Esempio di esercizio di esame

$$\frac{|5-2x|-7}{3-x} \geq 0$$

Per risolverlo serve

1) Saper risolvere disequazioni polinomiali di 1° grado.
2) Unire e intersecare soluzioni di disequazioni.
3) Studiare il segno di una frazione.
4) Risolvere disequazioni con valore assoluto.

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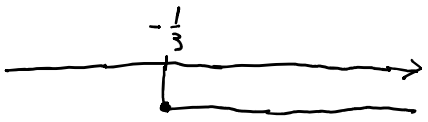
1) Disequazioni elementari di I grado

- $3x + 1 \geq 0$

  $3x \geq -1 \longrightarrow \left(\frac{-1}{3} = -\frac{1}{3}\right)$

  $x \geq -\frac{1}{3}$

Rappresentazione grafica delle soluzioni

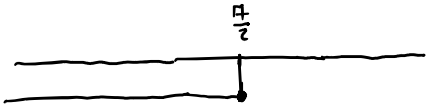


- $7 - 2x \geq 0$

  $-2x \geq -7$

  $x \leq \frac{-7}{-2}$

  $x \leq \frac{7}{2}$

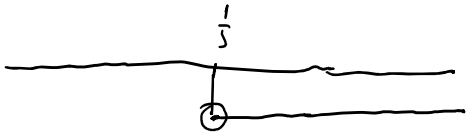


- $1 - 5x < 0$

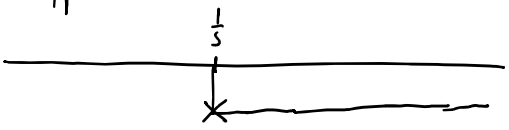
  $-5x < -1$

  $x > \frac{-1}{-5}$

  $x > \frac{1}{5}$



oppure



2) Unione e intersezione di soluzioni

Unione ($\vee$)

$2x - 1 \geq 0 \quad \vee \quad 3x + 1 < 0$

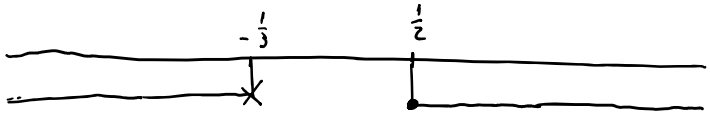
Si risolvono le due disequazioni e si uniscono le soluzioni (rappresentandole sulla stessa riga).

$2x - 1 \geq 0 \qquad \vee \qquad 3x + 1 < 0$

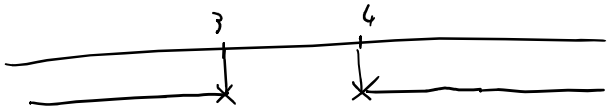
$2x \geq 1 \qquad\qquad 3x < -1$

$x \geq \frac{1}{2} \qquad\qquad x < -\frac{1}{3}$

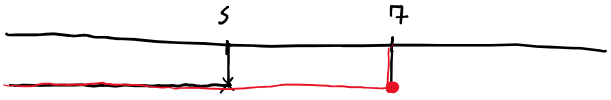
Soluzione: $x \geq \frac{1}{2} \quad \vee \quad x < -\frac{1}{3}$



- $x < 3 \quad \vee \quad x > 4$



- $x < 5 \quad \vee \quad x \leq 7$



si scrive anche più semplicemente come $x \leq 7$.

- $x < 2 \quad \vee \quad x \geq -1$

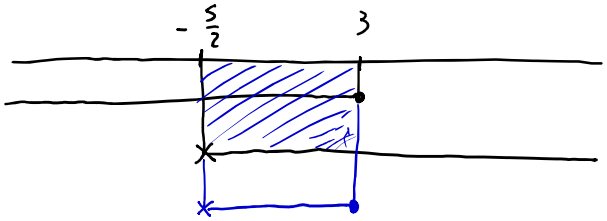
$\forall x \in \mathbb{R}$.

