$$\lim_{x \to -\infty} \left(\sqrt{x^2+5x} + x \right) = [+\infty - \infty]$$

$$= \lim_{x \to -\infty} \frac{\left(\sqrt{x^2+5x} + x\right)\left(\sqrt{x^2+5x} - x\right)}{\sqrt{x^2+5x} - x} =$$

$$\boxed{\begin{array}{l} a^2 - b^2 = \\ (a+b)(a-b) \end{array}}$$

$$= \lim_{x \to -\infty} \frac{\left(\sqrt{x^2+5x}\right)^2 - x^2}{\sqrt{x^2\left(1+\frac{5}{x}\right)} - x} =$$

$$= \lim_{x \to -\infty} \frac{\cancel{x^2} + 5x - \cancel{x^2}}{\underline{\underline{|x|}}\sqrt{1+\frac{5}{x}} - x} = \lim_{x \to -\infty} \frac{5x}{-x\sqrt{1+\frac{5}{x}} - x}$$

$$= \lim_{x \to -\infty} \frac{5\cancel{x}}{-\cancel{x}\left(\sqrt{1+\frac{5}{x}} + 1\right)} = -\frac{5}{2}$$

$$\lim_{x \to -\infty} \left(\sqrt{x^2-4} - \sqrt{x^2-4x+1} \right) = +\infty - \infty$$

$$= \lim_{x \to -\infty} \frac{\left(\sqrt{x^2-4} - \sqrt{x^2-4x+1}\right)\left(\sqrt{x^2-4} + \sqrt{x^2-4x+1}\right)}{\sqrt{x^2-4} + \sqrt{x^2-4x+1}} =$$

$$= \lim_{x \to -\infty} \frac{x^2-4-\left(x^2-4x+1\right)}{\sqrt{x^2\left(1-\frac{4}{x}\right)} + \sqrt{x^2\left(1-\frac{4}{x}+\frac{1}{x^2}\right)}} =$$

$$= \lim_{x \to -\infty} \frac{\cancel{x^2}-4-\cancel{x^2}+4x-1}{|x|\sqrt{1-\frac{4}{x}} + |x|\sqrt{1-\frac{4}{x}+\frac{1}{x^2}}} =$$

$$\lim_{x \to -\infty} \frac{4x - 5}{-x\sqrt{1-\frac{4}{x^2}} - x\sqrt{1-\frac{4}{x}+\frac{1}{x^2}}} =$$

$$= \lim_{x \to -\infty} \frac{\cancel{x}\left(4 - \frac{5}{x}\right)}{-\cancel{x}\underbrace{\left(\sqrt{1-\frac{4}{x^2}} + \sqrt{1-\frac{4}{x}+\frac{1}{x^2}}\right)}_{\downarrow \atop -2}} = -\frac{4}{2}$$

$$= -2$$

$$\lim_{x \to 2} \frac{3 - \sqrt{5x-1}}{4 - x^2} = \frac{0}{0}$$

$$= \lim_{x \to 2} \frac{(3 - \sqrt{5x-1})(3 + \sqrt{5x-1})}{(3 + \sqrt{5x-1})(4 - x^2)} =$$

$$= \lim_{x \to 2} \frac{3^2 - (\sqrt{5x-1})^2}{(3 + \sqrt{5x-1})(4 - x^2)} =$$

$$= \lim_{x \to 2} \frac{9 - (5x-1)}{(3 + \sqrt{5x-1})(4 - x^2)} =$$

$$= \lim_{x \to 2} \frac{9 - 5x + 1}{\left(3 + \sqrt{5x-1}\right)\left(4 - x^2\right)} =$$

$$= \lim_{x \to 2} \frac{10 - 5x}{\left(3 + \sqrt{5x+1}\right)\left(4 - x^2\right)} =$$

$$= \lim_{x \to 2} \frac{5\cancel{(2 - x)}}{\left(3 + \sqrt{5x-1}\right)(2 + x)\cancel{(2 - x)}} = \frac{5}{6 \cdot 4}$$

$$= \frac{5}{24}$$

LIMITI NOTEVOLI

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

$$\left[\quad \lim_{x \to \pm\infty} \frac{\sin x}{x} = 0 \right.$$

$$-1 \le \sin x \le 1 \qquad\qquad \to +\infty$$

$$\begin{array}{ccccc} -\dfrac{1}{x} & \le & \dfrac{\sin x}{x} & \le & \dfrac{1}{x} \\ \downarrow & & \downarrow & & \downarrow \\ 0 & & 0 & & 0 \end{array} \qquad x \to +\infty \quad \left. \right]$$

