$$\int x^3 \lg x \, dx$$

$$\boxed{\int f'(x)\, g(x)\, dx = f(x) \cdot g(x) - \int f(x)\, g'(x)\, dx}$$

$f'(x) = \lg x$ $\qquad$ $\int \lg x \, dx$?? NON

È LA SCELTA GIUSTA

$f'(x) = x^3$ $\qquad$ $f(x) = \dfrac{x^4}{4}$

$g(x) = \lg x$ $\qquad$ $g'(x) = \dfrac{1}{x}$

$$\int x^3 \lg x \, dx = \frac{x^4}{4} \cdot \lg x - \int \frac{\cancel{x^4}^{\,3}}{4} \cdot \frac{1}{\cancel{x}} \, dx =$$

$$= \frac{x^4}{4} \lg x - \frac{1}{4} \cdot \frac{x^4}{4} + c = \frac{x^4}{4} \cdot \left(\lg x - \frac{1}{4} \right) + c$$

- $\int \lg x \, dx = \int 1 \cdot \lg x \, dx = x \lg x - \int x \cdot \frac{1}{x} \, dx$

$f'(x) = 1 \qquad f(x) = x$

$g(x) = \lg x \qquad g'(x) = \frac{1}{x}$

$= x \lg x - \int 1 \, dx$

$= x \lg x - x + c$

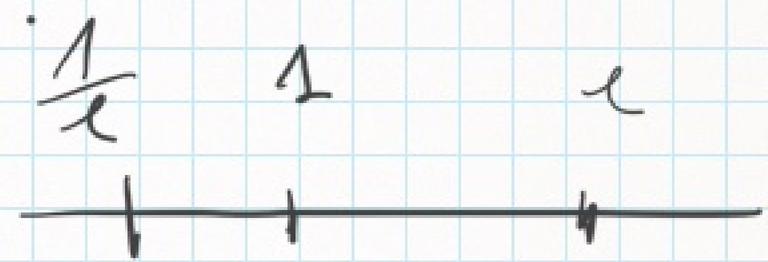
$= x(\lg x - 1) + c$

- $\int_{\frac{1}{e}}^{e} |\lg x| \, dx$

$\lg x \geq 0 \iff x \geq 1$

$$|\lg x| = \begin{cases} \lg x & \text{se } x \geq 1 \\ -\lg x & 0 < x \leq 1 \end{cases}$$

$[\frac{1}{e}, e]$



$$\int_{\frac{1}{e}}^{e} |\lg x|\,dx = \int_{\frac{1}{e}}^{1} |\lg x|\,dx + \int_{1}^{e} |\lg x|\,dx$$

$$= -\int_{\frac{1}{e}}^{1} \lg x\,dx + \int_{1}^{e} \lg x\,dx = -\left[x(\lg x - 1)\right]_{\frac{1}{e}}^{1} + \left[x(\lg x - 1)\right]_{1}^{e} =$$

$= -\left(-1 \cdot -\frac{1}{e}\left[\lg\frac{1}{e} - 1\right]\right)$

$\lg\left(\frac{1}{e}\right) =$

$= \lg e^{-1} =$

$= -1 \lg e = -1$

$+ e\left(\lg e - 1\right) + 1 \cdot =$

$= 1 + \frac{1}{e}(-2) + 0 + 1 \cdot =$

$= 2 - \frac{2}{e}$

$$\int \operatorname{arctg} x \, dx = \int 1 \cdot \operatorname{arctg} x \, dx$$

