$$\mathbb{C} = \{ x+iy \mid x \in \mathbb{R},\ y \in \mathbb{R} \}$$

i unità immaginaria

$$\boxed{i^2 = -1}$$

$z \in \mathbb{C}$ $\qquad z = x + iy$

x parte reale di z $\qquad \mathrm{Re}\, z$

y " immaginaria di z $\qquad \mathrm{Im}\, z$

$z = \mathrm{Re}\, z + i\, \mathrm{Im}\, z$ (RAPPRESENTAZIONE ALGEBRICA)

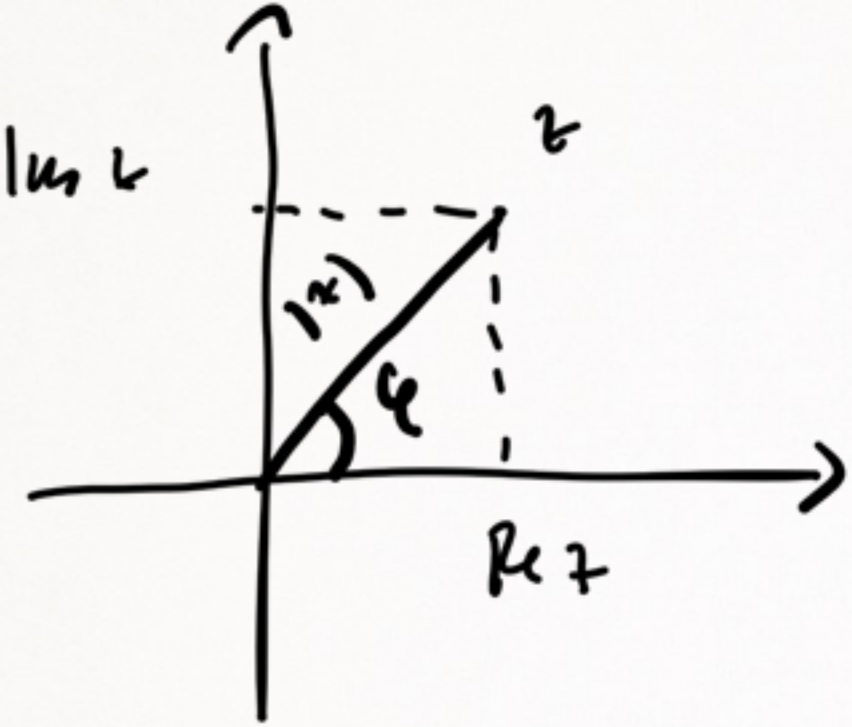
$z \in \mathbb{C}$ $\quad z = \mathrm{Re}\, z + i\, \mathrm{Im}\, z$

$\bar{z} = \mathrm{Re}\, z - i\, \mathrm{Im}\, z$ CONIUGATO di z

$$|z| = \underbrace{\sqrt{(\mathrm{Re}\, z)^2 + (\mathrm{Im}\, z)^2}}_{\text{RADICE QUADRATA IN } \mathbb{R}} \in \mathbb{R}$$

$\mathrm{Re}\, z \in \mathbb{R}$ $\quad |\bar{z}| = |z|$

$\mathrm{Im}\, z \in \mathbb{R}$



RAPPRESENTAZIONE TRIGONOMETRICA

$$z = |z| \left(\cos\varphi + i \sin\varphi \right)$$

$$\cos\varphi = \frac{\mathrm{Re}\, z}{|z|} \qquad \sin\varphi = \frac{\mathrm{Im}\, z}{|z|}$$

$$z = |z| \left(\cos(\varphi + 2k\pi) + i \sin(\varphi + 2k\pi) \right)$$

$$z = |z| (\cos \varphi + i \sin \varphi)$$

$$z' = |z'| (\cos \varphi' + i \sin \varphi')$$

$$z \cdot z' = |z| \cdot |z'| \left(\cos (\varphi + \varphi') + i \sin (\varphi + \varphi')\right)$$

$z' \neq 0$

$$\frac{z}{z'} = \frac{|z|}{|z'|} \left(\cos (\varphi - \varphi') + i \sin (\varphi - \varphi')\right)$$