Intersezione di soluzioni (sistemi)

$$\begin{cases} x - 3 \leq 0 \\ 2x + 5 > 0 \end{cases}$$

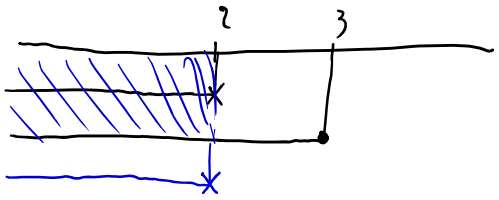
Se risolvono le due disuguaglianze e si prendono le regioni comuni agli insiemi delle soluzioni. Graficamente, si rappresentano su righe diverse le soluzioni delle due disequazioni e si prendono le regioni comuni.

$$\begin{cases} x - 3 \le 0 \iff x \le 3 \\ 2x + 5 > 0 \iff 2x > -5 \iff x > -\frac{5}{2} \end{cases}$$



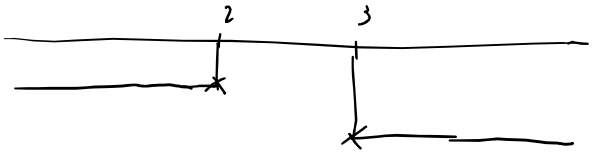
$-\frac{5}{2} < x \le 3$

- $\begin{cases} x < 2 \\ x \le 3 \end{cases}$



$x < 2$

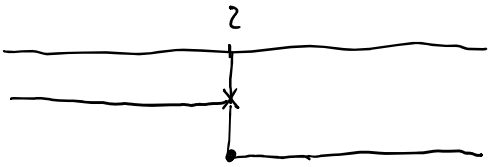
- $\begin{cases} x < 2 \\ x > 3 \end{cases}$



Il sistema non ha soluzioni.

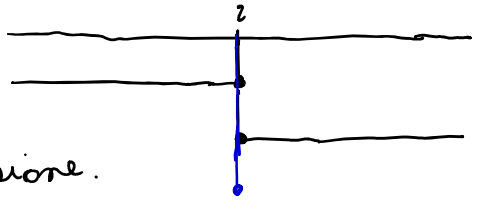
Non ci sono parti comuni.

- $\begin{cases} x < 2 \\ x \ge 2 \end{cases}$



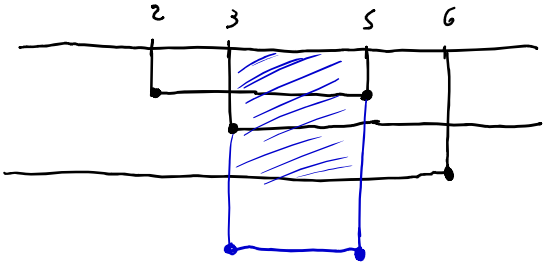
Non ci sono soluzioni.

- $\begin{cases} x \le 2 \\ x \ge 2 \end{cases}$



$x = 2$ è l'unica soluzione.

- $\begin{cases} 2 \le x \le 5 \\ x \ge 3 \\ x \le 6 \end{cases}$



$3 \le x \le 5$

3) Studio del segno di un prodotto o di una frazione.

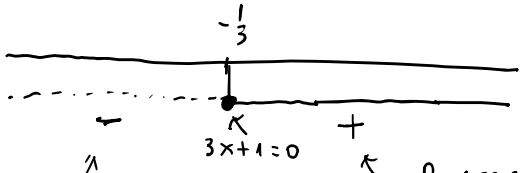
$3x + 1$

Per rappresentare il segno:

- Risolvo $3x+1 \geq 0$
- Rappresento la soluzione con una linea continua e la parte rimanente con linea tratteggiata. Può anche essere utile segnare con un pallino pieno gli zeri.

Segno di $3x+1$:

$3x+1 \geq 0$
$3x \geq -1$
$x \geq -\frac{1}{3}$



Regione in cui $3x+1 \geq 0$

Regione in cui $3x+1 < 0$

Studio del segno di un prodotto: si fa rappresentando i segni dei singoli fattori e "incrociando" i risultati

$(x-3)(x+2)(2x+1)$

Per rappresentare il segno:

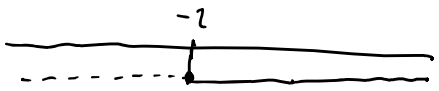
Segno di $x-3$:

$x-3 \geq 0 \iff x \geq 3$

segno di $x+2$

$x+2 \geq 0$
$x \geq -2$

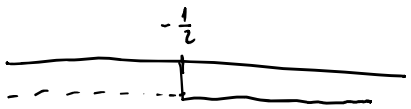
Segno di $x+2$:



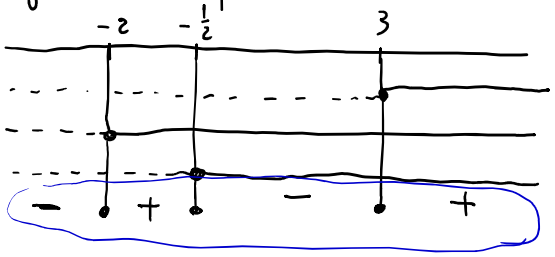
Segno di $2x+1$

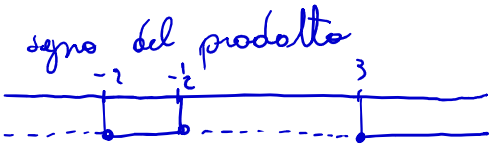
$2x+1 \geq 0$
$2x \geq -1$
$x \geq -\frac{1}{2}$

segno:



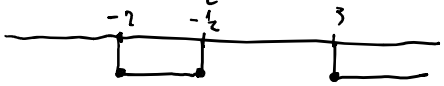
Segno del prodotto:





Una volta rappresentato il segno di un prodotto, si possono facilmente risolvere disequazioni:

$(x-3)(x+2)(2x+1) \geq 0 \iff -2 \leq x \leq -\frac{1}{2} \quad \vee \quad x \geq 3$



$(x-3)(x+2)(2x+1) \leq 0 \iff x \leq -2 \quad \vee \quad -\frac{1}{2} \leq x \leq 3$

- Studio del segno di una frazione:

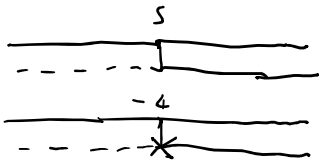
Si fa come per i prodotti ma bisogna ricordarsi che al denominatore non può essere 0.

$$\frac{x-5}{x+4}$$

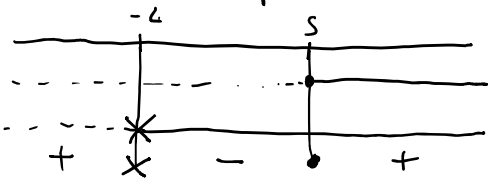
Numeratore: $x - 5 \geq 0 \iff x \geq 5$

Denominatore: $x + 4 > 0 \iff x > -4$

Non può essere 0

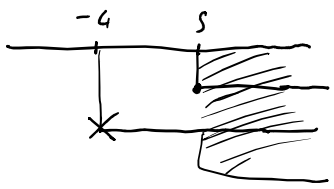


Segno della frazione:



E' importante non confondere lo studio del segno con la rappresentazione delle soluzioni o con la risoluzione dei sistemi

$$\begin{cases} x - 5 \geq 0 \\ x + 4 > 0 \end{cases} \iff \begin{cases} x \geq 5 \\ x > -4 \end{cases}$$



$x \geq 5$

Lo studio del segno permette di risolvere disequazioni in cui si impone che un prodotto / frazione sia > 0, ≥ 0, ≤ 0 o < 0.

ESEMPIO

$$\frac{1 - 3x}{5 - 2x} \leq 0$$

Si risolve facendo lo studio del segno della frazione $\frac{1-3x}{5-2x}$.

Numeratore: $1 - 3x \geq 0 \iff -3x \geq -1$

$$x \leq \frac{-1}{-3}$$

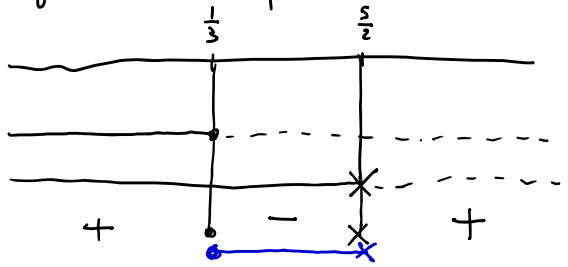
$$x \leq \frac{1}{3}$$

Denom.: $5 - 2x > 0 \iff 5 - 2x > 0$

$$-2x > -5$$

$$x < \frac{5}{2}$$

Segno della frazione:



Soluzione di $\frac{1-3x}{5-2x} \leq 0$: $\frac{1}{3} \leq x < \frac{5}{2}$

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4) Disequazioni con valore assoluto

$|a(x)| \begin{matrix} \geq \\ \leq \\ < \\ > \end{matrix} b(x)$

$$|a(x)| = \begin{cases} a(x) & \text{se } a(x) \geq 0 \\ -a(x) & \text{se } a(x) < 0 \end{cases}$$

(lo stesso vale per gli altri tipi di disequazioni)

La disequazione $|a(x)| \geq b(x)$ è equivalente a:

$$\begin{cases} a(x) \geq 0 \\ a(x) \geq b(x) \end{cases} \quad \vee \quad \begin{cases} a(x) < 0 \\ -a(x) \geq b(x) \end{cases}$$

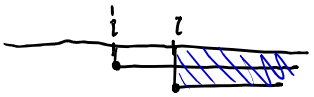
ESEMPIO

$|2x-1| \geq 3$

① $\begin{cases} 2x-1 \geq 0 \\ 2x-1 \geq 3 \end{cases}$ $\vee$ ② $\begin{cases} 2x-1 < 0 \\ -(2x-1) \geq 3 \end{cases}$

① $\begin{cases} x \geq \frac{1}{2} \\ 2x \geq 4 \end{cases}$

$\begin{cases} x \geq \frac{1}{2} \\ x \geq 2 \end{cases}$



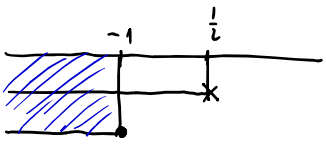
$x \geq 2$

② $\begin{cases} x < \frac{1}{2} \\ -2x+1 \geq 3 \end{cases}$ *

* $-2x \geq 2$

$x \leq -1$

$\begin{cases} x < \frac{1}{2} \\ x \leq -1 \end{cases}$



$x \leq -1$

Soluzione: $x \geq 2 \quad \vee \quad x \leq -1$.

ESERCIZIO

$$\frac{|2x-1| - 2}{x-5} \geq 0$$

Denominatore

$x - 5 > 0 \iff x > 5$ (segno del denominatore )

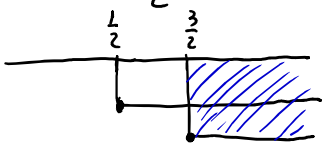
Numeratore:

$|2x-1| - 2 \geq 0$

① $\begin{cases} 2x-1 \geq 0 \\ 2x-1-2 \geq 0 \end{cases}$ $\vee$ $\begin{cases} 2x-1 < 0 \\ -(2x-1)-2 \geq 0 \end{cases}$ ②

$\begin{cases} x \geq \frac{1}{2} \\ 2x-3 \geq 0 \end{cases}$

$\begin{cases} x \geq \frac{1}{2} \\ x \geq \frac{3}{2} \end{cases}$



$x \geq \frac{3}{2}$

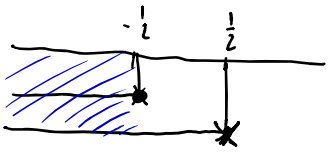
$\begin{cases} x < \frac{1}{2} \\ -2x+1-2 \geq 0 \end{cases}$

$-2x-1 \geq 0$

$-2x \geq 1$

$x \leq -\frac{1}{2}$

$\begin{cases} x < \frac{1}{2} \\ x \leq -\frac{1}{2} \end{cases}$

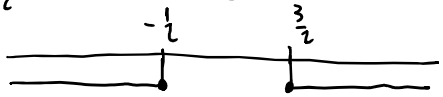


$x \leq -\frac{1}{2}$

Soluzione di

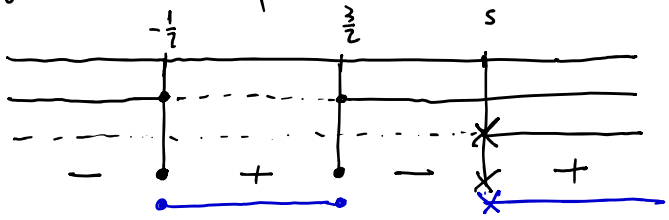
$|2x-1| - 2 \geq 0$

è $x \geq \frac{3}{2} \vee x \leq -\frac{1}{2}$



Segno del numeratore:

Segno della frazione:



Soluzione finale: $-\frac{1}{2} \leq x \leq \frac{3}{2} \quad \vee \quad x > 5$

ESERCIZIO

$$\frac{|5-2x|-7}{3-x} \geq 0$$

- Numeratore:

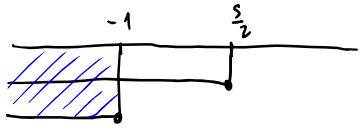
$|5-2x|-7 \geq 0$

$$\begin{cases} 5-2x \geq 0 \\ 5-2x-7 \geq 0 \end{cases} \quad \vee \quad \begin{cases} 5-2x < 0 \\ -(5-2x)-7 \geq 0 \end{cases}$$

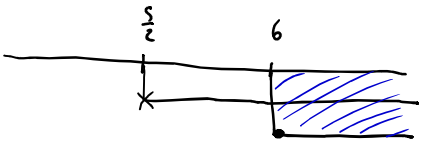
$$\begin{cases} -2x \geq -5 \\ -2x-2 \geq 0 \end{cases} \qquad \begin{cases} -2x < -5 \\ -5+2x-7 \geq 0 \end{cases}$$

$$\begin{cases} x \leq \frac{-5}{-2} \\ -2x \geq 2 \end{cases} \qquad \begin{cases} x > \frac{5}{2} \\ 2x-12 \geq 0 \end{cases}$$

$$\begin{cases} x \leq \frac{5}{2} \\ x \leq -1 \end{cases} \qquad \begin{cases} x > \frac{5}{2} \\ x \geq 6 \end{cases}$$



$x \leq -1$



$x \geq 6$

$x \leq -1 \quad \vee \quad x \geq 6$

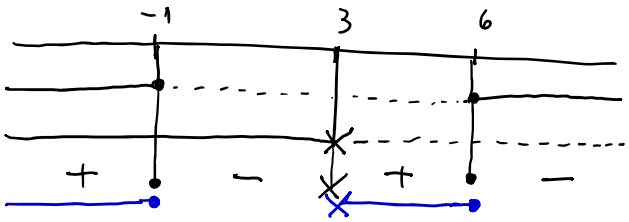
Denominatore:

$3-x > 0$

$-x > -3$

$x < 3$

Segno della frazione:



≥ 0

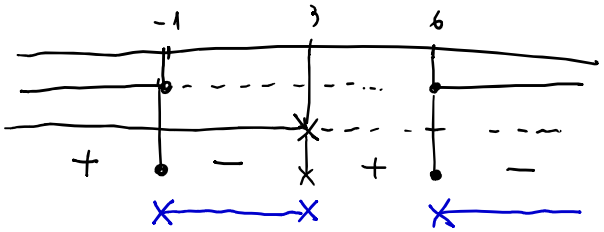
$x \leq -1 \quad \vee \quad 3 < x \leq 6$

$$\frac{|5-2x|-7}{3-x} < 0$$

- Num
$|5-2x|-7 \geq 0 \quad \dots\dots \quad x \leq -1 \quad \vee \quad x \geq 6$

- Den: $x < 3$

Segno



$<$

Soluzione $\quad -1 < x < 3 \quad \vee \quad x > 6.$

Nota: Lo studio del segno si può usare solo in disequazioni del tipo $\frac{n(x)}{d(x)} \geq 0$, ≤ 0, > 0, < 0

$\frac{x+1}{x-2} \geq 1$ → Non possiamo fare subito lo studio del segno.

$$\frac{x+1}{x-2} - 1 \geq 0$$

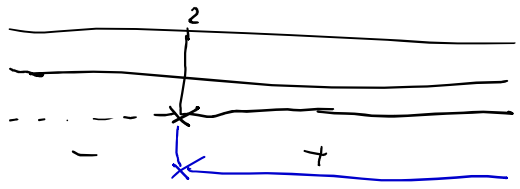
$$\frac{x+1-(x-2)}{x-2} \geq 0$$

$$\frac{x+1-x+2}{x-2} \geq 0$$

$$\frac{3}{x-2} \geq 0$$

$3 \geq 0$ sempre

$x-2 > 0 \iff x > 2$



ESERCIZI

1) $\dfrac{|3x-1|-2}{x-3} \geq 0$ $\quad \left(-\frac{1}{3} \leq x \leq 1 \ \vee \ x > 3\right)$

