

$$f, g: A \subset \mathbb{R} \rightarrow \mathbb{R}$$

$$x_0 \in D(A)$$

$$\lim_{x \rightarrow x_0} f(x) = \pm \infty = \lim_{x \rightarrow x_0} g(x)$$

Si ol'è che f è infinito ol'orol'è
impio e g è x_0 e

$$\lim_{x \rightarrow x_0} \frac{f(x)}{g(x)} = \pm \infty$$

Si dice che f è un inferiore a g in x_0

$$\text{e} \quad \lim_{x \rightarrow x_0} \frac{f(x)}{g(x)} = 0$$

Infer, f e g sono infiniti della stessa

ordine

$$\lim_{x \rightarrow x_0} \frac{f(x)}{g(x)} = K \in \mathbb{R}, \quad K \neq 0$$

.

$$\lim_{x \rightarrow +\infty} \underbrace{3x^3 - 5x^2}_{\text{}} = +\infty - \infty$$

$$= \lim_{x \rightarrow +\infty} x^3 \left(3 - \frac{5}{x} \right) = +\infty$$

$$\frac{5}{+\infty} = 0$$

OSJ $\lim_{x \rightarrow +\infty} 3x^3 - 5x^2 =$

$$= \lim_{x \rightarrow +\infty} 3x^3$$

(for "maximization" $5x^2$)

$$\lim_{x \rightarrow -\infty} \frac{3x^3 - 5x^2}{x + 3} =$$

$$\approx \lim_{x \rightarrow -\infty} \frac{3x^3}{x} = \lim_{x \rightarrow -\infty} 3x^2 = +\infty$$

$$\lim_{x \rightarrow +\infty} \frac{3x^5 - 7x^2 + x}{x^8 - 7x} =$$

$$\approx \lim_{x \rightarrow +\infty} \frac{3x^5}{x^8} = \lim_{x \rightarrow +\infty} \frac{3}{x^3} = \frac{3}{+\infty} = 0$$

$$\lim_{x \rightarrow -\infty} \frac{x^4 + 7x}{3x^4 + 1} =$$

$$= \lim_{x \rightarrow -\infty} \frac{\cancel{x^4}}{3\cancel{x^4}} = \frac{1}{3}$$

REGOLA GENERALE

$P(x)$, $Q(x)$ polinom.

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

$$Q(x) = b_m x^m + b_{m-1} x^{m-1} + \dots + b_1 x + b_0$$

$$\lim_{x \rightarrow \pm \infty} \frac{P(x)}{Q(x)} = \begin{cases} \pm \infty & n > m \\ 0 & n < m \\ \frac{a_n}{b_m} & n = m \end{cases}$$

Indice

$$\lim_{x \rightarrow \pm \infty}$$

$$\frac{x^m}{x^n}$$

=

$$\begin{cases} \pm \infty & m > n \\ 0 & m < n \\ 1 & m = n \end{cases}$$

come più è alta la potenza n
più velocemente la funzione $f(x) = x^n$
va a $\pm \infty$

OSS $f, g: A \subset \mathbb{R} \rightarrow \mathbb{R}, \quad x_0 \in D(A)$

$$\lim_{x \rightarrow x_0} f(x) = +\infty = \lim_{x \rightarrow x_0} g(x)$$

f infinito di ordine superiore a g in x_0

$$\lim_{x \rightarrow x_0} (f(x) - g(x)) = +\infty - \infty.$$

$$= \lim_{x \rightarrow x_0} f(x) \left(1 - \frac{g(x)}{f(x)} \right) = +\infty$$

