

NUMERI COMPLESSI I

$$\textcircled{*} \quad \underline{x^2 + 1 = 0}$$

NON HA SOLUZIONI
REALI

ESISTE UN INSIEME NUMERICO
IN CUI $\textcircled{*}$ ABBIA SOLUZIONI?

UNITÀ IMMAGINARIA i È VERIFICA

$$i^2 = -1$$

NON È UN NUMERO
REALE

Un numero complesso

$$z = x + iy$$

$$x, y \in \mathbb{R}$$

L'insieme dei numeri complessi si indica
con $\mathbb{C}$

$$\mathbb{C} = \{ x + iy \mid x, y \in \mathbb{R} \}$$

$$z = x + iy$$

x " PARTE REALE " di z $\operatorname{Re} z$

y " PARTE IMMAGINARIA " di z $\operatorname{Im} z$

$$z = \operatorname{Re} z + i \operatorname{Im} z$$

$$z = 4 - 3i$$

$$4 = \operatorname{Re} z \quad -3 = \operatorname{Im} z$$

oss $\operatorname{Re} z, \operatorname{Im} z \in \mathbb{R}$

$$z = \operatorname{Re} z + i \operatorname{Im} z$$

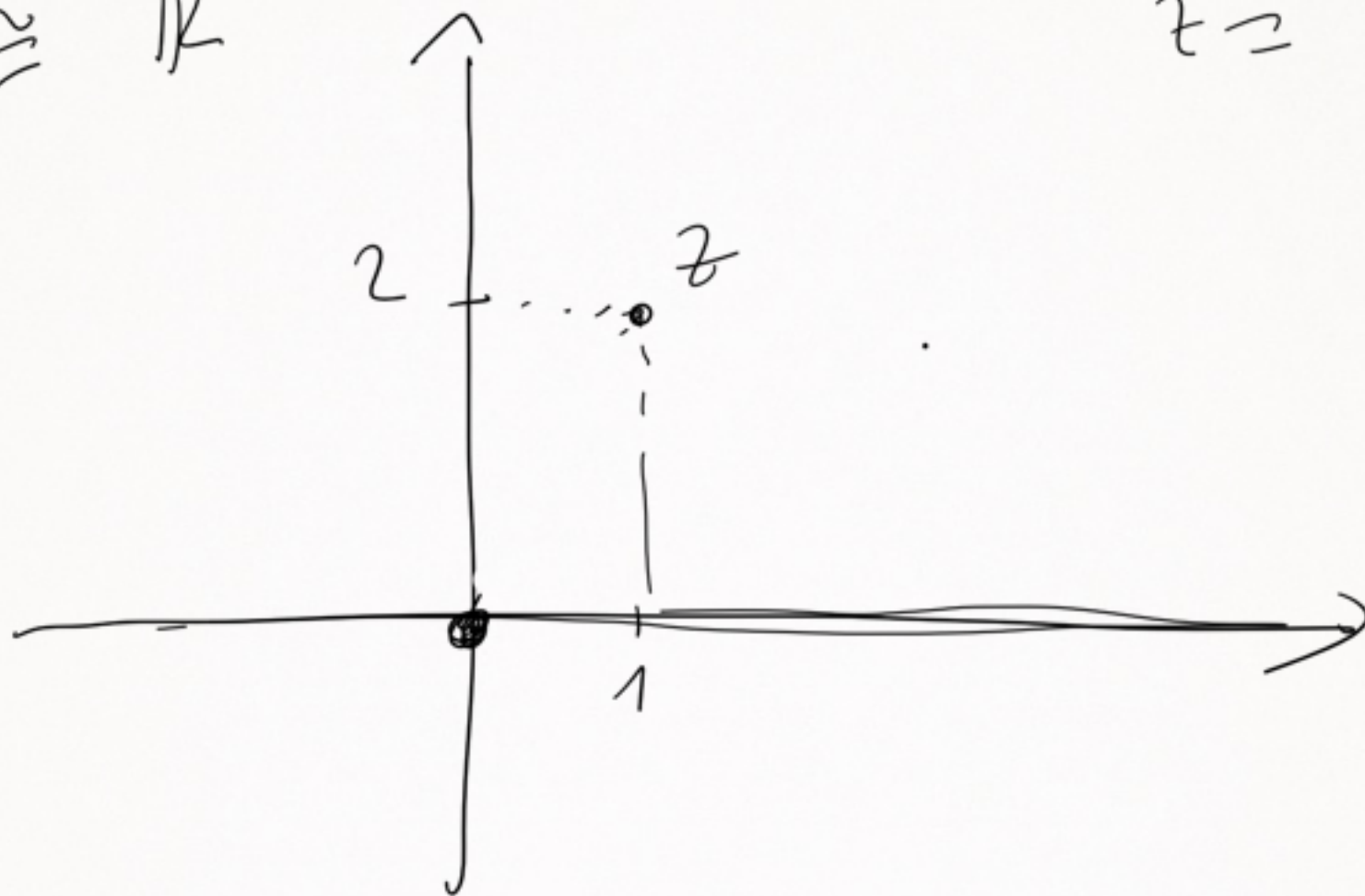
z is uniquely determined by $(\operatorname{Re} z, \operatorname{Im} z)$

$$z = 4 - 3i$$

z is uniquely determined by $(\overset{\downarrow}{4}, \overset{\downarrow}{-3})$