- $$\boxed{\lim_{x \to 0} \frac{\sin x}{x} = 1}$$

- $$\lim_{x \to 0} \frac{\operatorname{tg} x}{x} = \lim_{x \to 0} \frac{\sin x}{\cos x \cdot x} =$$

$$= \lim_{x \to 0} \frac{\sin x}{x} \cdot \frac{1}{\cos x} = 1 \cdot 1 = 1$$

- $$\boxed{\lim_{x \to 0} \frac{\operatorname{tg} x}{x} = 1}$$

- $$\lim_{x\to 0} \frac{\arcsin x}{x} = 1$$

$$\lim_{x\to 0} \frac{\arcsin x}{x}$$

$y = \arcsin x$

$x = \sin y$

$x \to 0$

$y \to 0$

$$= \lim_{y\to 0} \frac{y}{\sin y} = 1.$$

$$\boxed{\lim_{x\to 0} \frac{\arcsin x}{x} = 1}$$

- $$\boxed{\lim_{x \to 0} \frac{\operatorname{arctg} x}{x} = 1}$$

$$\boxed{\cos^2 x + \sin^2 x = 1}$$

- $$\lim_{x \to 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}$$

$$\lim_{x \to 0} \frac{1 - \cos x}{x^2} = \lim_{x \to 0} \frac{(1 - \cos x)(1 + \cos x)}{x^2 (1 + \cos x)} =$$

$$= \lim_{x \to 0} \frac{1 - \cos^2 x}{x^2 (1 + \cos x)} = \lim_{x \to 0} \frac{\sin^2 x}{x^2} \cdot \frac{1}{1 + \cos x}$$

$$= \lim_{x \to 0} \frac{\sin x}{x} \cdot \frac{\sin x}{x} \cdot \frac{1}{1 + \cos x} = \frac{1}{2}$$

- $$\boxed{\lim_{x\to 0} \frac{1-\cos x}{x^2} = \frac{1}{2}}$$

- $$\boxed{\lim_{x\to 0} \frac{a^x-1}{x} = \ln a}$$ $\forall a>0$, $a\neq 1$

- $$\boxed{\lim_{x\to 0} \frac{e^x-1}{x} = 1} = (\ln e)$$

- $$\boxed{\lim_{x\to 0} \frac{\lg_a(x+1)}{x} = \frac{1}{\ln a}}$$ $\forall a>0$, $a\neq 1$

$$\boxed{\cdot \lim_{x \to 0} \frac{\ln(1+x)}{x} = 1}$$

$$\cdot \boxed{\lim_{x \to 0} \frac{(1+x)^{a} - 1}{x} = a} \qquad \begin{matrix} a > 0 \\ a \neq 1 \end{matrix}$$

$$\lim_{x \to 0} \frac{\sin x + 3x + \operatorname{tg} x}{e^{x} - 1} = \frac{0}{0}$$

$$= \lim_{x \to 0} \frac{\frac{\sin x + 3x + \operatorname{tg} x}{x} \cdot \cancel{x}}{\frac{e^{x} - 1}{x} \cdot \cancel{x}} =$$

$$= \lim_{x \to 0} \frac{\dfrac{\sin x}{x} + \dfrac{3\cancel{x}}{\cancel{x}} + \dfrac{\operatorname{tg} x}{x}}{\dfrac{e^x - 1}{x}} =$$

$$= \lim_{x \to 0} \frac{\overbrace{\dfrac{\sin x}{x}}^{\to 1} + 3 + \overbrace{\dfrac{\operatorname{tg} x}{x}}^{\to 1}}{\underbrace{\dfrac{e^x - 1}{x}}_{\to 1}} = 5$$