METODO IN EVIDENZA L'INFINITO
DI ORDINE SUPERIORE

1° STESSA COSA n

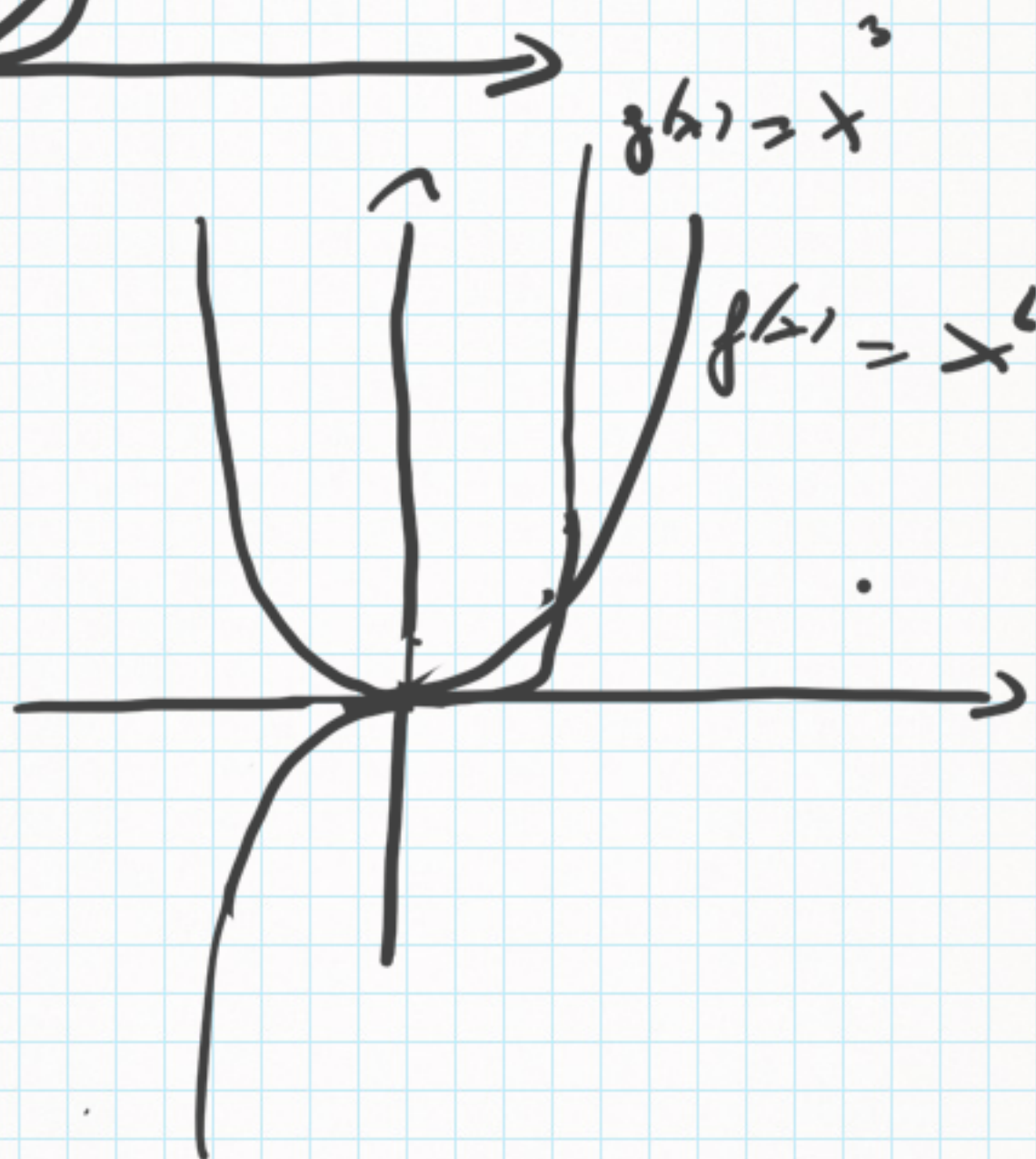
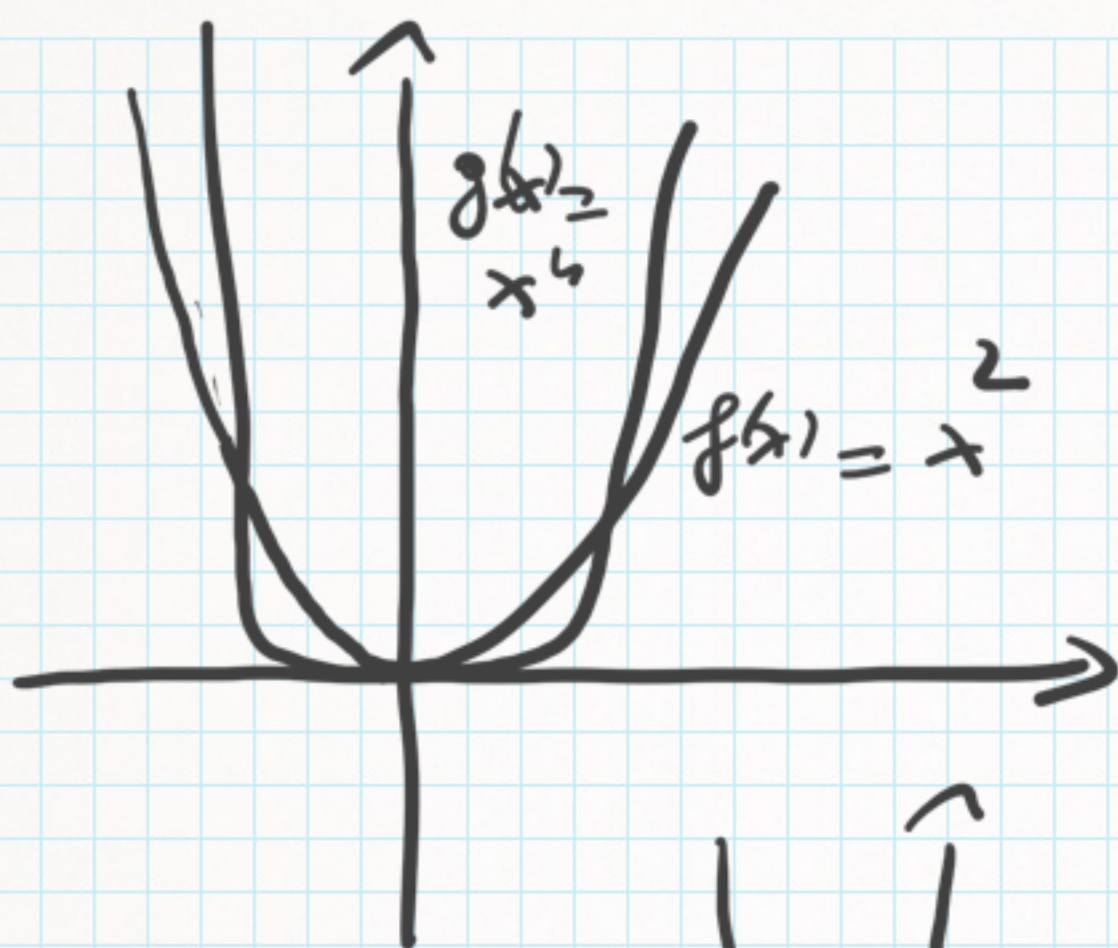
$$\lim_{x \rightarrow x_0} f(x) = -\infty = \lim_{x \rightarrow x_0} g(x)$$