$$\mathbb{C} \cong \mathbb{R}^2$$

$$z = 1 + 2i$$



$$x \in \mathbb{R}$$

$$x = x + 0 \cdot i$$

$$iy$$

$$iy = 0 + iy \quad (\text{imag. part})$$

$$\underline{0} =$$

$$0 + i0$$

$$(0, 1) \in \mathbb{R}^2$$

$$(2, 1)$$

$$(0, 1) + (2, 1) = (0+2, 1+1)$$

$$= (2, 2)$$

Im allgemeinen

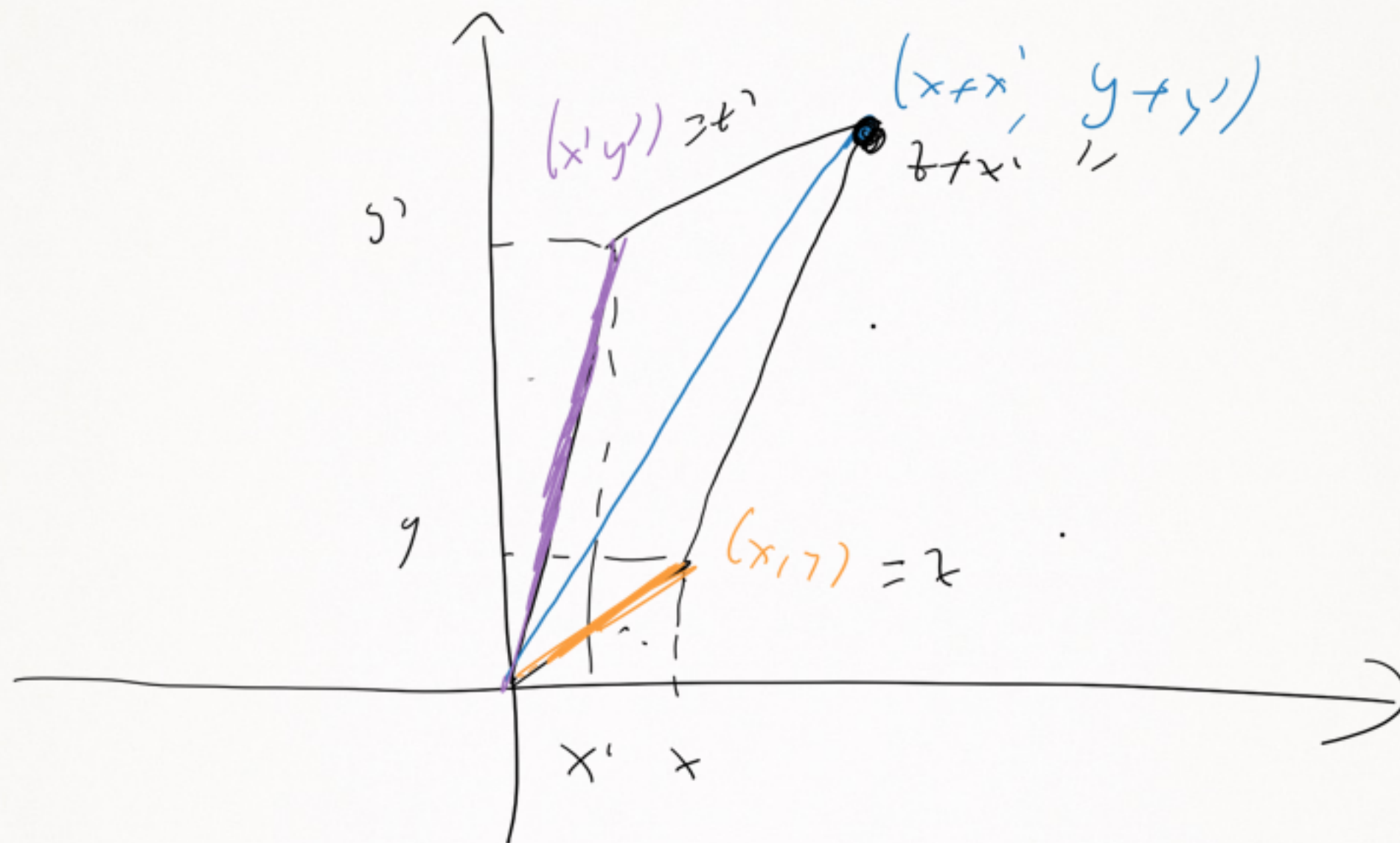
$$(x, y)$$

$$\in \mathbb{R}^2$$

$$(x', y')$$

$$(x, y) + (x', y') =$$

$$= (x+x', y+y')$$



$$z = x + iy$$

$$z' = x' + iy'$$

$$z + z' = (x + x') + i(y + y')$$

$$z = 4 - 3i$$

$$z' = 7 + 3i$$

$$z + z' = (4 + 7) + i(-3 + 3) = 11 + 0i$$

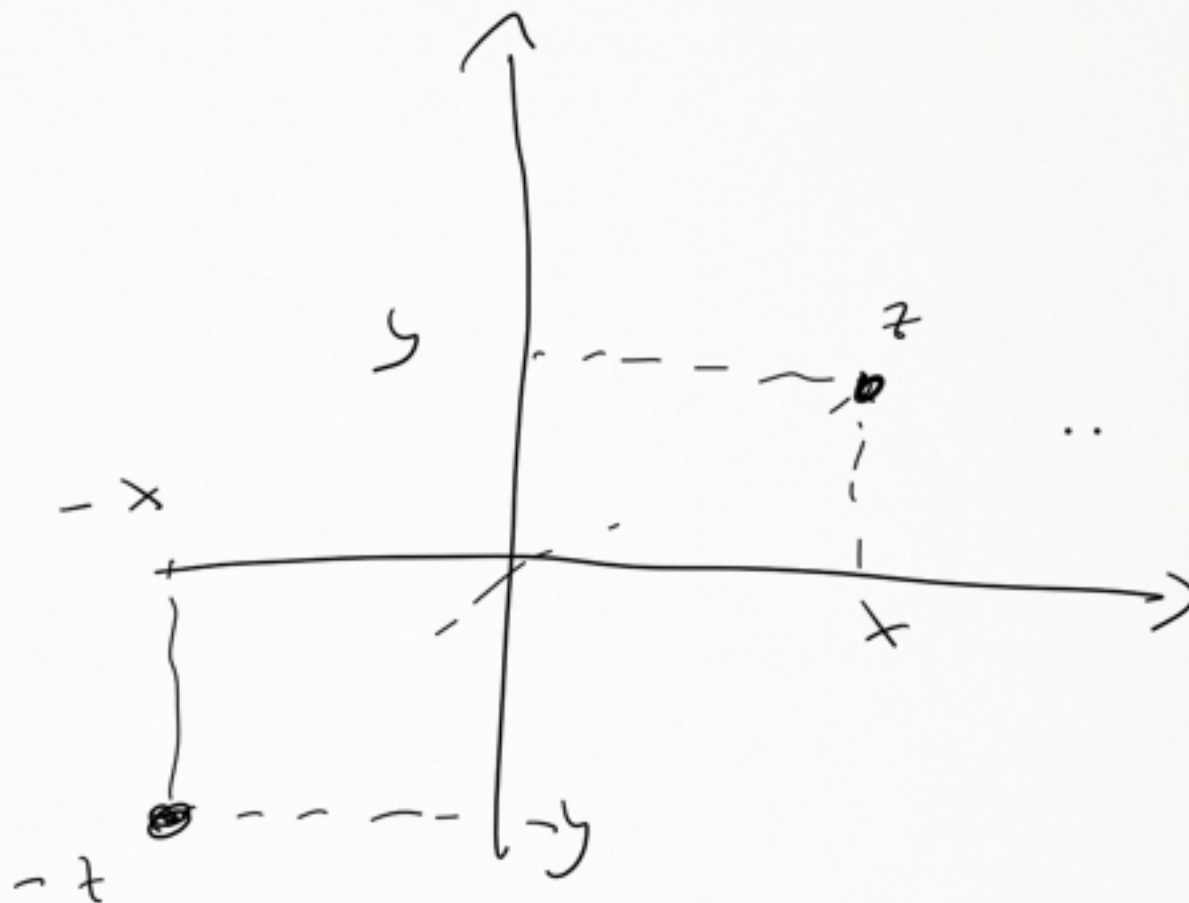
$$z = 11 - 2i$$

$$z' = 4 - 7i$$

$$z + z' = (11 + 4) + i(-2 - 7) = 15 - 9i$$

$$z = x + iy$$

$$-z = -x - iy$$



$$z = x + iy$$

$$z' = x' + iy'$$

$$z - z' = z + (-z') =$$

$$= (x - x') + i(y - y')$$

$$z = 4 - 3i$$

$$z' = 11 - 2i$$

$$\begin{aligned} z - z' &= (4 - 11) + i(-3 + 2) = \\ &= -7 - i \end{aligned}$$

$$z + 0_4 = z = 0_4 + z$$

0_4 "ELEMENTO
NEUTRO
PER LA
SOMMA"

$$0_4 = 0 + i \cdot 0$$