2) $\dfrac{1-|2x-3|}{x-9} \leq 0$ $\quad \left(1 \leq x \leq 2 \ \vee \ x > 9\right)$

3) $\dfrac{|x-2|}{x-3} \geq 2$ $\quad \left(3 < x \leq 4\right)$

4) $\dfrac{x-3}{|2x-1|-x} > 0$ $\quad \left(\frac{1}{3} < x < 1 \ \vee \ x > 3\right)$

Soluzioni degli esercizi

1) $\dfrac{|3x-1|-2}{x-3} \geq 0$

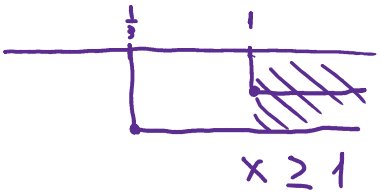
- Numeratore:

$|3x-1|-2 \geq 0$

$\begin{cases} 3x-1 \geq 0 \\ 3x-1-2 \geq 0 \end{cases}$ $\vee$ $\begin{cases} 3x-1 < 0 \\ -(3x-1)-2 \geq 0 \end{cases}$

$\begin{cases} x \geq \frac{1}{3} \\ 3x-3 \geq 0 \end{cases}$

$\begin{cases} x \geq \frac{1}{3} \\ x \geq 1 \end{cases}$



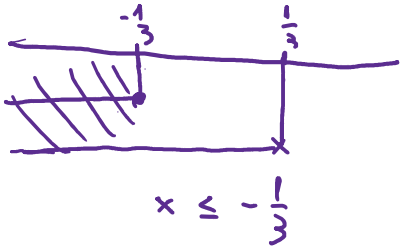
$x \geq 1$

$\begin{cases} x < \frac{1}{3} \\ -3x+1-2 \geq 0 \end{cases}$

$\begin{cases} x < \frac{1}{3} \\ -3x-1 \geq 0 \end{cases}$

$\begin{cases} x < \frac{1}{3} \\ -3x \geq 1 \end{cases}$

$\begin{cases} x < \frac{1}{3} \\ x \leq -\frac{1}{3} \end{cases}$



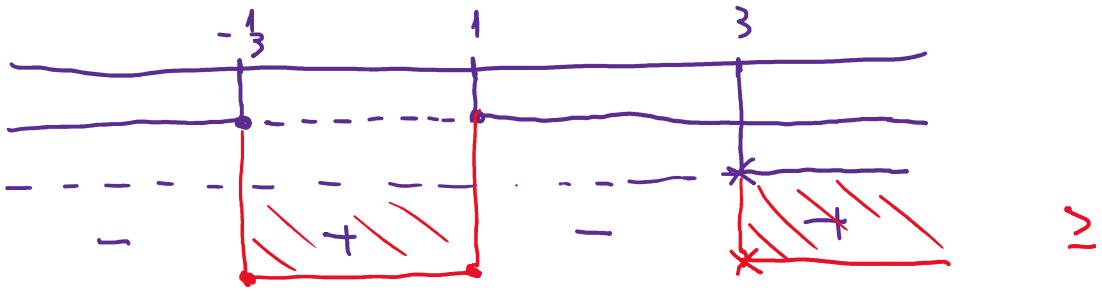
$x \leq -\frac{1}{3}$

Numeratore $x \geq 1 \quad \vee \quad x \leq -\frac{1}{3}$

- Denominatore:

$x - 3 > 0 \iff x > 3$

- Studio del segno:



Soluzione: $-\frac{1}{3} \leq x \leq 1 \quad \vee \quad x > 3$

2) $\dfrac{1 - |2x - 3|}{x - 9} \leq 0$

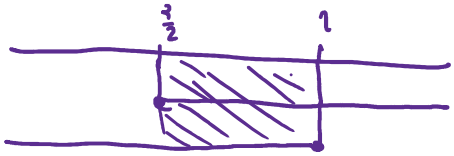
- Numeratore:

$1 - |2x - 3| \geq 0$

$$\begin{cases} 2x - 3 \geq 0 \\ 1 - (2x - 3) \geq 0 \end{cases} \quad \vee \quad \begin{cases} 2x - 3 < 0 \\ 1 + (2x - 3) \geq 0 \end{cases}$$

$$\begin{cases} x \geq \frac{3}{2} \\ 4 - 2x \geq 0 \end{cases}$$

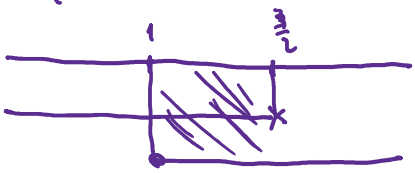
$$\begin{cases} x \geq \frac{3}{2} \\ x \leq 2 \end{cases}$$



$$\frac{3}{2} \leq x \leq 2$$

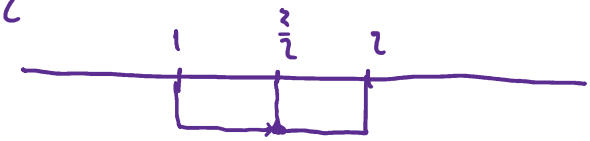
$$\begin{cases} x < \frac{3}{2} \\ 2x - 2 \geq 0 \end{cases}$$

$$\begin{cases} x < \frac{3}{2} \\ x \geq 1 \end{cases}$$



$$1 \leq x < \frac{3}{2}$$

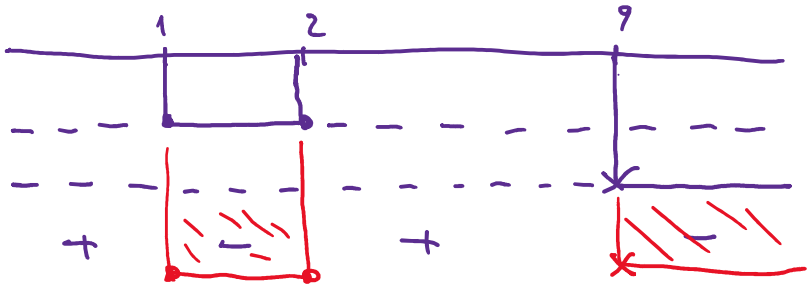
$$\frac{3}{2} \leq x \leq 2 \quad \vee \quad 1 \leq x < \frac{3}{2}$$



$$1 \leq x \leq 2$$

Denominatore: $x - 9 > 0 \Leftrightarrow x > 9$

Studio del segno:



$\leq$

$$1 \leq x \leq 2 \quad \vee \quad x > 9$$

3) $\dfrac{|x-2|}{x-3} \geq 2$

$$\frac{|x-2|}{x-3} - 2 \geq 0$$

$$\frac{|x-2| - 2(x-3)}{x-3} \geq 0$$

$$\frac{|x-2| - 2x + 6}{x-3} \geq 0$$

Iniziamo lo studio del segno:

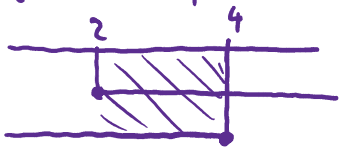
Numeratore: $|x-2| - 2x + 6 \geq 0$

$$\begin{cases} x - 2 \geq 0 \\ x - 2 - 2x + 6 \geq 0 \end{cases} \vee \begin{cases} x - 2 < 0 \\ -(x-2) - 2x + 6 \geq 0 \end{cases}$$