$$\lim_{x \rightarrow x_0} (f(x) - g(x)) = -\infty$$

1° definizione

$$\lim_{x \rightarrow x_0} (f(x) - g(x)) = \lim_{x \rightarrow x_0} f(x)$$

(Ve ben anche $\lim_{x \rightarrow x_0} (f(x) + g(x))$ purché il risultato del limite ne $+\infty - \infty$)



Quanto alto vai andar por fusão.
del tipo

$$g(x) = x^\alpha$$

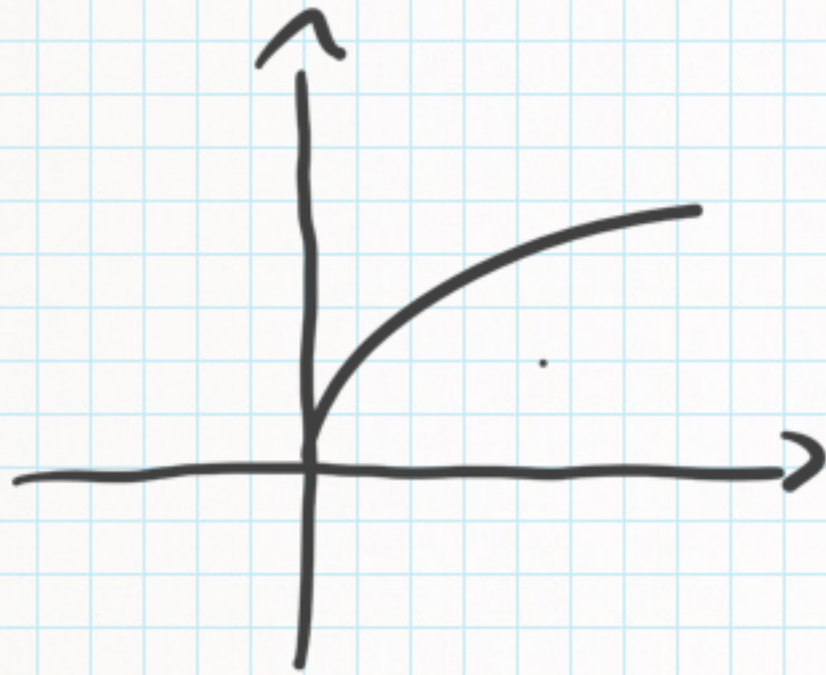
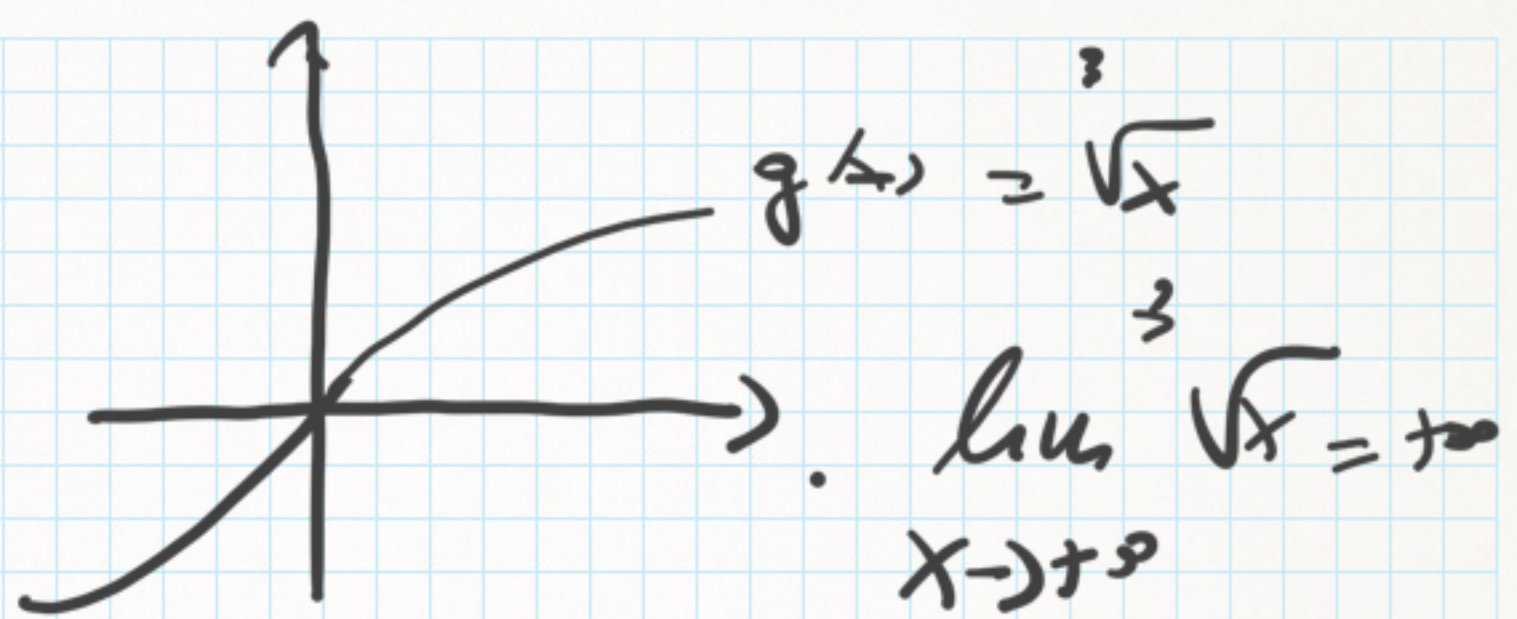
$$\alpha \in \mathbb{R}$$