FORMULA DI DE MOIVRE

$$z = |z| (\cos \varphi + i \sin \varphi) \qquad n \geq 1$$

$$z^n = |z|^n (\cos n\varphi + i \sin n\varphi)$$

ES

$$z = \frac{1}{i-1} + \frac{1}{2}\underbrace{\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^{18}}$$

$$\mathrm{Re}\, z, \quad \mathrm{Im}\, z, \quad \bar{z}, \quad |z|$$

$$\frac{1}{i-1} = \frac{(i+1)}{(i-1)(i+1)} = \frac{i+1}{i^2-1} =$$

$$= \frac{i+1}{-1-1} = -\frac{1}{2}(i+1) =$$

$$= -\frac{1}{2} - \frac{1}{2}i$$

$$z' = \frac{\sqrt{3}}{2} + \frac{i}{2} \qquad |z'| = \sqrt{\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2} =$$

$$= \sqrt{\frac{3}{4} + \frac{1}{4}} = 1$$

$$\cos \varphi' = \frac{\operatorname{Re} z'}{|z'|} = \frac{\sqrt{3}}{2}$$

$$\varphi' = \frac{\pi}{6}$$

$$\sin \varphi' = \frac{\operatorname{Im} z'}{|z'|} = \frac{1}{2}$$

$$z' = \cos \frac{\pi}{6} + i \sin \frac{\pi}{6}$$

$$(z')^{18} = |z'|^{18}\left(\cos 18\varphi' + i \sin 18\varphi'\right)$$

$$= \cos\left(18 \cdot \frac{\pi}{6}\right) + i \sin\left(18 \cdot \frac{\pi}{6}\right) =$$

$$= \cos 3\pi + i \sin 3\pi =$$

$$= \cos(\pi + 2\pi) + i \sin(\pi + 2\pi)$$

$$= \cos \pi + i \sin \pi = -1$$

Ricapitolando

$$z = \frac{1}{i-1} + \frac{1}{2}\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^{18} =$$

$$= -\frac{1}{2} - \frac{1}{2}i - \frac{1}{2} = -1 - \frac{1}{2}i$$

$$\operatorname{Re} z = -1 \qquad \operatorname{Im} z = -\frac{1}{2}$$

$$\bar{z} = -1 + \frac{1}{2}i$$

$$|z| = \sqrt{(-1)^2 + \left(-\frac{1}{2}\right)^2} = \frac{\sqrt{5}}{2}$$

$z^2 + 2\bar{z} = 1$ $\qquad z = x + iy$

$$(x+iy)^2 + 2(x - iy) = 1$$

$$\underbrace{x^2 - y^2 + 2ixy + 2x - 2iy - 1}_{} = 0$$

$$x^2 - y^2 + 2x - 1 + i(2xy - 2y) = 0$$

$$\begin{cases} x^2 - y^2 + 2x - 1 = 0 \\ \underbrace{2xy - 2y}_{} = 0 \end{cases}$$

$$\begin{cases} x^2-y^2+2x-1=0 \\ 2y(x-1)=0 \end{cases}$$

$$\begin{cases} x^2+2x-1=0 \\ y=0 \end{cases} \quad \checkmark \qquad\qquad \begin{cases} \cancel{1}-y^2+2\cancel{-1}=0 \\ x=1 \end{cases}$$

$$\begin{cases} x=-1\pm\sqrt{2} \\ y=0 \end{cases} \qquad\qquad \begin{cases} y=\pm\sqrt{2} \\ x=1 \end{cases}$$

$z_1 = -1 - \sqrt{2}$

$z_2 = -1 + \sqrt{2}$

$z_3 = 1 + \sqrt{2}\, i$

$z_4 = 1 - \sqrt{2}\, i$

RADICI n-SIME DI UN NUMERO COMPLESSO

$z, z' \in \mathbb{C}$

$$z = |z| (\cos \varphi + i \sin \varphi)$$

$$z' = |z'| (\cos \varphi' + i \sin \varphi')$$

$$z = z' \iff \begin{cases} |z| = |z'| \text{ e} \\ \varphi = \varphi' + 2k\pi, \quad k \in \mathbb{Z} \end{cases}$$

<u>DEF</u>. Sia $z \in \mathbb{C}$, $n \geq 1$

Si definisce RADICE n-esima di z ogni $z' \in \mathbb{C}$ t.c. $\underbrace{(z')^n = z}$.

$$z = |z| (\cos \varphi + i \sin \varphi)$$

$$z' = |z'| (\cos \varphi' + i \sin \varphi')$$

voglio capire come sono fatti

$|z'|$ e φ'

$$(z')^n = z$$

$$(z')^n = |z'|^n (\cos n\varphi' + i \sin n\varphi')$$

$$= \underbrace{|z| (\cos \varphi + i \sin \varphi)}_{z}$$

$$(z')^n = z \iff \begin{cases} |z'|^n = |z| \\ n\varphi' = \varphi + 2k\pi \end{cases}$$

$$\iff \begin{cases} \underline{|z'| = \sqrt[n]{|z|} \quad \in \mathbb{R}} \\ \varphi' = \dfrac{\varphi + 2k\pi}{n} \end{cases}$$

$$z' = \sqrt[n]{|z|}\left(\cos\left(\frac{\varphi + 2k\pi}{n}\right) + i\sin\left(\frac{\varphi + 2k\pi}{n}\right)\right)$$