$$z = x + iy$$

$$z' = x' + iy'$$

$$\begin{aligned} z \cdot z' &= (x + iy)(x' + iy') = \\ &= x \cdot x' + ix y' + ix' y + i^2 y y' = \end{aligned}$$

$$\begin{aligned}
 x \cdot x' + i x y' + i x' y - y y' &= \\
 &= (x \cdot x' - y y') + i (x y' + x' y)
 \end{aligned}$$

RICORDANDO, il numero complesso
 $z \cdot x = \bar{z} \cdot x$.

$$Re(z \cdot x) = x \cdot x' - y y'$$

$$Im(z \cdot x) = x y' + x' y$$

$$z_0 = 1 + i \cdot 0$$

$$z \cdot z_0 = z = z_0 \cdot z \quad \forall z \in \mathbb{C}$$

ELEMENTO NEUTRO PER LA MOLTIPLICAZIONE

$$z = x + iy$$

$$z \neq 0$$

$$\frac{1}{z} = \frac{1}{x + iy} = \frac{(x - iy)}{(x + iy)(x - iy)} =$$

$$= \frac{(x - iy)}{x^2 - (i^2 y^2)} =$$

$$= \frac{x - iy}{x^2 - (-1)y^2} = \frac{x - iy}{x^2 + y^2} =$$

$$= \frac{x}{x^2 + y^2} + i \left(\frac{-y}{x^2 + y^2} \right)$$

$$\begin{aligned} (a+b)(a-b) \\ = a^2 - b^2 \end{aligned}$$

INVERSE

DI 7

$$z, z' \in \mathbb{C}$$

$$z' \neq 0_{\mathbb{C}}$$

$$\frac{z}{z'} = z \cdot \left(\frac{1}{z'} \right)$$

ESERPIO;