$$\lim_{x \to 0} \frac{\sin 3x - \arcsin 2x}{x \cdot tg(2x)} =$$

$$= \lim_{x \to 0} \frac{1 \; 2x}{tg(2x) \; 2x} \left[3\,\frac{\sin 3x}{3x} - 2\,\frac{\arcsin 2x}{2x} \right]$$

$$\left\lceil \lim_{x \to 0} \frac{\sin 3x}{3x} \overset{y = 3x}{=\!=} \lim_{y \to 0} \frac{\sin y}{y} = 1 \right\rfloor$$

$$\left\lceil \lim_{x \to 0} \frac{\arcsin 2x}{2x} \overset{y = 2x}{=\!=} \lim_{y \to 0} \frac{\arcsin y}{y} = 1 \right\rfloor$$

$$\left[\ \lim_{x\to 0}\frac{2x}{tg\,2x} \overset{y=2x}{=\!=} \lim_{y\to 0}\frac{y}{tg\,y} = 1\ \right]$$

RICAPITOLANDO

$$\lim_{x\to 0}\frac{\sin 3x - \sin 2x}{x\,tg\,2x} =$$

$$= \lim_{x\to 0}\underbrace{\frac{2x}{tg(2x)}}_{\downarrow\ 1}\cdot\frac{1}{2x}\left[\underbrace{3\,\frac{\sin 3x}{3x} - 2\,\frac{\sin 2x}{2x}}_{\downarrow\ 1}\right]$$

$$\lim_{x \to 0} \frac{1}{2x} \quad \text{NON ESISTE}$$

In questo
$$\lim_{x \to 0^+} \frac{1}{2x} = \frac{1}{0^+} = +\infty$$

$$\lim_{x \to 0^-} \frac{1}{2x} = \frac{1}{0^-} = -\infty$$

$$\lim_{x \to 0^+} \frac{\sin 3x - \arcsin 2x}{x \operatorname{tg} 2x} = +\infty$$

$$\lim_{x \to 0^-} \frac{\sin 3x - \arcsin 2x}{x \operatorname{tg} 2x} = -\infty$$

$$\lim_{x\to 0} (e^x + x)^{\frac{1}{x}} = 1^{\infty}$$

$$= \lim_{x\to 0} e^{\underbrace{\frac{1}{x}\lg(e^x+x)}}$$

$$y = \frac{\lg(e^x+x)}{x}$$

$$\boxed{\lim_{y\to ?} e^y} = \lim_{y\to l} e^y = e^l$$

Devo risolvere

$$\lim_{x\to 0} \frac{\lg(e^x+x)}{x} =$$

$$e^{\frac{1}{x}\lg(e^x+x)} = e^{\lg(e^x+x)^{\frac{1}{x}}} = (e^x+x)^{\frac{1}{x}}$$

Si sfrutta la proprietà che

$$\lg_a x^\alpha = \alpha \lg_a x$$

$$= \lim_{x \to 0} \frac{\ln(\overbrace{e^x + x - 1} + 1)}{x} \qquad \circledast$$

⌜

$$\lim_{y \to 0} \frac{\ln(y+1)}{y} = 1$$

$$\lim_{x \to 0} \frac{\ln(e^x + x - 1 + 1)}{e^x + x - 1} = \qquad y = e^x + x - 1$$

$$= \lim_{y \to 0} \frac{\lg(y+1)}{y} = 1$$ ⌟

$$= \lim_{x \to 0} \frac{\lg(e^x + x - 1 + 1)}{x} =$$

$$= \lim_{x \to 0} \underbrace{\frac{\lg(e^x + x - 1 + 1)}{e^x + x - 1}}_{\to 1} \cdot \frac{e^x + x - 1}{x}$$

$$= \lim_{x \to 0} \frac{\lg(e^x + x - 1 + 1)}{e^x + x - 1} \cdot \left[\underbrace{\frac{e^x - 1}{x}}_{\to 1} + \underbrace{\frac{\cancel{x}}{\cancel{x}}}_{=1} \right]$$

$$= 2$$

$$\lim_{x \to 0} \frac{\lg_2 (3x^2 - x + 1)}{\sin^3 x} =$$

$$= \lim_{x \to 0} \frac{\lg_2 (3x^2 - x + 1)}{\left[\frac{\sin x}{x} \cdot x\right]\left[\frac{\sin x}{x} \cdot x\right]\left[\frac{\sin x}{x} \cdot x\right]}$$

$$= \lim_{x \to 0} \frac{\frac{\lg_2 (3x^2 - x + 1)}{3x^2 - x} \cdot (3x^2 - x)}{\frac{\sin^3 x}{x^3} \cdot x^3} \quad \circledast$$