$k \in \mathbb{Z}$

Per avere radici distinte,
bisogna prendere $k = 0, 1, \ldots, n-1$
da formula finale per ottenere le
n radici n-esime DISTINTE di z è

$$z_k = \sqrt[n]{|z|} \left(\cos \frac{\varphi + 2k\pi}{n} + i \sin \frac{\varphi + 2k\pi}{n}\right)$$

$k = 0, 1, \ldots, n-1$

Risolvere la seguente equazione in $\mathbb{C}$

$$z^4 + 4 = 0$$

$$z^4 = -4$$

TROVARE LE RADICI QUARTE DEL NUMERO COMPLESSO $\underline{\underline{-4}}$

$$\cancel{\sqrt[4]{-4}}$$

① SCRIVERE -4 IN FORMA TRIGONOMETRICA

$$-4 = 4(\cos\pi + i\sin\pi)$$

$$|-4| = \sqrt{(-4)^2 + 0^2} = \sqrt{16} = 4$$

$$\cos\varphi = \frac{-4}{4} = -1$$

$$\varphi = \pi$$

$$\sin\varphi = \frac{0}{4} = 0$$

$$z_k = \sqrt[4]{4}\left(\cos\left(\frac{\pi + 2k\pi}{4}\right) + i\sin\left(\frac{\pi + 2k\pi}{4}\right)\right)$$

$$k = 0, 1, 2, 3$$

$$z_0 = = \sqrt{2}\left(\cos\left(\frac{\pi + 0}{4}\right) + i\sin\left(\frac{\pi + 0}{4}\right)\right) =$$

$$= \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right) =$$

$$\sqrt{2}\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right) = 1 + i$$

$$z_1 = \sqrt{2}\left(\cos\frac{\pi+2\pi}{4} + i\sin\frac{\pi+2\pi}{4}\right) =$$

$$= \sqrt{2}\left(\cos\frac{3}{4}\pi + i\sin\frac{3}{4}\pi\right) =$$

$$\sqrt{2}\left(-\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right) = -1+i$$

$$z_2 = \sqrt{2}\left(\cos\frac{\pi+4\pi}{4} + i\sin\frac{\pi+4\pi}{4}\right) =$$

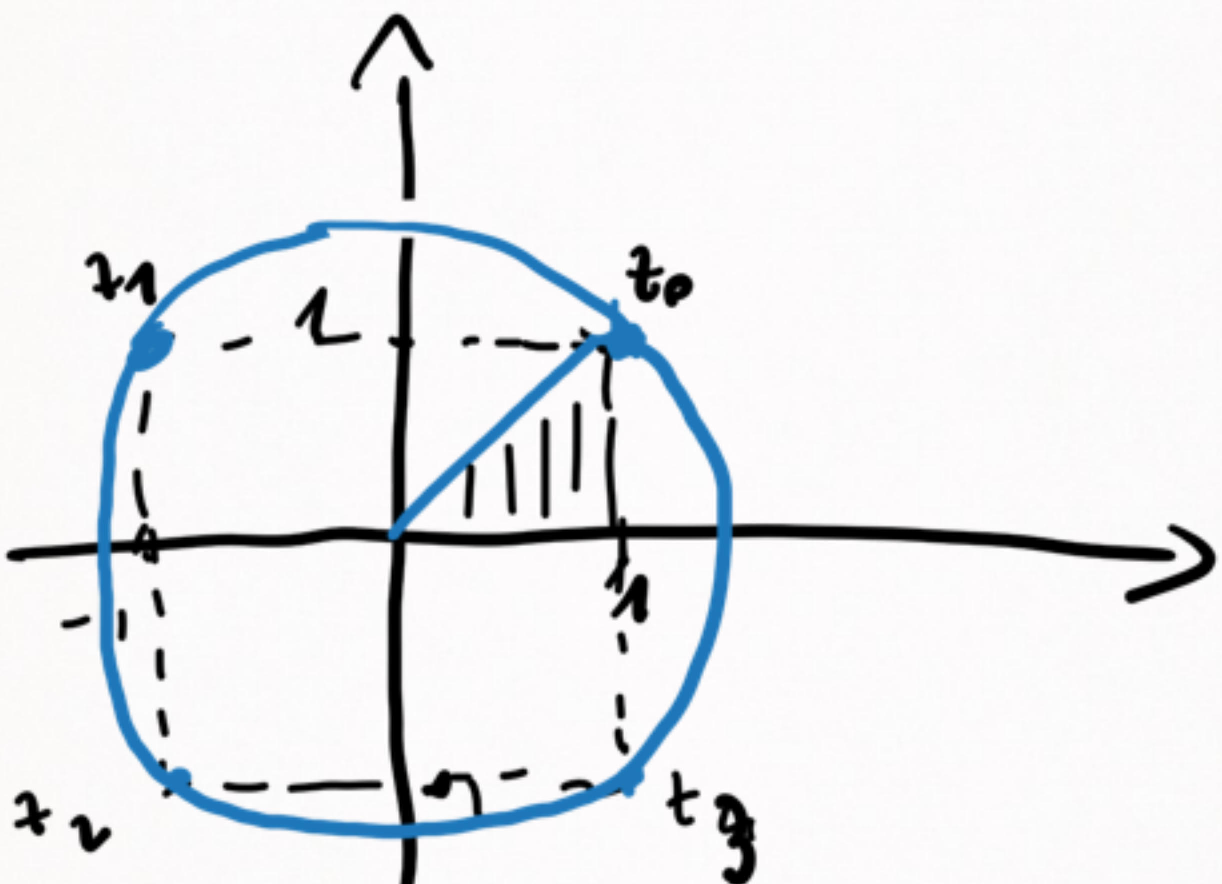
$$= \sqrt{2}\left(\cos\frac{5}{4}\pi + i\sin\frac{5}{4}\pi\right) =$$