$f'(x) = 1 \qquad f(x) = x$

$g(x) = \operatorname{arctg} x \qquad g'(x) = \frac{1}{1+x^2}$

$$\int \operatorname{arctg} x \, dx = x \operatorname{arctg} x - \int \frac{x}{1+x^2} \, dx =$$

$$= x \operatorname{arctg} x - \frac{1}{2} \int \frac{2x}{1+x^2} \, dx =$$

$$1+x^2=t$$

$$= x \operatorname{arctg} x - \frac{1}{2} \int \frac{1}{t} \, dt =$$

$$x \operatorname{arctg} x - \frac{1}{2} \lg\left(|\underbrace{1+x^2}_{>0}| \right) + c =$$

$$= x \operatorname{arctg} x - \frac{1}{2} \lg\left(1+x^2\right) + c$$

⌜ oss $\displaystyle\int \frac{f'(x)}{f(x)}\,dx = \lg|f(x)| + c$ ⌟

$$\int \lg^2 x\,dx = \int 1\cdot \lg^2 x\,dx$$

$f'(x) = 1 \qquad f(x) = x$

$g(x) = \lg^2 x \qquad g'(x) = 2\lg x \cdot \frac{1}{x}$

$$\int \lg^2 x\,dx = x\lg^2 x - \int 2\lg x\cdot\cancel{\frac{1}{x}}\cdot\cancel{x}\,dx =$$

$$= x\lg^2 x - 2\int \lg x\,dx = x\lg^2 x - 2x(\lg x - 1) + c$$

$$\int e^x \cos x \, dx$$

$f'(x) = e^x \qquad f(x) = e^x$

$g'(x) = \cos x \qquad g(x) = -\sin x$

$$\int e^x \cos x \, dx = e^x \cos x + \int e^x \sin x \, dx$$

$f'(x) = e^x \qquad f(x) = e^x$

$g(x) = \sin x \qquad g'(x) = \cos x$

$$e^x \cos x + e^x \sin x - \int e^x \cos x \, dx$$

$$\int e^x \cos x \, dx = e^x \cos x + e^x \sin x - \int e^x \cos x \, dx$$

$$2 \int e^x \cos x \, dx = e^x \cos x + e^x \sin x + C$$

$$\int e^x \cos x \, dx = \frac{e^x (\cos x + \sin x)}{2} + C$$

$$\int \sin(\lg x) \, dx$$

$$\lg x = t$$

$$e^{\lg x} = e^t$$

$$x = e^t$$

$$\int \sin(\lg x)\,dx = \int \sin t \; e^t\,dt =$$

$$\int e^t \sin t\,dt.$$

$\lg x = t$

$e^{\lg x} = e^t$

$x = e^t$

$\frac{dx}{dt} = (e^t)' = e^t$

$dx = e^t\,dt$

DEVO RISOLVERE

$t = \lg x$

$$\int e^t \cdot \sin t\,dt =$$

$f'(t) = e^t$ $\quad f(t) = e^t$

$g(t) = \sin t$ $\quad g'(t) = \cos t$.

$$\int e^t \cdot \sin t \, dt = e^t \sin t - \int e^t \cos t \, dt$$

$$f'(t) = e^t \qquad f(t) = e^t$$

$$g'(t) = \cos t \qquad f'(t) = -\sin t$$

$$= e^t \sin t - e^t \cos t - \int e^t \sin t \, dt$$

Allora

$$2\int e^t \sin t \, dt = \frac{e^t \sin t - e^t \cos t}{2} + C$$

$$(t = \lg x)$$

$$\int \cos^2 t \, dt$$

$$\cos^2\left(\frac{x}{2}\right) = \frac{1+\cos x}{2} \qquad \sin^2\left(\frac{x}{2}\right) = \frac{1-\cos x}{2}$$

$$\cos^2 t = \frac{1+\cos(2t)}{2} \qquad \sin^2 t = \frac{1-\cos 2t}{2}$$

$$\int \cos^2 t \, dt = \int \frac{1+\cos(2t)}{2} \, dt =$$

$$= \int \frac{1}{2} \, dt + \frac{1}{2} \int \cos(2t) \, dt =$$

$$= \frac{1}{2} t + \frac{1}{2} \int \cos(2t)\, dt =$$

$$2t = s$$

$$\int \cos(2t)\, dt = \frac{1}{2} \int 2 \cos(2t)\, dt$$

$$ds = \underline{2\, dt}$$

$$= \frac{1}{2} \int \cos s \, ds = + \frac{1}{2} \sin s + c = + \frac{1}{2} \sin 2t + c$$