$$x \geq 0$$

$$\alpha > 0$$

$$\lim_{x \rightarrow +\infty} \frac{\sqrt[3]{x+3}}{\sqrt{x+2}} =$$

$$= \lim_{x \rightarrow +\infty} \frac{(x+3)^{\frac{1}{3}}}{(x+2)^{\frac{1}{2}}} = \lim_{x \rightarrow +\infty} \frac{x^{\frac{1}{3}} \left(1 + \frac{3}{\sqrt[3]{x}}\right)}{x^{\frac{1}{2}} \left(1 + \frac{2}{\sqrt{x}}\right)}$$



$$h(x) = \sqrt{x}$$

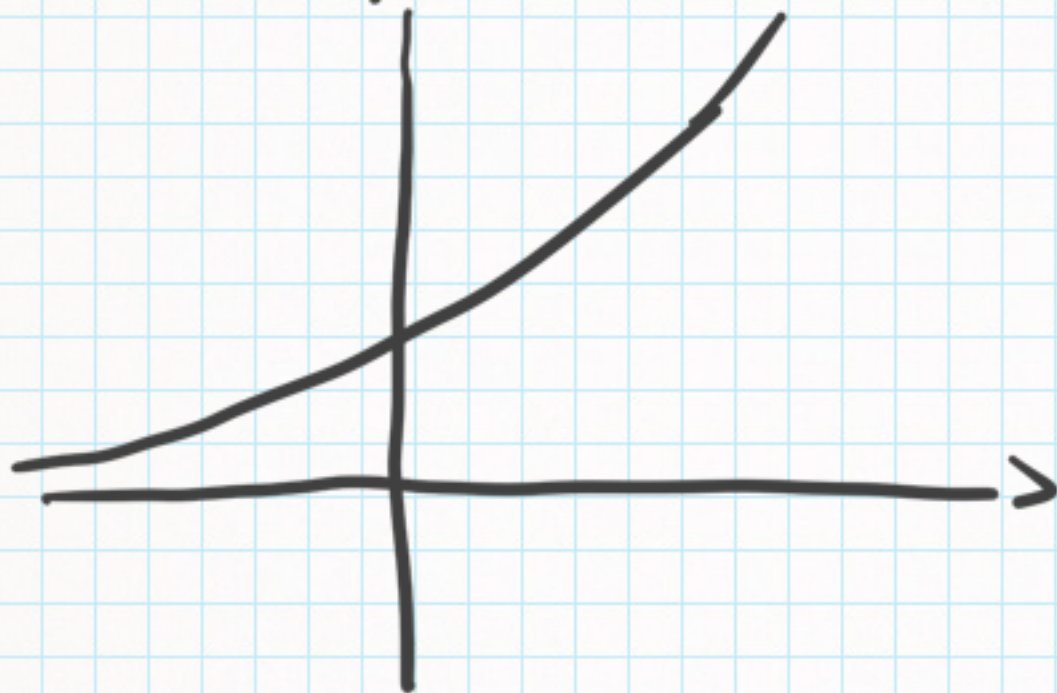
$$= \lim_{x \rightarrow +\infty} \frac{x^{\frac{1}{3}}}{x^{\frac{1}{2}}} = 0$$