$$= \sqrt{2}\left(-\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}\right) = -1-i$$

$$z_3 = \sqrt{2}\left(\cos\left(\frac{\pi + 6\pi}{4}\right) + i \sin\left(\frac{\pi + 6\pi}{4}\right)\right)$$

$$= \sqrt{2}\left(\cos\frac{7}{4}\pi + i \sin\frac{7}{4}\pi\right) =$$

$$= \sqrt{2}\left(\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}\right) = 1 - i$$



In generale le radici n-sime di un complesso z sono i vertici del poligono regolare di n lati iscritto nella circonferenza di centro $(0,0)$ e raggio $\sqrt[n]{|z|}$

RISOLVERE LA SEGUENTE EQUAZIONE

$$z^3 = (1+i)^5$$

(1) SCRIVO $1+i$ in forma trigonometrica

$$1+i = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$

$$|1+i| = \sqrt{1+1} = \sqrt{2}$$

$$\cos\varphi = \frac{Re\, z}{|z|} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\sin\varphi = \frac{Im\, z}{|z|} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\boxed{\varphi = \frac{\pi}{4}}$$

(2) CALCOLO $(1+i)^5$

⌈ FORMULA DI DE MOIVRE

$$z^n = |z|^n (\cos n\varphi + i \sin n\varphi)$$ ⌋

$$(1+i)^5 = \underbrace{(\sqrt{2})^5}\left(\cos \underbrace{\frac{5}{4}}\pi + i \sin \frac{5}{4}\pi\right)$$

$$|(1+i)^5| = 2^{\frac{5}{2}}$$

$$\varphi = \frac{5}{4}\pi$$

$$\boxed{z^3 = (1+i)^5}$$

(3) CALCOLARE LE RADICI TERZE

$$z_k = \sqrt[3]{|z|}\left(\cos \frac{\varphi+2k\pi}{3} + i \sin \frac{\varphi+2k\pi}{3}\right)$$

$$k = 0, 1, 2$$

$$z_0 = \sqrt[3]{2^{\frac{5}{2}}}\left(\cos\frac{\frac{5}{4}\pi + 0}{3} + i\sin\frac{\frac{5}{4}\pi + 0}{3}\right)$$

$$= 2^{\frac{5}{6}}\left(\cos\frac{5}{12}\pi + i\sin\frac{5}{12}\pi\right)$$

$$z_1 = 2^{\frac{5}{6}}\left(\cos\frac{\frac{5}{4}\pi + 2\pi}{3} + i\sin\frac{\frac{5}{4}\pi + 2\pi}{3}\right)$$

$$= 2^{\frac{5}{6}}\left(\cos\frac{13}{12}\pi + i\sin\frac{13}{12}\pi\right)$$

$$z_2 = 2^{\frac{5}{6}}\left(\cos\left(\frac{\frac{5}{4}\pi + 4\pi}{3}\right) + i\sin\left(\frac{\frac{5}{4}\pi + 4\pi}{3}\right)\right)$$