TROVARE parti reali e parti immaginarie di

$$z = \frac{1+2i}{1+3i} = \frac{(1+2i)(1-3i)}{(1+3i)(1-3i)} =$$

$$= \frac{1-3i+2i-6i^2}{(1)^2 - (3i)^2} = \frac{1+6+i(-3+2)}{1+9} =$$

$$\frac{7-i}{10} = \frac{7}{10} - \frac{1}{10}i$$

$$\operatorname{Re} z = \frac{7}{10}$$

$$\operatorname{Im} z = -\frac{1}{10}$$

$$z = \frac{1-i}{1+i} = \frac{(1-i)^2}{(1+i)(1-i)} =$$

$$= \frac{1 - 2i + i^2}{1 - i^2} =$$

$$(a+bi)^2 = a^2 + 2abi + b^2$$

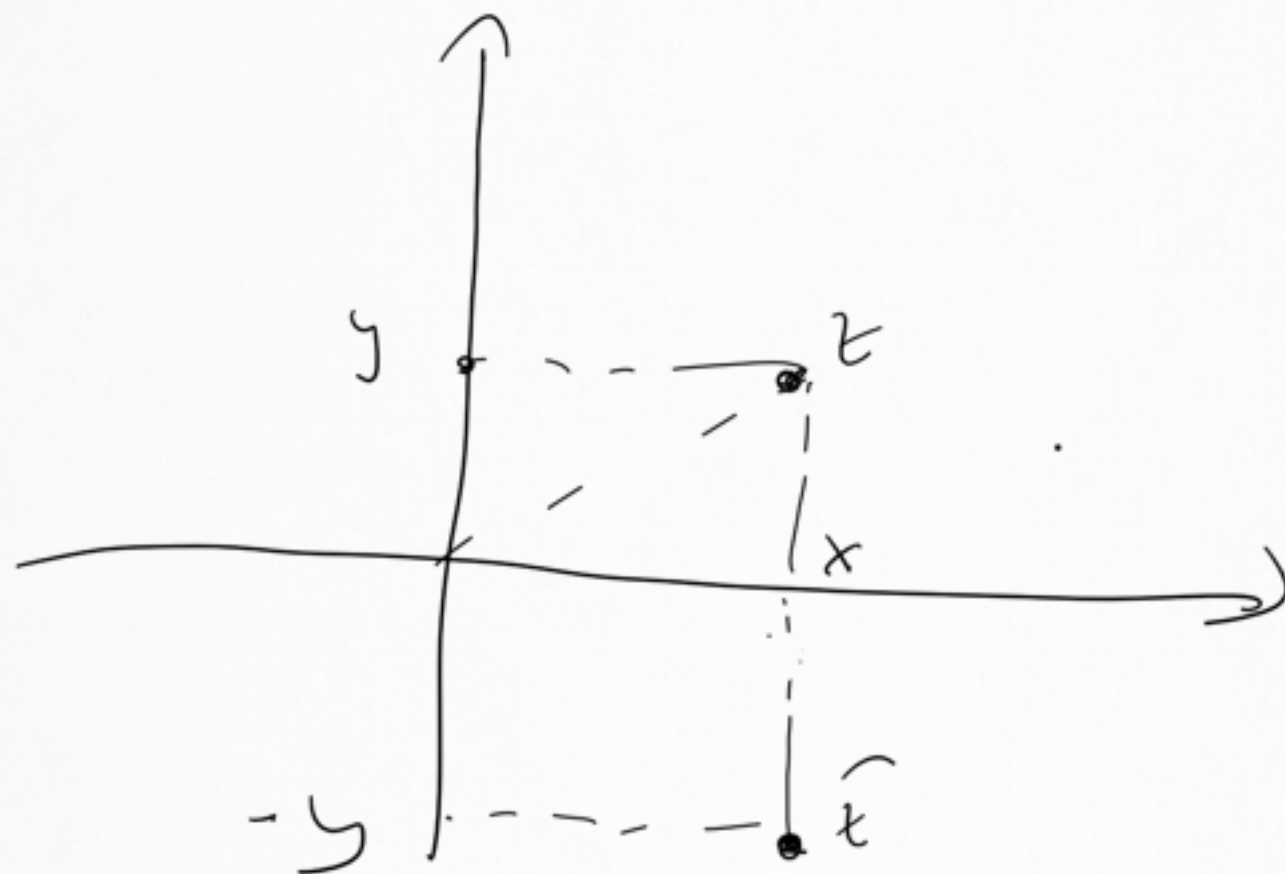
$$= \frac{\cancel{1} - 2i - \cancel{1}}{1 + 1} = -\frac{2i}{2} = -i$$