$$\begin{cases} x \geq 2 \\ -x + 4 \geq 0 \end{cases}$$

$$\begin{cases} x \geq 2 \\ -x \geq -4 \end{cases}$$

$$\begin{cases} x \geq 2 \\ x \leq 4 \end{cases}$$



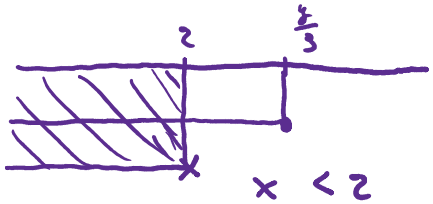
$2 \leq x \leq 4$

$$\begin{cases} x < 2 \\ -x + 2 - 2x + 6 \geq 0 \end{cases}$$

$$\begin{cases} x < 2 \\ -3x + 8 \geq 0 \end{cases}$$

$$\begin{cases} x < 2 \\ -3x \geq -8 \end{cases}$$

$$\begin{cases} x < 2 \\ x \leq \frac{8}{3} \end{cases}$$



$x < 2$

$2 \leq x \leq 4 \quad \vee \quad x < 2$

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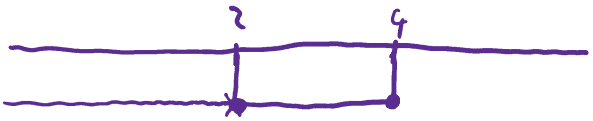
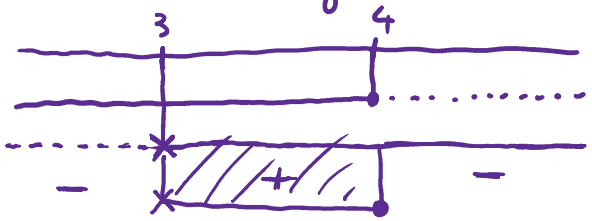


$x \leq 4$

$|x-2| - 2x + 6 \geq 0 \iff x \leq 4$

Denominatore: $x - 3 > 0 \iff x > 3$

Studio del segno:



Soluzione: $3 < x \leq 4$.

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4) $\dfrac{x-3}{|2x-1| - x} > 0$

Studio del segno:

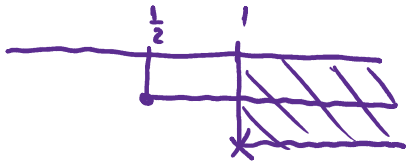
Numeratore $x - 3 \geq 0 \iff x \geq 3$

Denominatore: $|2x-1| - x > 0$

$$\begin{cases} 2x - 1 \geq 0 \\ 2x - 1 - x > 0 \end{cases} \quad \vee \quad \begin{cases} 2x - 1 < 0 \\ -(2x-1) - x > 0 \end{cases}$$

$$\begin{cases} x \geq \frac{1}{2} \\ x - 1 > 0 \end{cases}$$

$$\begin{cases} x \geq \frac{1}{2} \\ x > 1 \end{cases}$$



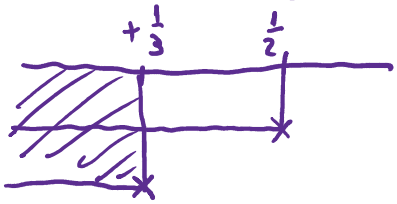
$x > 1$

$$\begin{cases} x < \frac{1}{2} \\ -2x + 1 - x > 0 \end{cases}$$

$$\begin{cases} x < \frac{1}{2} \\ -3x + 1 > 0 \end{cases}$$

$$\begin{cases} x < \frac{1}{2} \\ -3x > -1 \end{cases}$$

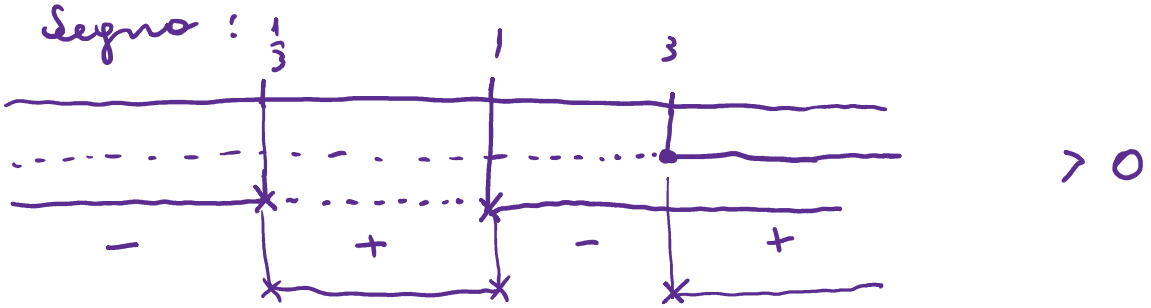
$$\begin{cases} x < \frac{1}{2} \\ x < 1\frac{1}{3} \end{cases}$$



$x < +\frac{1}{3}$

$x > 1 \quad \vee \quad x < +\frac{1}{3}$

Segno :



Soluzione : $\frac{1}{3} < x < 1 \quad \vee \quad x > 3.$