$$\left(\frac{1}{3} < \frac{1}{2}\right)$$

$$\lim_{x \rightarrow +\infty} (\sqrt{x+3} - \sqrt[3]{x+5}) = +\infty - \infty$$

$= +\infty$ (puoversi l'infinito
di ordine maggiore)

$$f(x) = a^x \quad |a| > 1$$



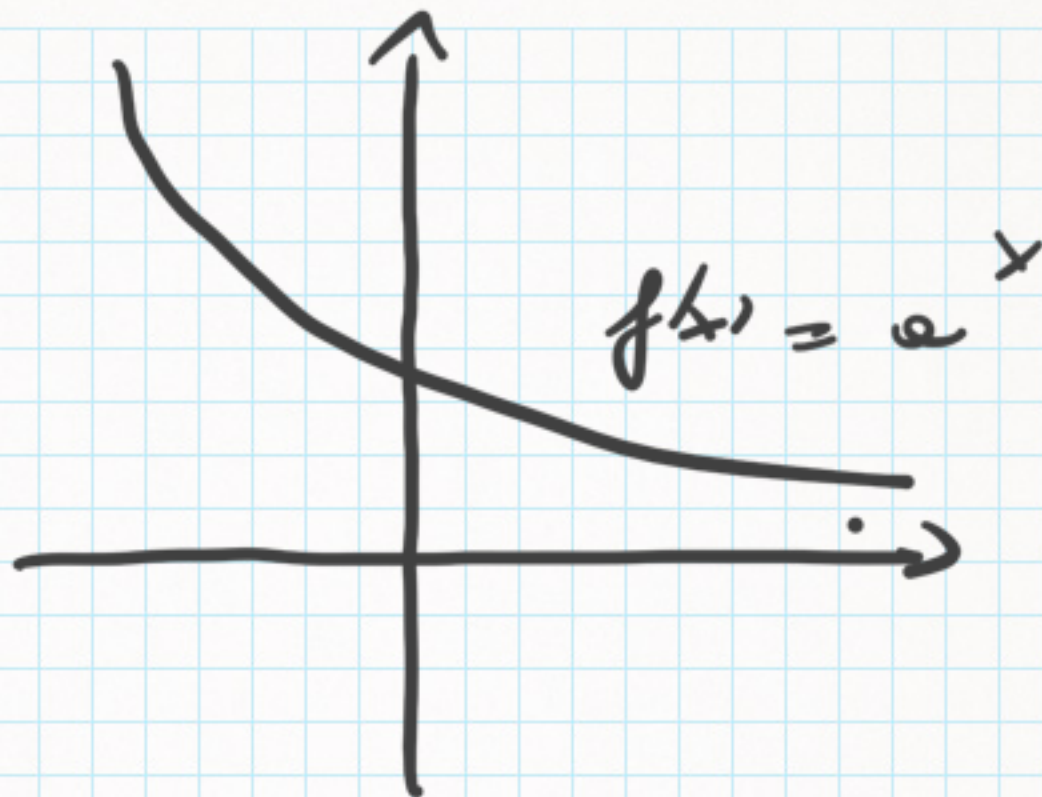
$$\lim_{x \rightarrow +\infty} a^x = +\infty$$

$$\lim_{x \rightarrow +\infty} \frac{e^x}{x^\alpha} = +\infty \quad \forall \alpha > 0$$

$$\lim_{x \rightarrow +\infty} (e^x - 3x^2 + 1) = +\infty - \infty$$

$$= \lim_{x \rightarrow +\infty} e^x \left(1 - \underbrace{\frac{3x^2}{e^x}}_{\downarrow 0} + \underbrace{\frac{1}{e^x}}_{\downarrow 0} \right) = +\infty$$

$$0 < a < 1$$



$$\lim_{x \rightarrow -\infty} e^x = +\infty$$

$$\lim_{x \rightarrow -\infty} \frac{e^x}{x^2} = 0$$

slowly he runs
 x^2

In general, in all the $f(x) = e^x$
 is an infinity of order coming
 greater, since is an infinity
 of order superior a qualunque
 potenza