$\operatorname{Re} z = 0 \quad \operatorname{Im} z = -1$

$$z = x + iy$$

CONJUGATO DI z

$$\overline{z} = x - iy$$



$$z \in \mathbb{C}$$

$$\overline{\overline{z}} = z$$

$$\overline{z + z'} = \overline{z} + \overline{z'}$$

$$z = x + iy$$

$$z' = x' + iy'$$

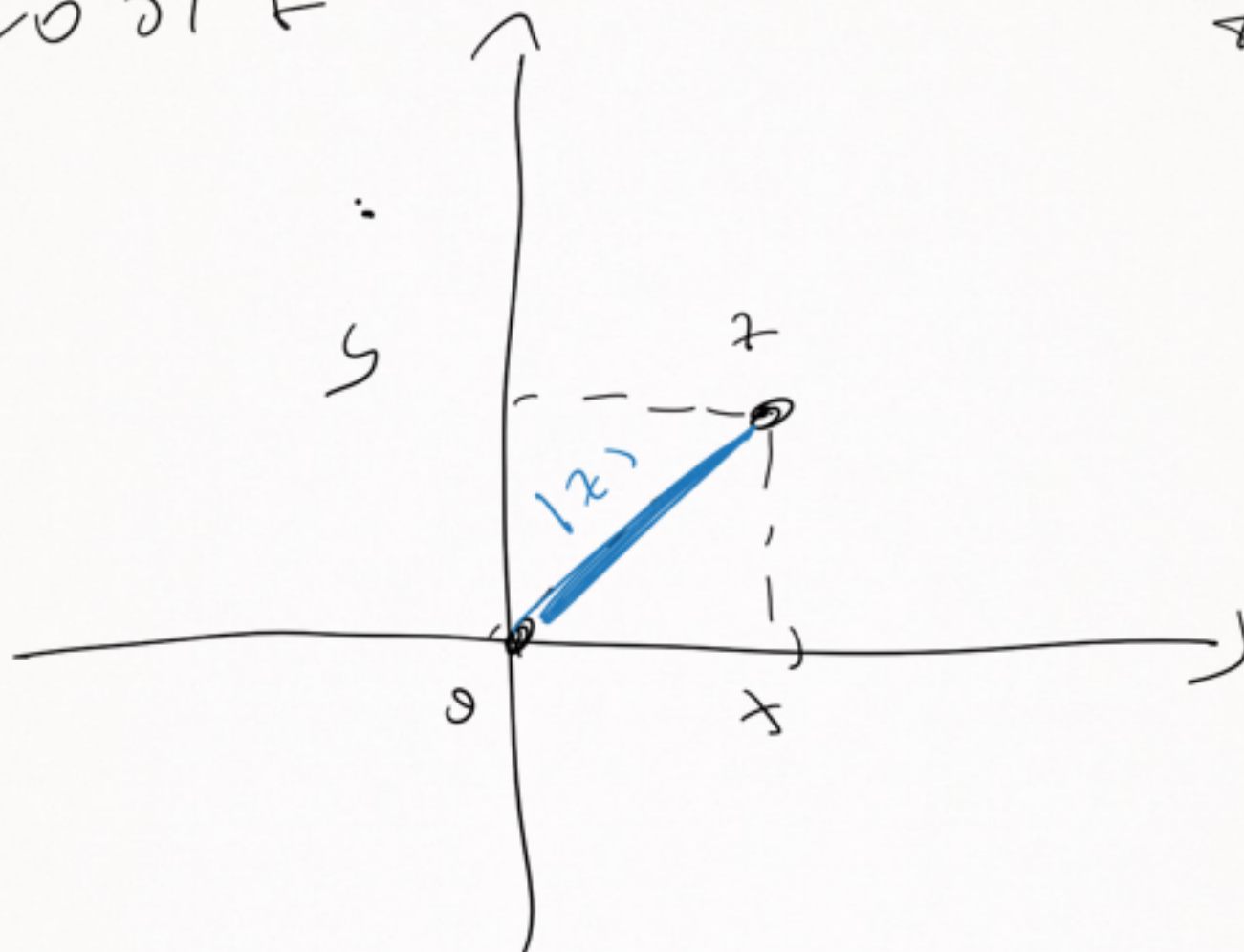
$$z + z' = (x + x') + i(y + y')$$

$$\overline{z + z'} = (x + x') - i(y + y') =$$

$$= (x - iy) + (x' - iy') = \bar{z} + \bar{z}'$$

Modulo z

$$z = x + iy$$



$$|z| = \sqrt{x^2 + y^2}$$

Modulo di z

$$z = 3 - 2i$$

$$|z| = \sqrt{3^2 + (-2)^2} = \sqrt{9 + 4} = \sqrt{13}$$

$$|\operatorname{Re} z|$$

$$|\operatorname{Im} z|$$

$$\cdot \text{VALORE ASSOLUTO}$$

$$z = \operatorname{Re} z + i \operatorname{Im} z$$

$$|z|$$

$$\text{MODULO DI } z$$

$$|z| \in \mathbb{R}.$$

PROPRIETÀ

$$z, z' \in \mathbb{C}$$

$$|z \cdot z'| = |z| \cdot |z'|$$

$$\left| \frac{z}{z'} \right| = \frac{|z|}{|z'|}$$

$$z' \neq 0$$

$$|z + z'| \leq |z| + |z'| \quad \text{DISUGUAGLIANZA TRIANGOLARE}$$

$$z \cdot \bar{z} = |z|^2 = (\operatorname{Re} z)^2 + (\operatorname{Im} z)^2$$

For $z \neq 0$ and $z \in \mathbb{C}$

$$\frac{1}{z} = \frac{\bar{z}}{z \cdot \bar{z}} = \frac{\bar{z}}{|z|^2}$$

$$z \neq 0$$

Def

$$z = x + iy$$

$$z' = x' + iy'$$

$$z = (x, y) \quad z' = (x', y')$$

$$z = z' \quad (\Rightarrow)$$

$$\operatorname{Re} z = \operatorname{Re} z'$$

$$\operatorname{Im} z = \operatorname{Im} z'$$

RISOLVERE LA SEGUENTE EQUAZIONE
in $\mathbb{C}$

$$z^2 + z + 1 = |z|^2$$



(TROVARE PER QUALI $z \in \mathbb{C}$ LA

~~(*)~~ $z \in \mathbb{C}$ VERIFICA)

$$|z| = \sqrt{\operatorname{Re} z^2 + \operatorname{Im} z^2}$$

$$z = x + iy$$

$$(x + iy)^2 + x + iy + 1 = x^2 + y^2$$

$$x^2 + 2ixy + (iy)^2 + x + iy + 1 - x^2 - y^2 = 0$$

$$= \cancel{x^2} + \underbrace{2ixy}_{\text{m}} - \underbrace{y^2}_{\text{m}} + \underbrace{x}_{\text{m}} + \underbrace{iy}_{\text{m}} + 1 - \cancel{x^2} - \underbrace{y^2}_{\text{m}} = 0$$

$$\underbrace{-2y^2 + x + 1}_{\text{blue}} + i(\underbrace{2xy + y}_{\text{orange}}) = 0 = \underbrace{0 + i \cdot 0}_{\text{orange}}$$

$$\begin{cases} -2y^2 + x + 1 = 0 \\ 2xy + y = 0 \end{cases}$$

$$\begin{cases} x = 2y^2 - 1 \\ \underline{2(2y^2 - 1)y + y = 0} \end{cases}$$

$$2(2y^2 - 1)y + y = 0$$

$$y(2(2y^2 - 1) + 1) = 0$$

$$\Leftrightarrow y = 0 \quad \vee \quad 4y^2 - 2 + 1 = 0$$

$$y = 0 \quad \vee \quad y^2 = \frac{1}{4}$$

$$y = 0 \quad \vee \quad y = \pm \frac{1}{2}$$

$$\begin{cases} x = y^2 - 1 \\ y = 0 \end{cases}$$

$$\begin{cases} x = -1 \\ y = \pm \frac{1}{2} \end{cases}$$

$$\boxed{z = -1}$$

$$\begin{cases} x = 2y^2 - 1 \\ y = -\frac{1}{2} \end{cases}$$

$$\begin{cases} x = \frac{1}{2} - 1 = -\frac{1}{2} \\ y = -\frac{1}{2} \end{cases}$$

