{3} \right[=$$

$$= \left[\frac{1}{5}, \frac{3}{7} \right] \setminus \left\{ \frac{1}{3} \right\}$$



$$x \in \left[\frac{1}{5}, \frac{3}{7} \right], x \neq \frac{1}{3}$$

$$|f(x)| < c \quad c \in \mathbb{R}$$

• $c < 0$ NESSUNA SOLUZIONE

• $c = 0$ NESSUNA SOLUZIONE

$$|f(x)| \leq 0 \Rightarrow f(x) = 0$$

$c > 0$

$$|f(x)| < c \Leftrightarrow$$

$$-c < f(x) < c \Leftrightarrow \begin{cases} f(x) > -c \\ f(x) < c \end{cases}$$

$$|f(x)| < c$$

$$c > 0$$

$$\begin{cases} f(x) \geq 0 \\ f(x) < c \end{cases}$$

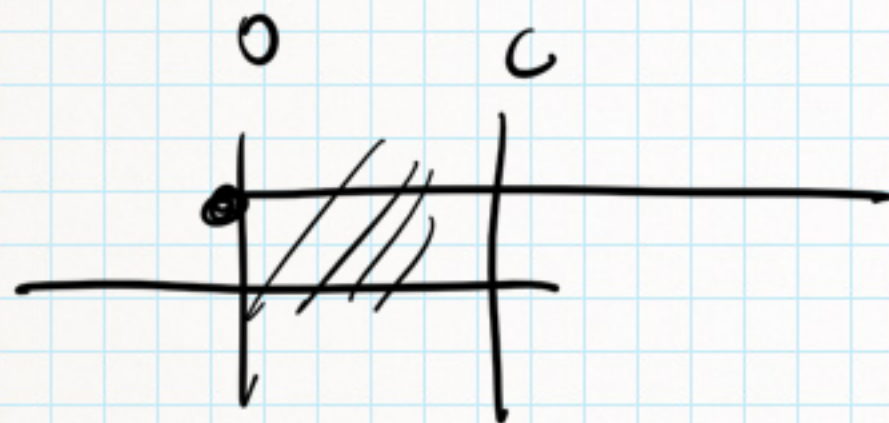
$\vee$

$$\begin{cases} f(x) < 0 \\ -f(x) < c \end{cases}$$

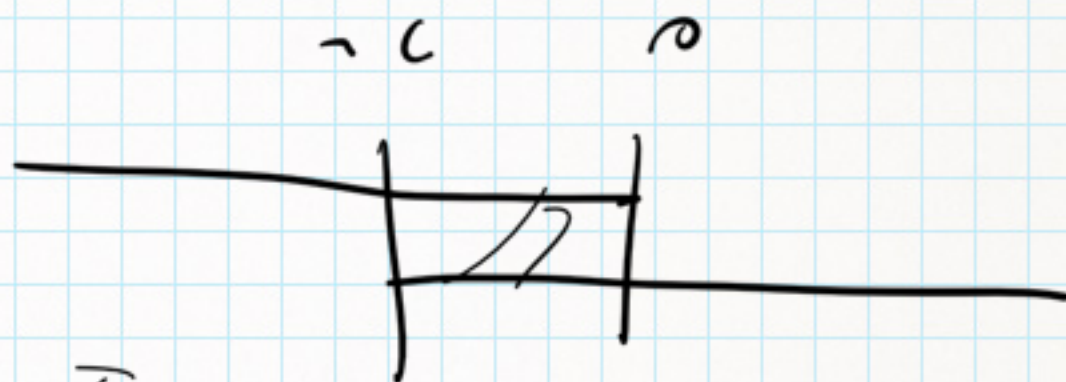
$$\begin{cases} f(x) \geq 0 \\ f(x) < c \end{cases}$$