$$= 2^{\frac{5}{6}}\left(\cos\frac{21}{12}\pi + i\sin\frac{21}{12}\pi\right)$$

$$z^2 = -\sqrt{3} + i$$

(1) SCRIVERE $-\sqrt{3} + i$ in forma trigonometrica

$$|-\sqrt{3} + i| = \sqrt{(-\sqrt{3})^2 + 1^2} = \sqrt{3+1} = 2$$

$$\cos \varphi = \frac{\operatorname{Re} z}{|z|} = -\frac{\sqrt{3}}{2}$$

$$\sin \varphi = \frac{\operatorname{Im} z}{|z|} = \frac{1}{2}$$

$$\varphi = \frac{5}{6}\pi$$

$$-\sqrt{3} + i = 2\left(\cos \frac{5}{6}\pi + i \sin \frac{5}{6}\pi\right)$$

$$z_k = \sqrt{|z|}\left(\cos\frac{\varphi+2k\pi}{2} + i\sin\frac{\varphi+2k\pi}{2}\right)$$

$$k = 0, 1$$

$$z_0 = \sqrt{2}\left(\cos\frac{\frac{5}{6}\pi+0}{2} + i\sin\frac{\frac{5}{6}\pi+0}{2}\right)$$

$$= \sqrt{2}\left(\cos\frac{5}{12}\pi + i\sin\frac{5}{12}\pi\right)$$

$$z_1 = \sqrt{2}\left(\cos\frac{\frac{5}{6}\pi+2\pi}{2} + i\sin\left(\frac{\frac{5}{6}\pi+2\pi}{2}\right)\right)$$

$$= \sqrt{2}\left(\cos\left(\frac{5}{12}\pi+\pi\right) + i\sin\left(\frac{5}{12}\pi+\pi\right)\right) =$$

$$= \sqrt{2}\left(-\cos\frac{5}{12}\pi - i\sin\frac{5}{12}\pi\right) =$$

$$= -z_0$$

$$\boxed{\begin{aligned}\cos(\varphi+\pi) &= -\cos\varphi \\ \sin(\varphi+\pi) &= -\sin\varphi\end{aligned}}$$

È VERO SEMPRE CHE LE RADICI QUADRATE DI UN NUMERO COMPLESSO SONO UNA L'OPPOSTO DELL'ALTRA

$a z^2 + b z + c = 0 \; (*)$ $\qquad a \in \mathbb{R}$

$\qquad\qquad b, c \in \mathbb{C}$

le formule risolutive per $(*)$

$$\boxed{w_{1,2} = \frac{-b + z_k}{2a}} \qquad k = 0, 1$$

ove z_k sono le radici quadrate complesse di $b^2 - 4ac$

$z^2 - z + 1 = 0$

$b^2 - 4ac = 1 - 4 = -3$

DEVO CALCOLARE LE RADICI QUADRATE DI -3

$$-3 = 3(\cos\pi + i\sin\pi)$$

$$z_K = \sqrt{|z|}\left(\cos\left(\frac{\varphi+2k\pi}{2}\right) + i\sin\left(\frac{\varphi+2k\pi}{2}\right)\right)$$

$$z_0 = \sqrt{3}\left(\cos\frac{\pi+0}{2} + i\sin\frac{\pi+0}{2}\right) =$$

$$= \sqrt{3}\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right) = \sqrt{3}\,i$$

$$z_1 = -z_0 = -\sqrt{3}\,i$$

$$z^2 - z + 1 = 0$$

$$w_1 = \frac{1+\sqrt{3}i}{2}$$

$$w_2 = \frac{1-\sqrt{3}i}{2}$$

$$z^4 + (1-i)z^2 - i = 0$$

$$z^2 = t$$

$$t^2 + (1-i)t - i = 0$$

$$b^2 - 4ac = (1-i)^2 + 4i = 1 - 1 - 2i + 4i =$$

$= 2i$ $\qquad b^2 - 4ac = 2i$

DEVO CALCOLARE LE RADICI

- QUADRATE DI $2i$

- APPLICARE LA FORMULA RISOLUTIVA PER LE EQUAZIONI DI II GRADO

(t_1, t_2)

$z^2 = t$

- $z^2 = t_1$ $\qquad$ $z^2 = t_2$

TEOREMA FONDAMENTALE DELL'ALGEBRA

Se $a_n z^n + a_{n-1} z^{n-1} + \dots + a_1 z + a_0 = 0$

un'equazione in $\mathbb{C}$.

Essa ammette <u>SEMPRE</u> n

soluzioni (anche non distinte)