$$\boxed{z = -\frac{1}{2} - \frac{1}{2}i}$$

$$\begin{cases} x = 2y^2 - 1 \\ y = \frac{1}{2} \end{cases}$$

$$\begin{cases} x = 2 \cdot \frac{1}{4} - 1 = -\frac{1}{2} \\ y = \frac{1}{2} \end{cases}$$

$$\boxed{z = -\frac{1}{2} + \frac{1}{2}i}$$

DETERMINE, NUMBER COMPLEX,

$$z \in \mathbb{C} \quad \text{TL}$$

$$(z + 3i)^2 - 7|z|^2 - 3\operatorname{Im} z = 0$$

$$z = x + iy$$

$$(x + iy + 3i)^2 - 7(x^2 + y^2) - 3y = 0$$

$$(x + i(y+3))^2 - 7x^2 - 7y^2 - 3y = 0$$

$$\begin{aligned} x^2 - (y+3)^2 + 2ix(y+3) - 7x^2 - 7y^2 - 3y \\ = 0 \end{aligned}$$

$$\underbrace{x^2}_{\lambda^2} - \underbrace{y^2}_{\lambda^2} - \underbrace{9}_{\lambda^2} - \underbrace{6y}_{\lambda^2} + 2ixy + 6ix - \underbrace{7x^2}_{\lambda^2} - \underbrace{7y^2}_{\lambda^2} - \underbrace{3y}_{\lambda^2} = 0$$

$$\underbrace{-6x^2 - 8y^2 - 9 - 9y + 2ixy + 6ix}_{\lambda^2} = 0$$

$$\lambda^2 (2xy + 6x) = 0$$

$$\left\{ \begin{array}{l} -6x^2 - 8y^2 - 9 - 9y = 0 \\ 2xy + 6x = 0 \end{array} \right.$$

$$\begin{cases} -6x^2 - 8y^2 - 3 - 3y = 0 \\ 2x(y+3) = 0 \end{cases}$$

$$\begin{cases} // \\ 2x = 0 \end{cases}$$

v

$$\begin{cases} // \\ y + 3 = 0 \end{cases}$$

$$\begin{cases} -8y^2 - 9y - 3 = 0 \\ x = 0 \end{cases}$$

~~$$\begin{cases} 8y^2 + 9y + 3 = 0 \\ \Delta < 0 \end{cases}$$~~

IMPOSSIBLE

y1
y2

$$\begin{cases} -6x^2 - 7x - 3 + 17 = 0 \\ y = -3 \end{cases}$$

~~$$\begin{cases} -6x^2 - 54x = 0 \\ \text{no sol.} \\ y = -3 \end{cases}$$~~

$$z^2 = \operatorname{Re} z + i(\operatorname{Im} z)^2 - |z|$$

$$z = x + iy$$

$$(x + iy)^2 = x + iy^2 - \sqrt{x^2 + y^2}$$

$$x^2 - y^2 + 2ixy = x - y^2 + \sqrt{x^2 + y^2} = 0$$

$$x^2 - y^2 - x + \sqrt{x^2 + y^2} + i(2xy - y^2) = 0$$

$$\begin{cases} x^2 - y^2 - x + \sqrt{x^2 + y^2} = 0 \\ 2xy - y^2 = 0 \end{cases}$$

$$\begin{cases} x^2 - y^2 - x + \sqrt{x^2 + y^2} = 0 \end{cases}$$

$$\begin{cases} y(2x - y) = 0 \end{cases}$$

$$\begin{array}{l} \text{①} \begin{cases} \underbrace{x^2 - x + \sqrt{x^2}}_v = 0 \\ y = 0 \end{cases} \quad \begin{cases} // \\ 2x - y = 0 \end{cases} \quad \text{②} \end{array}$$

$$x^2 - x + \sqrt{x^2} = 0$$

$$\boxed{\sqrt{x^2} = |x|}$$

$$x^2 - x + |x| = 0$$

$$\begin{cases} x \geq 0 \\ x^2 - \cancel{x} + \cancel{x} = 0 \end{cases}$$

$$\begin{cases} \cancel{x} \geq 0 \\ x^2 = 0 \end{cases}$$

$$\boxed{x = 0}$$

$$\vee \begin{cases} x < 0 \\ x^2 - \cancel{x} - \cancel{x} = 0 \end{cases}$$

↓

$$\begin{cases} \cancel{x} < 0 & \text{NON ACCETT.} \\ x^2 - 2x = 0 \\ x(x-2) = 0 \\ x = 0 \vee x = 2 \end{cases}$$

$$\begin{cases} x = 0 \\ y = 0 \end{cases}$$

$$z = 0$$

$$\textcircled{2} \begin{cases} x^2 - y^2 - x + \sqrt{x^2 + y^2} = 0 \\ 2x - y = 0 \end{cases}$$

$$\begin{cases} x^2 - 4x^2 - x + \sqrt{5x^2} = 0 \\ y = 2x \end{cases}$$

$$-3x^2 - x + \sqrt{5} |x| = 0$$

$$\left\{ \begin{array}{l} x \geq 0 \\ -3x^2 - x + \sqrt{5} x = 0 \end{array} \right.$$

↓

$$\left\{ \begin{array}{l} x \geq 0 \\ x(-3x - 1 + \sqrt{5}) = 0 \end{array} \right.$$

$$x = 0 \vee x = \frac{\sqrt{5} - 1}{3} > 0$$

$$\vee \left\{ \begin{array}{l} x < 0 \\ -3x^2 - x - \sqrt{5} x = 0 \end{array} \right.$$

↑

ACCETTABILI

$$\begin{cases} x=0 \\ y=2x=0 \end{cases}$$

$$z=0$$

$$\begin{cases} x = \frac{\sqrt{5}-1}{3} \\ y = 2 \left(\frac{\sqrt{5}-1}{3} \right) \end{cases}$$

$$t = \frac{\sqrt{5}-1}{3} + 2 \frac{\sqrt{5}-1}{3} i$$

$$x < 0$$

$$-3x^2 - x - \sqrt{5}x = 0$$

$$-x(3x + 1 + \sqrt{5}) = 0 \rightarrow \text{ACQUAIBILE}$$

~~$x < 0 \vee$~~
$$x = \frac{-1 - \sqrt{5}}{3}$$

$$\begin{cases} x = -\frac{1-\sqrt{5}}{3} \\ y = 2x = 2\left(-\frac{1-\sqrt{5}}{3}\right) \end{cases}$$