l'integrale si può risolvere per parti

$$\int \cos^2 x \, dx = \int \cos x \cdot \cos x \, dx =$$

$$f'(x) = \cos x \qquad f(x) = \sin x$$

$$g(x) = \cos x \qquad g'(x) = -\sin x$$

$$= \sin x \cdot \cos x + \int \sin^2 x \, dx =$$

$$\boxed{\begin{aligned} &\cos^2 x + \sin^2 x = 1 \\ &\sin^2 x = 1 - \cos^2 x \end{aligned}}$$

$$= \sin x \cos x + \int (1 - \cos^2 x) \, dx$$

$$= \sin x \cos x + \int 1 \, dx - \int \cos^2 x \, dx$$

$$= \sin x \cos x + x - \int \cos^2 x \, dx$$

$$\int \cos^2 x \, dx = \sin x \cos x + x - \int \cos^2 x \, dx$$

$$\cancel{2} \int \cos^2 x \, dx = \frac{x + \sin x \cos x}{2} + c$$

$$\int \sin^2 x \, dx$$ (NEI DUE MODI)

ALCUNI INTEGRALI PER SOSTITUZIONE

$$\int f\left(x, \sqrt[n]{\frac{ax+b}{cx+d}}\right) dx$$

la sostituzione è: $t = \sqrt[n]{\frac{ax+b}{cx+d}}$

$$\int \frac{1}{(x+9)\sqrt{x+1}} \, dx$$

$$t = \sqrt{x+1}$$

$$t^2 = x+1$$

$$x = \underbrace{t^2 - 1}$$

$$dx = 2t\,dt$$

$$\int \frac{1}{(x+9)\sqrt{x+1}}\,dx = \int \frac{2\cancel{t}\,dt}{(t^2-1+9)\,\cancel{t}}$$

$$= \int \frac{2}{t^2+8}\,dt = \int \frac{2}{8\left(\frac{t^2}{8}+1\right)}\,dt =$$

$$= \frac{1}{4} \int \frac{1}{\left(\frac{t}{\sqrt{8}}\right)^2+1}\,dt =$$

$$= \frac{1}{4} \cdot \sqrt{8} \int \frac{\frac{1}{\sqrt{8}}}{\left(\frac{t}{\sqrt{8}}\right)^2 + 1} \, dt =$$

$$= \frac{\sqrt{8}}{4} \operatorname{arctg} \frac{t}{\sqrt{8}} + c \underset{t = \sqrt{x+1}}{=} \frac{\sqrt{8}}{4} \cdot \operatorname{arctg} \sqrt{x+1} + c$$

$$\int f(\sin x, \cos x) \, dx$$

$$t = \operatorname{tg} \frac{x}{2}$$

FORMULE PARAMETRICHE

$$\sin x = \frac{2t}{1+t^2} \qquad \cos x = \frac{1-t^2}{1+t^2}$$

$$t = \operatorname{tg} \frac{x}{2}$$

$$\frac{x}{2} = \operatorname{arctg} t$$

$$x = 2 \operatorname{arctg} t$$

$$dx = \frac{2}{1+t^2}\, dt$$

$$\int \frac{3+\sin x}{1+\cos x}\, dx$$

$$t = \operatorname{tg} \frac{x}{2}$$

$$\sin x = \frac{2t}{1+t^2}, \quad \cos x = \frac{1-t^2}{1+t^2}$$

$$dx = \frac{2}{1+t^2}\, dt$$

$$\int \frac{3+\sin x}{1+\cos x}\,dx = \int \frac{3 \quad \frac{2t}{1+t^2}}{1+\frac{1-t^2}{1+t^2}} \cdot \frac{2}{t^2+1}\,dt$$

$$= \int \frac{\frac{3t^2+2t+3}{1+t^2}}{\frac{\cancel{t^2}+1+1-\cancel{t^2}}{1+t^2}} \cdot \frac{2}{t^2+1}\,dt =$$