CONFRONTO TRA ESPONENZIALI

$$\begin{aligned} & \lim_{x \rightarrow +\infty} \frac{3^x + 5x}{4^x - x^2} = \\ & = \lim_{x \rightarrow +\infty} \frac{3^x \left(1 + \frac{5}{3^x}\right)}{4^x \left(1 - \frac{x^2}{4^x}\right)} = \lim_{x \rightarrow +\infty} \frac{3^x}{4^x} \end{aligned}$$

$$\Rightarrow \lim_{x \rightarrow +\infty} \left(\frac{3}{4}\right)^x = 0$$

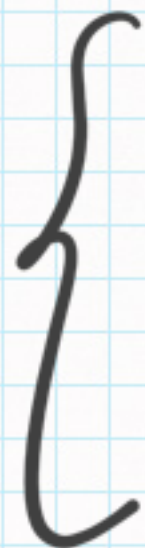
REGOLA GENERALE

$$a, b > 1 \quad a \neq b$$

$$\lim_{x \rightarrow +\infty}$$

$$\frac{a^x}{b^x}$$

=



$+\infty$

$a > b$

0

$a < b$

$$\lim_{x \rightarrow +\infty} 3 + x^2 - 2^x = +\infty$$

(perché 3^x è un infinito
di ordine superiore a 2^x e x)

$$\lim_{x \rightarrow -\infty} \frac{\left(\frac{1}{3}\right)^x}{\left(\frac{1}{4}\right)^x} = \lim_{x \rightarrow -\infty} \left(\frac{\frac{1}{3}}{\frac{1}{4}} \right)^x =$$

$$= \lim_{x \rightarrow -\infty} \left(\frac{1}{3} \cdot 4 \right)^x = \lim_{x \rightarrow -\infty} \left(\frac{4}{3} \right)^x = 0$$

$$\lim_{x \rightarrow -\infty} \frac{\left(\frac{1}{4}\right)^x}{\left(\frac{1}{3}\right)^x} = +\infty$$

In general, $0 < a, b < 1$,

$$\lim_{x \rightarrow -\infty} \frac{a^x}{b^x} = \begin{cases} 0 & a > b \\ +\infty & a < b \end{cases}$$

$$f(x) = e^{\alpha x}$$
$$g(x) = e^{\beta x}$$

$$\lim_{x \rightarrow +\infty} e^{\alpha x} = +\infty = \lim_{x \rightarrow +\infty} e^{\beta x}$$

$$\alpha > 0$$
$$\alpha, \beta > 0$$

$$\lim_{x \rightarrow +\infty} \frac{e^{3x}}{e^{2x}} = \lim_{x \rightarrow +\infty} e^{3x-2x} =$$

$$= \lim_{x \rightarrow +\infty} e^x = +\infty.$$

$$\lim_{x \rightarrow +\infty} \frac{e^{\alpha x}}{e^{\beta x}} = \begin{cases} +\infty & \alpha > \beta \\ 0 & \alpha < \beta \end{cases}$$

$$\lim_{x \rightarrow +\infty} \frac{3^x - x}{3^{3x} + 2} =$$

$$= \lim_{x \rightarrow +\infty} \frac{3^x}{3^{3x}} = \lim_{x \rightarrow +\infty} 3^{x-3x} =$$

$$= \lim_{x \rightarrow +\infty} 3^{-2x} = \lim_{x \rightarrow +\infty} \frac{1}{3^{2x}} =$$