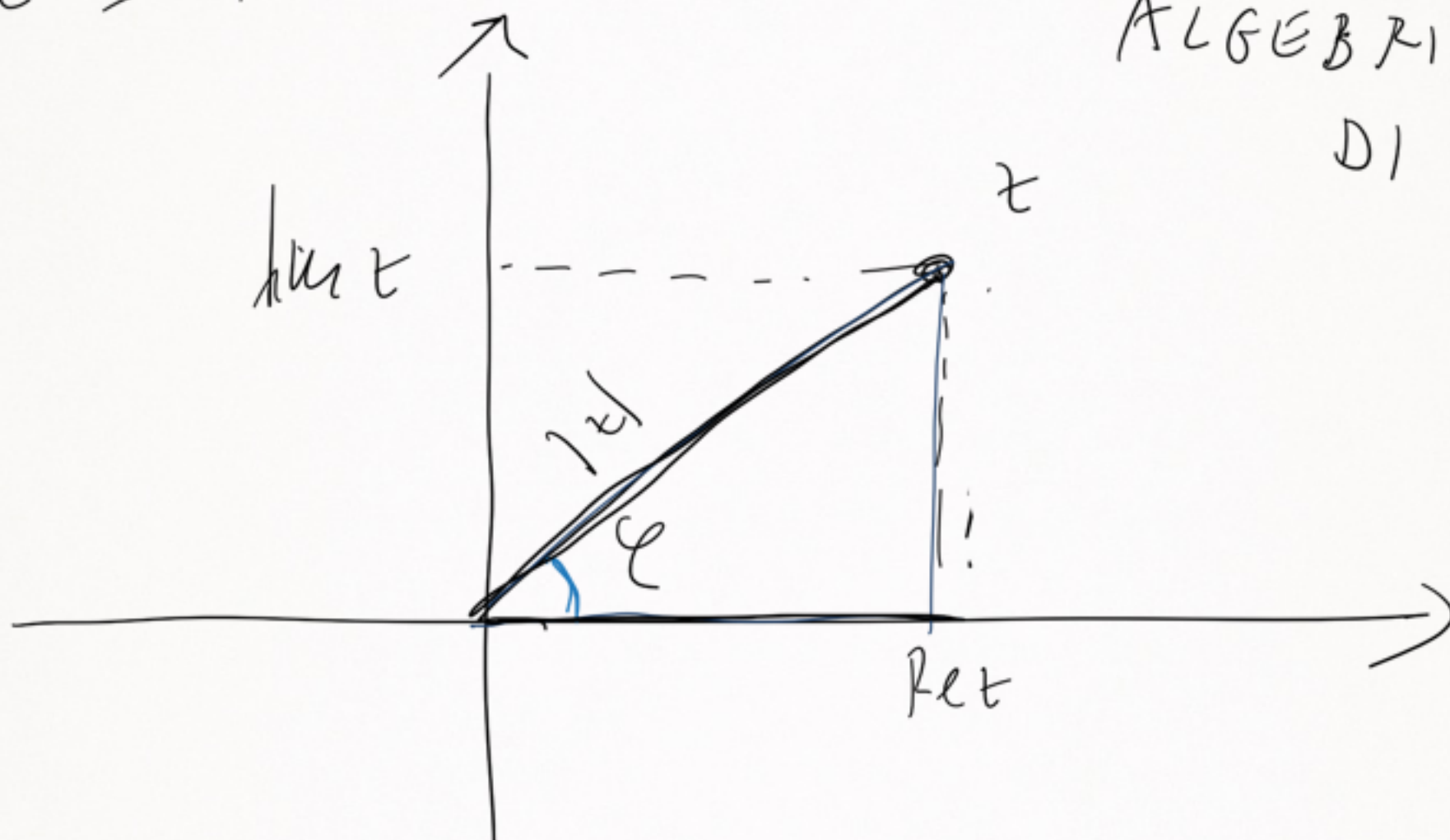
$$z = \frac{-1-\sqrt{5}}{3} + 2\left(-\frac{1-\sqrt{5}}{3}\right)i$$

$$z \in \mathbb{C}$$

$$z = \operatorname{Re} z + i \operatorname{Im} z$$

(REPRESENTATIONS
ALGEBRA

DI z



$$\operatorname{Re} z = |z| \cos \varphi \Rightarrow$$

$$\cos \varphi = \frac{\operatorname{Re} z}{|z|}$$

$$\operatorname{Im} z = |z| \sin \varphi \Rightarrow$$

$$\sin \varphi = \frac{\operatorname{Im} z}{|z|}$$

$$z = |z| \cos \varphi + i |z| \sin \varphi =$$

$$= |z| (\cos \varphi + i \sin \varphi)$$

$\underbrace{\hspace{10em}}$
 RAPPRESENTAZIONE TRIGONOMETRICA
 DI z

φ ARGOMENTO DI z

$\varphi + 2k\pi$ ARGOMENTO DI z

$$z = 1 + i$$

REPRESENTATION
TRIGONOMETRIC

$$\operatorname{Re} z = 1$$

$$\operatorname{Im} z = 1$$

$$|z| = \sqrt{(\operatorname{Re} z)^2 + (\operatorname{Im} z)^2} = \sqrt{1 + 1} = \sqrt{2}$$

$$\cos \varphi = \frac{\operatorname{Re} z}{|z|} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\varphi = \frac{\pi}{4}$$

$$\sin \varphi = \frac{\operatorname{Im} z}{|z|} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$(45^\circ)$$