$\vee$

$$\begin{cases} f(x) < 0 \\ f(x) > -c \end{cases}$$



$$[0, c[\quad \vee \quad]-c, 0[$$





$$[-c, 0] \cup [0, c] = [-c, c]$$

$$f(x) \in [-c, c] \Leftrightarrow -c < f(x) < c$$

$$|x^2 - x| < 2 \quad (\Leftrightarrow) \quad -2 < x^2 - x < 2$$

$$\begin{cases} x^2 - x < 2 \\ x^2 - x > -2 \end{cases}$$

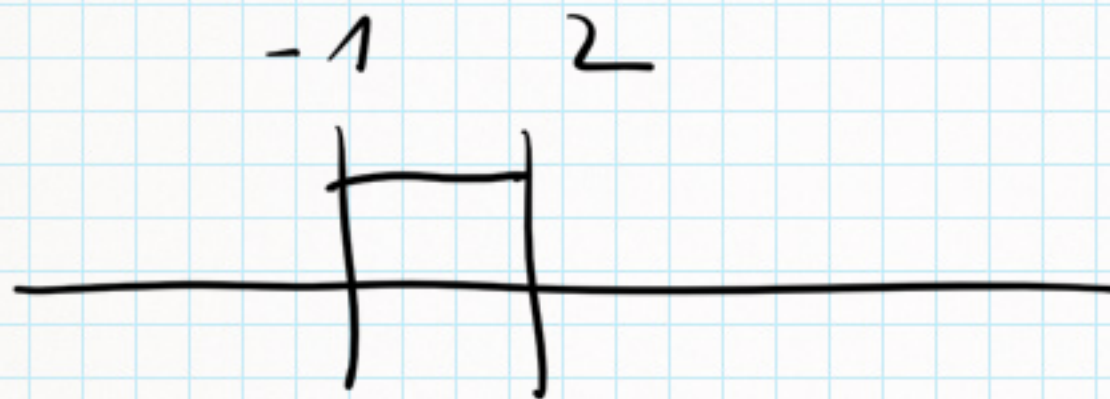
$$\begin{cases} x^2 - x - 2 < 0 \\ x^2 - x + 2 > 0 \\ \Delta < 0 \end{cases}$$