$$= \frac{1}{+\infty} = 0$$

large miracle n

$$e^{\alpha x} \\ e^{\beta x}$$

$$0 < \alpha < 1, ??$$

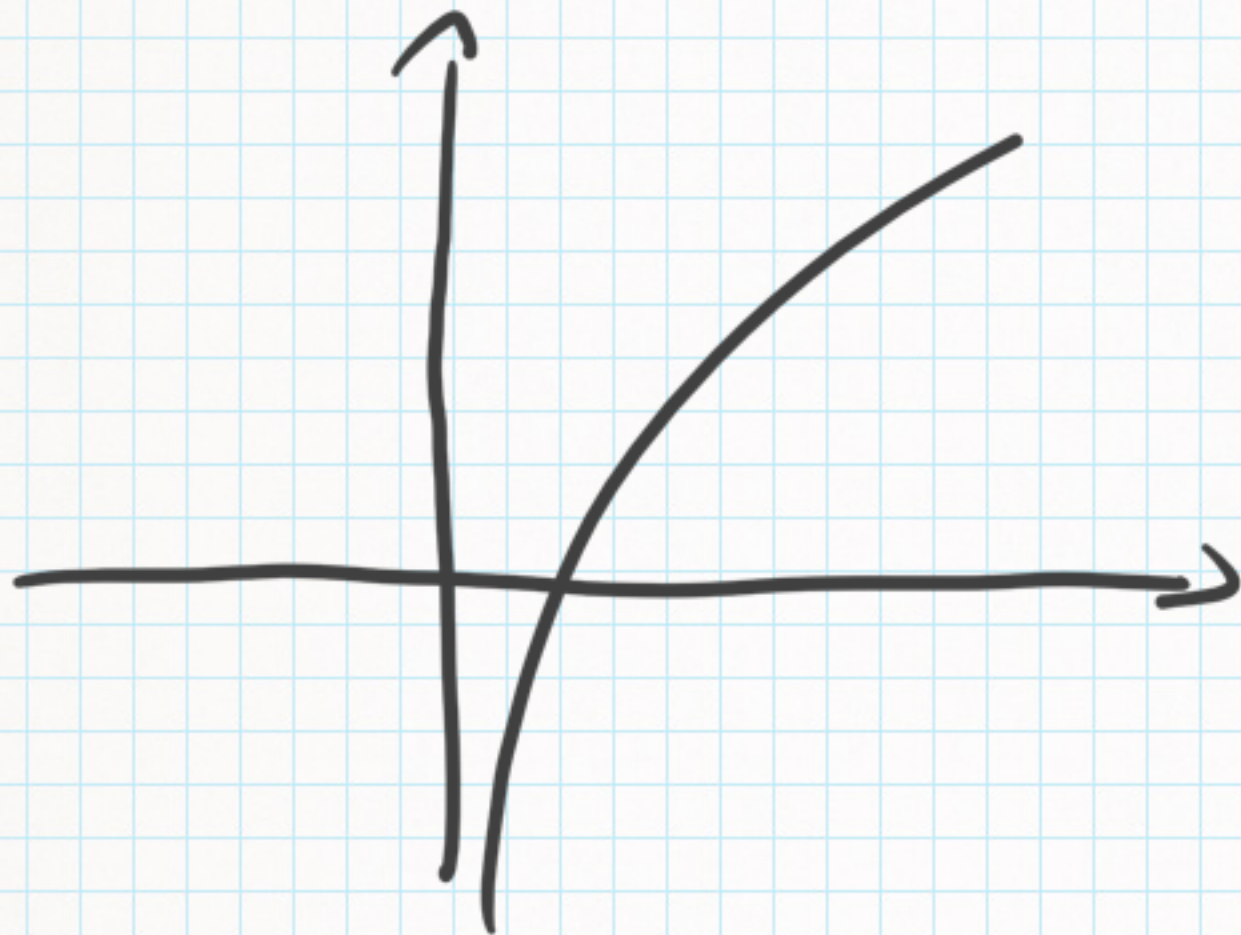
$$\alpha, \beta > 0$$

$$\lim_{x \rightarrow -\infty} \frac{e^{\alpha x}}{e^{\beta x}} =$$

$$\left\{ \begin{array}{l} \\ \end{array} \right. ?$$

$$f(x) = \lg_e x$$

$$a > 1$$



$$\lim_{x \rightarrow +\infty} \lg_e x = +\infty$$

$$\lim_{x \rightarrow 0^+} \lg_e x = -\infty$$

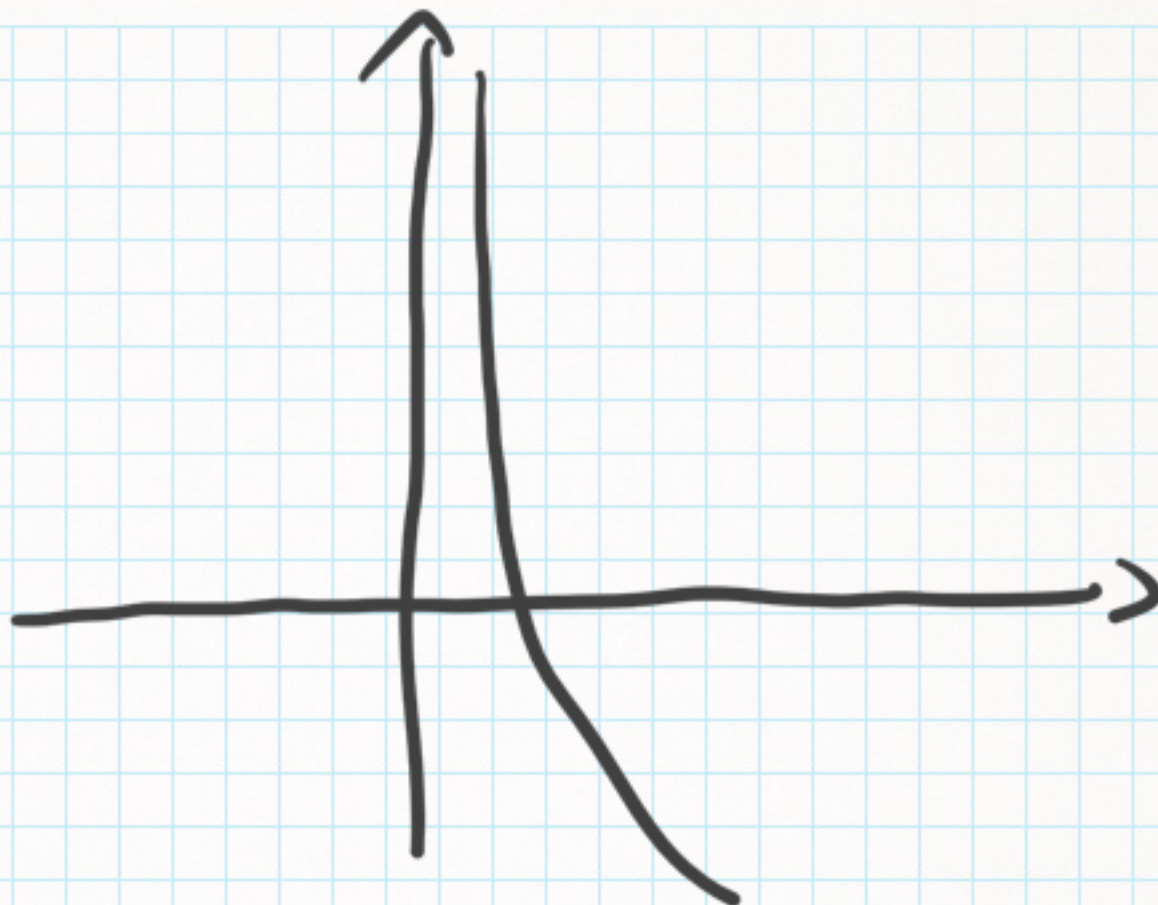
$$\lim_{x \rightarrow +\infty} \frac{\log_e x}{x^\alpha} = 0 \quad \forall \alpha > 0$$