$\lg 2 \leftarrow$

$\to 1$

$$\ulcorner \lim_{y\to 0} \frac{\lg_2(y+1)}{y} = \lg 2$$

$$\lim_{x\to 0} \frac{\lg_2(3x^2-x+1)}{3x^2-x} \overset{y=3x^2-x}{=\!=}$$

$$\lim_{y\to 0} \frac{\lg_2(y+1)}{y} = \lg 2 \lrcorner$$

$$\circledast = \lg 2 \lim_{x\to 0} \frac{3x^2-x}{x^3} =$$

$$= \lg 2 \lim_{x\to 0} \frac{\cancel{x}(3x-1)}{\cancel{x}x^2} = \lg 2 \lim_{x\to 0} \frac{3x-1}{x^2} =$$

$$\underbrace{\lg 2}_{>0} \quad \frac{(-1)}{0^+} = -\infty$$

- $$\lim_{x \to 0} \frac{3^{x^2} - \cos x}{\operatorname{tg}^2 x} =$$

$$\lim_{x \to 0} \frac{3^{x^2} - 1 + 1 - \cos x}{\frac{\operatorname{tg}^2 x}{x^2} \cdot x^2} =$$

$\lg 3 \leftarrow$ $\qquad\qquad\qquad\qquad$ $\to \frac{1}{2}$

$$= \lim_{x \to 0} \frac{\frac{3^{x^2} - 1}{x^2} \cdot x^2 + \frac{1 - \cos x}{x^2} \cdot x^2}{\frac{\operatorname{tg}^2 x}{x^2} \cdot x^2} \quad \circledast$$

$1 \leftarrow$

$$\left\ulcorner \lim_{x\to 0} \frac{3^{x^2}-1}{x^2} \overset{y=x^2}{=\!=} \lim_{y\to 0} \frac{3^y-1}{y} = \lg 3 \right\lrcorner$$

$$\circledast \lim_{x\to 0} \frac{\lg 3 \cdot x^2 + \frac{1}{2} x^2}{x^2} =$$

$$= \lim_{x\to 0} \frac{\left(\lg 3 + \frac{1}{2}\right) \cancel{x^2}}{\cancel{x^2}} = \lg 3 + \frac{1}{2}$$

$$\lim_{x\to 0} \frac{\sin^2 x - (1-\cos x)}{1-\cos x} =$$

$$= \lim_{x \to 0} \frac{\dfrac{\sin^2 x - (1-\cos x)}{x^2}}{\dfrac{1-\cos x}{x^2}} =$$

$$= \lim_{x \to 0} \frac{\underbrace{\dfrac{\sin^2 x}{x^2}}_{\to 1} - \underbrace{\left(\dfrac{1-\cos x}{x^2}\right)}_{\to \frac{1}{2}}}{\underbrace{\dfrac{1-\cos x}{x^2}}_{\to \frac{1}{2}}} =$$

$$= \frac{1 - \frac{1}{2}}{\frac{1}{2}} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1$$

METODO ALTERNATIVO $\quad \boxed{\sin^2 x + \cos^2 x = 1}$

$$\lim_{x \to 0} \frac{\sin^2 x - (1-\cos x)}{1-\cos x} =$$

$$= \lim_{x \to 0} \frac{1-\cos^2 x - (1-\cos x)}{1-\cos x} =$$

$$= \lim_{x \to 0} \frac{(1-\cos x)(1+\cos x) - (1-\cos x)}{1-\cos x}$$

$$= \lim_{x \to 0} \frac{\cancel{(1-\cos x)}(\cancel{1}+\cos x - \cancel{1})}{\cancel{1-\cos x}} = \lim_{x \to 0} \cos x$$

$$= 1$$