$$= \int \frac{3t^2+2t+3}{\cancel{t^2+1}} \cdot \frac{\cancel{t^2+1}}{\cancel{2}} \cdot \frac{\cancel{2}}{t^2+1}\,dt$$

$$= \int \frac{3t^2+2t+3}{t^2+1}\,dt$$

$$\begin{array}{lll|l} 3t^2 & +2t & +3 & t^2+1 \\ \cline{4-4} 3t^2 & & 3 & 3 \\ \cline{1-3} // & 2t & / & \end{array}$$

quoziente

resto

$$\frac{3t^2+2t+3}{t^2+1} = \overset{\text{quoziente}}{3} + \frac{\overset{\text{resto}}{2t}}{\underbrace{t^2+1}_{\text{divisore}}}$$

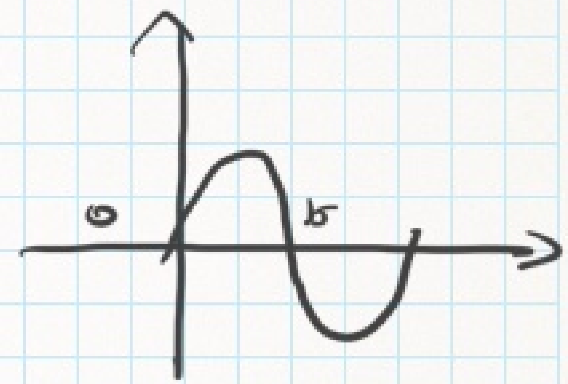
$$\int \frac{3t^2+2t+3}{t^2+1}\,dt = \int \left(3 + \frac{2t}{t^2+1}\right) dt =$$

$$= \int 3\,dt + \int \frac{2t}{t^2+1}\,dt = 3t + \lg(t^2+1) + c$$

$$= 3\,\mathrm{tg}\frac{x}{2} + \lg\left(\left(\mathrm{tg}\frac{x}{2}\right)^2+1\right) + c$$

$$\int f(x, \sqrt{1-x^2})\,dx$$

$-1 \le x \le 1$



$x = \cos t \qquad t \in [0, \pi]$

$$\sqrt{1-x^2} = \sqrt{1-\cos^2 t} = |\sin t| = \sin t$$

$$dx = -\sin t\,dt$$

$$\int \frac{\sqrt{1-x^2}}{x}\,dx = \int \frac{\overbrace{\sin t}^{\sqrt{1-x^2}}\ \overbrace{(-\sin t)}^{dx}}{\underbrace{\cos t}_{x}}\,dt =$$

$$= -\int \frac{\sin^2 t}{\cos t}\,dt = -\int \frac{1-\cos^2 t}{\cos t}\,dt =$$

$$= -\int \left(\frac{1}{\cos t} - \frac{\cos^2 t}{\cos t} \right) dt =$$

$$= -\int \frac{1}{\cos t}\, dt + \underbrace{\int \cos t\, dt} =$$

$$= \underbrace{-\int \frac{1}{\cos t}\, dt} + \sin t + c$$

DEVO RISOLVERE

$$\int \frac{1}{\cos t}\, dt$$

(USO LE FORMULE PARAMETRICHE)

$$s = \operatorname{tg}\frac{t}{2} \Rightarrow \quad \frac{t}{2} = \operatorname{arctg} s \Rightarrow$$

$$t = 2 \operatorname{arctg} s$$

$$\cos t = \frac{1-s^2}{1+s^2} \qquad dt = \frac{2}{1+s^2}\, ds$$

$$\boxed{\begin{aligned} x &= \varphi(t) \\ dx &= \varphi'(t)\, dt \end{aligned}}$$

$$\int \frac{1}{\cos t}\, dt =$$

$$= \int \frac{1}{\underbrace{\frac{1-s^2}{1+s^2}}_{\cos t}} \cdot \overbrace{\frac{2}{1+s^2}\, ds}^{dt} =$$

$$= \int \frac{\cancel{1+s^2}}{1-s^2} \cdot \frac{2}{\cancel{1+s^2}}\, ds = \underline{\int \frac{2}{1-s^2}\, ds}$$

??