$f(x) = \log_e x$ è un infinito
 per $x \rightarrow +\infty$ o l'altro converge
 piccolo

Allo stesso modo

$$\lim_{x \rightarrow 0^+} \frac{\log_e x}{\left(\frac{1}{x}\right)^\alpha} = 0 \quad \forall \alpha > 0$$

$$0 < x < 1$$



$$\lim_{x \rightarrow 0^+} \ln x = -\infty$$

$$\lim_{x \rightarrow +\infty} \ln x = +\infty$$

una funzione di natura continua
piccola

$$\lim_{x \rightarrow +\infty} (\sqrt{3x^2+1} - x) = +\infty - \infty$$

$$= \lim_{x \rightarrow +\infty} \left(\sqrt{x^2 \left(3 + \frac{1}{x^2} \right)} - x \right) =$$

$$= \lim_{x \rightarrow +\infty} \left(1 \times 1 \sqrt{3 + \frac{1}{x^2}} - x \right) =$$

$$= \lim_{x \rightarrow +\infty} \left(x \left(\sqrt{3 + \frac{1}{x^2}} - 1 \right) \right) =$$

$$= \lim_{x \rightarrow +\infty} x \left(\sqrt{3 + \frac{1}{x^2}} - 1 \right) = +\infty$$

$\nearrow$ $\sqrt{3} - 1$

$$\lim_{x \rightarrow +\infty} (\sqrt{x^2 + x} - x) = +\infty - \infty$$

$$= \lim_{x \rightarrow +\infty} \left(\sqrt{x^2 \left(1 + \frac{1}{x}\right)} - x \right)$$

$$= \lim_{x \rightarrow +\infty} \left(|x| \sqrt{1 + \frac{1}{x}} - x \right) =$$

$$= \lim_{x \rightarrow +\infty} \left(x \sqrt{1 + \frac{1}{x}} - x \right) =$$

$$= \lim_{x \rightarrow +\infty} x \left(\underbrace{\sqrt{1 + \frac{1}{x}}}_{\rightarrow 1} - 1 \right) = +\infty \cdot 0$$

$$a^2 - b^2 = (a+b)(a-b)$$

$$a = \sqrt{x^2 + x}$$

$$b = x$$

$$\lim_{x \rightarrow +\infty} \sqrt{x^2 + x} - x =$$

$$= \lim_{x \rightarrow +\infty} \frac{(\sqrt{x^2 + x} - x)(\sqrt{x^2 + x} + x)}{\sqrt{x^2 + x} + x} =$$

$$= \lim_{x \rightarrow +\infty} \frac{(\sqrt{x^2 + x})^2 - x^2}{\sqrt{x^2 + x} + x} =$$

$$\Rightarrow \lim_{x \rightarrow +\infty} \frac{\cancel{x^2} + x - \cancel{x^2}}{\sqrt{x^2(1 + \frac{1}{x})} + x} =$$

$$= \lim_{x \rightarrow +\infty} \frac{x}{|x| \sqrt{1 + \frac{1}{x}} + x} =$$

$$= \lim_{x \rightarrow +\infty} \frac{\cancel{x}}{\cancel{x} \left(\sqrt{1 + \frac{1}{x}} + 1 \right)} = \frac{1}{2}$$

$\downarrow$
 1

$$\lim_{x \rightarrow -\infty} \sqrt{x^2 - 3x} + x = +\infty - \infty$$

$$= \lim_{x \rightarrow -\infty} \frac{(\sqrt{x^2 - 3x} + x) / (\sqrt{x^2 - 3x} - x)}{\sqrt{x^2 - 3x} - x}$$

$$= \lim_{x \rightarrow -\infty} \frac{\cancel{x} - 3x - \cancel{x}}{\sqrt{x^2(1 - \frac{3}{x})} - x} =$$

$$= \lim_{x \rightarrow -\infty} \frac{-3x}{|x| \sqrt{1 - \frac{3}{x}} - x}$$