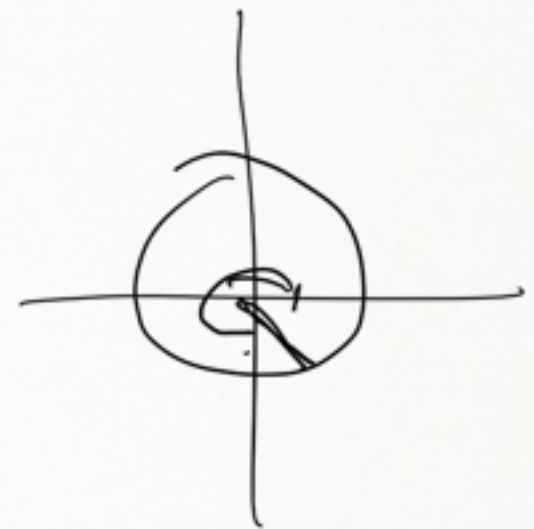
$$z = \sqrt{2} \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)$$

$$z = 1 - \sqrt{3} i$$

REPRESENTS
TRIANGLE

$$|z| = \sqrt{1+3} = \sqrt{4} = 2$$

$$\cos \theta = \frac{\operatorname{Re} z}{|z|} = \frac{1}{2}$$



$$\sin \theta = \frac{\operatorname{Im} z}{|z|} = \frac{-\sqrt{3}}{2} \quad \theta = \frac{5}{3} \pi$$

$$z = 2 \left(\cos \frac{5}{3} \pi + i \sin \frac{5}{3} \pi \right)$$

$$z = 1 + i$$

$$z^{100} = (1 + i)^{100}$$

MULTIPLY COMPLEX NUMBERS,
 WRITE IN FORM TRIGONOMETRIC

$$z = |z| (\cos \theta + i \sin \theta)$$

$$z' = |z'| (\cos \theta' + i \sin \theta')$$

$$\begin{aligned} z \cdot z' &= |z| (\cos \theta + i \sin \theta) |z'| (\cos \theta' + i \sin \theta') \\ &= |z| \cdot |z'| (\cos \theta \cos \theta' - \sin \theta \sin \theta' + i (\sin \theta \cos \theta' + \cos \theta \sin \theta')) \end{aligned}$$

$$\begin{aligned}
 &= |z| \cdot |z'| \left(\underbrace{\cos \theta \cos \theta' - \sin \theta \sin \theta'}_{\cos(\theta + \theta')} + i \underbrace{(\sin \theta \sin \theta' + \cos \theta \cos \theta')}_{\sin(\theta + \theta')} \right) \\
 &= |z| \cdot |z'| \left(\cos(\theta + \theta') + i \sin(\theta + \theta') \right)
 \end{aligned}$$

$z \cdot z'$ is also complex.

to $|z \cdot z'| = |z| \cdot |z'|$

the arguments are same
 arg of $z \cdot z'$

$$z = 1 + i = \sqrt{2} \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)$$

$$z' = 1 - 3i = 2 \left(\cos \frac{5}{3}\pi + i \sin \frac{5}{3}\pi \right)$$

$$zz' = 2\sqrt{2} \left(\cos \left(\frac{\pi}{4} + \frac{5}{3}\pi \right) + i \sin \left(\frac{\pi}{4} + \frac{5}{3}\pi \right) \right)$$

$$z^2 = z \cdot z = |z|^2 \left(\cos 2\ell + i \sin 2\ell \right)$$

$n > 1$

$$z^n = |z|^n \left(\cos n\ell + i \sin n\ell \right)$$

FORMULA DI DE MOIVRE

$$(1+i)^{100} = (\sqrt{2})^{100} \left(\cos \frac{100}{4} \pi + i \sin \frac{100}{4} \pi \right)$$

$$\left. \begin{array}{l} |z| = \sqrt{2} \\ \varphi = \frac{\pi}{4} \end{array} \right) = 2^{50} \left(\cos(25\pi) + i \sin(25\pi) \right)$$

$$= 2^{50} \left(\cos(24\pi + \pi) + i \sin(24\pi + \pi) \right)$$

$$= 2^{50} \left(\cos(\underbrace{12 \cdot 2 \pi}_{24\pi} + \pi) + i \sin(12 \cdot 2\pi + \pi) \right)$$

$$= 2^{50} \left(\underset{-1}{\cos \pi} + i \underset{0}{\sin \pi} \right) = -2^{50}$$

$$z = |z| (\cos \varphi + i \sin \varphi)$$

$$e^{i\varphi} = \cos \varphi + i \sin \varphi$$

$$\underline{z = |z| e^{i\varphi}}$$

RAPPRESENTAZIONE
ESPOENENTIALE di z

$$z = |z| e^{i\varphi}$$

$$z' = |z'| e^{i\varphi'}$$

$$i(\varphi + \varphi')$$

$$z \cdot z' = |z| \cdot |z'| e^{i(\varphi + \varphi')}$$

$$z^m = |z|^m e^{im\varphi}$$

$$z = |z| e^{i\varphi}$$

FORMULA DI DE MOIVRE
IN RAPPRESENTAZIONE
ESPO NENZIALE

(ES)

$$z = -1$$

FORMA

TRIGONOMETRICA

$$|z| = \sqrt{(\operatorname{Re} z)^2 + (\operatorname{Im} z)^2} = \sqrt{1 + 0} = 1$$

$$\cos \varphi = \frac{\operatorname{Re} z}{|z|} = \frac{-1}{1} = -1$$

$$\varphi = \pi$$

$$\sin \varphi = \frac{\operatorname{Im} z}{|z|} = \frac{0}{1} = 0$$

$$\cos \pi + i \sin \pi = -1$$

PROB. TRIG

$$e^{i\pi} = -1$$

$$e^{i\pi} + 1 = 0$$

$$(\text{PROB. ES } \neq)$$

$$(\text{FORMULA DI EULERO})$$