$$\begin{cases} x^2 - x - 2 < 0 \\ \forall x \in \mathbb{R} \end{cases}$$

$$\begin{cases} -1 < x < 2 \\ \forall x \in \mathbb{R} \end{cases}$$

$$x_{1,2} = \frac{1 \pm \sqrt{9}}{2} = \frac{1 \pm 3}{2}$$

2
-1



$$-1 < x < 2$$

$$|x^2 - 1| + 2x \geq 1$$

$$|x^2 - 1| \geq 1 - 2x$$

$$\begin{cases} x^2 - 1 \geq 0 \\ x^2 - 1 + 2x \geq 1 \end{cases}$$

$$\vee \begin{cases} x^2 - 1 < 0 \\ -(x^2 - 1) + 2x \geq 1 \end{cases}$$

$$\begin{cases} x \leq -1 \vee x \geq 1 \\ x^2 + 2x - 2 \geq 0 \end{cases}$$

$$\vee \begin{cases} -1 < x < 1 \\ -x^2 + 1 + 2x \geq 1 \end{cases}$$

$$\begin{cases} x \leq -1 \vee x \geq 1 \\ x^2 + 2x - 2 \geq 0 \end{cases}$$

$\bigcirc$

$$\left\{ \begin{array}{l} -1 < x < 1 \\ x^2 - 2x \leq 0 \end{array} \right.$$

$$x^2 + 2x - 2 \geq 0$$

$$x_{1,2} = \frac{-2 \pm \sqrt{12}}{4}$$

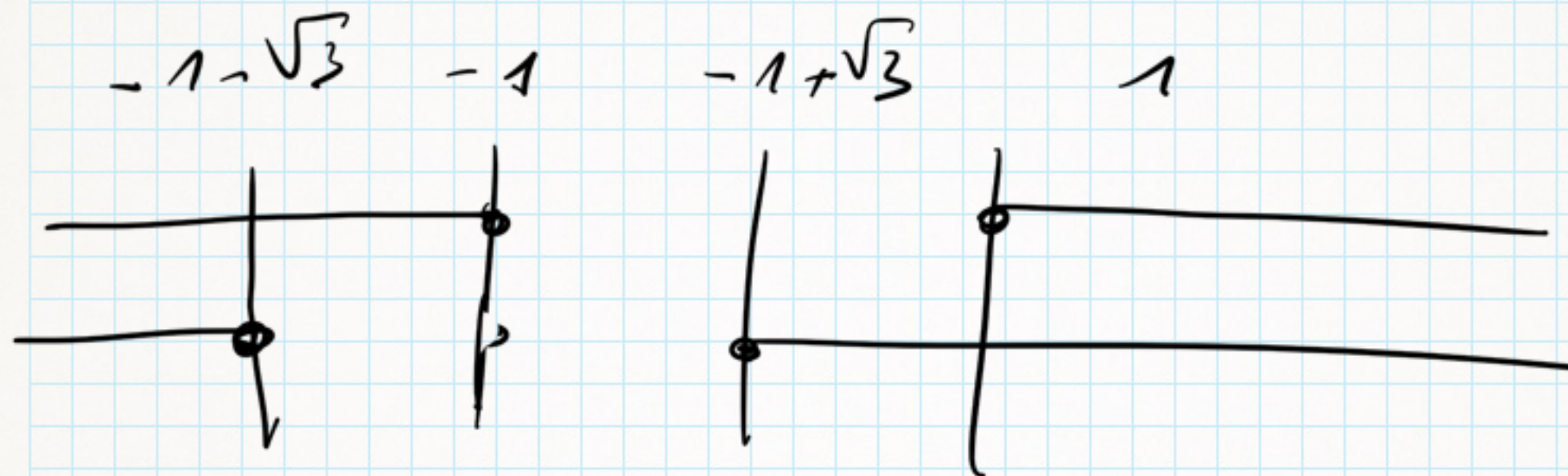
$$x^2 - 2x = 0$$

$$x(x-2) = 0$$

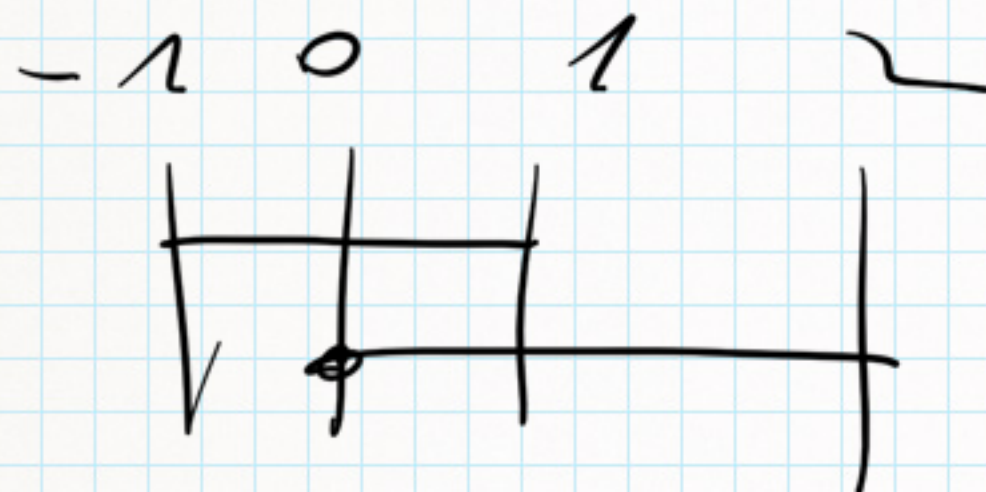
$$x = 0 \vee x = 2$$

$$\begin{cases} x \leq -1 \vee x \geq 1 \\ x \leq \frac{-2-\sqrt{12}}{2} \vee x \geq \frac{-2+\sqrt{12}}{2} \\ \quad \text{"} \quad \quad \quad \text{"} \\ \quad -1-\sqrt{3} \quad \quad -1+\sqrt{3} \end{cases}$$

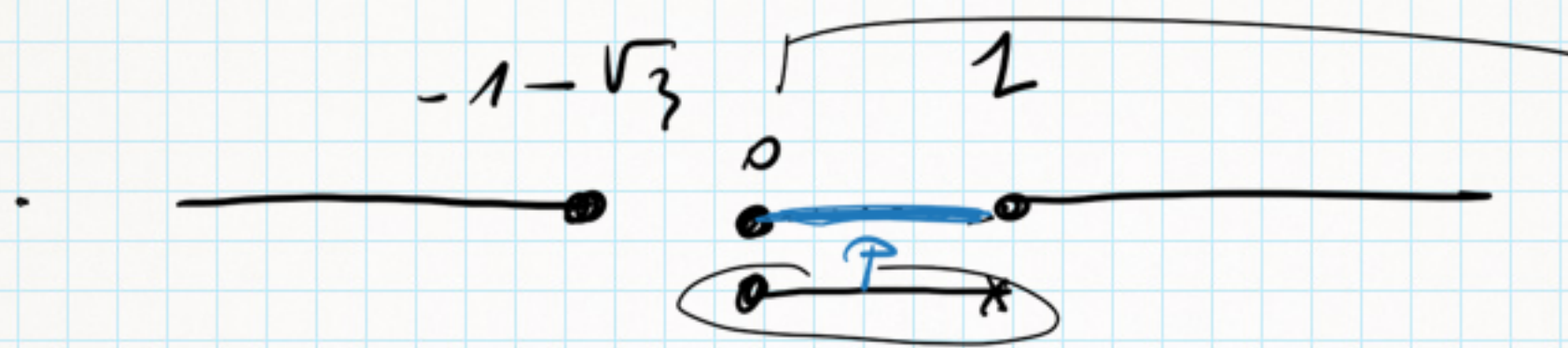
$$\begin{cases} -1 < x < 1 \\ 0 \leq x \leq 2 \end{cases}$$



$$x \leq -1-\sqrt{3} \quad \vee \quad x \geq 1$$



$$0 < x < 1$$



$$[-\infty, 1 - \sqrt{3}] \cup [0, +\infty]$$

EQUATIONI $\in$ DISEQUATIONI

IRRATIONALI

$$\sqrt[n]{f(x)} = g(x)$$

n DISPARI

$$\sqrt[3]{-8} = -2$$

$$x < y$$

$$x^m < y^m$$

$$\forall x, y \in \mathbb{R}$$

monoton.

$$\sqrt[3]{x^3 - 1} \geq (x - 1)$$

$$\left(\sqrt[3]{x^3 - 1}\right)^3 \geq (x - 1)^3$$

$$\left(\sqrt[3]{x-1}\right)^3 \geq (x-1)^3$$

$$\cancel{x-1}^3 \geq \cancel{x-1}^3 - 3x^2 + 3x$$

$$3x^2 - 3x \geq 0$$

$$x^2 - x \geq 0$$

$$x \leq 0 \vee x \geq 1$$

$$x^2 - x \geq 0$$

$$x = 0 \vee x \geq 1$$

