UNIVERSITÀ DEGLI STUDI DI BARI ALDO MORO
CONSIGLIO INTERCLASSE IN MATEMATICA

COURSE OF STUDY	**TWO-YEAR MASTER OF SCIENCE PROGRAMME IN MATHEMATICS**
ACADEMIC YEAR	**2023-2024**
ACADEMIC SUBJECT	**ADVANCED GEOMETRY 2**

General information	
Programme year	Second
Term	Second semester (February 26, 2024 – May 31, 2024)
European Credit Transfer and Accumulation System credits (ECTS)	7
SSD	MAT/03 – Geometry
Language	Italian
Mode of attendance	Not mandatory

Lecturers	
Name and surname	Antonio Lotta
E-mail	antonio.lotta@uniba.it
Telephone	+39 080 544 2682
Department and office	Department of Mathematics, room 2 third floor
Virtual meeting room	Microsoft Teams
Web page	https://www.dm.uniba.it/en/members/lotta
Office hours	By appointment via e-mail

Work schedule				
	Total	Lectures	Hands-on learning	Self-study
Hours	175	56		119
ECTS credits	7	7		

Learning objectives	
	Acquiring knowledge of some advanced topics of modern Riemannian Geometry, especially concerning homogeneous and symmetric spaces and some results concerning the relationship between curvature and topology, providing the necessary background for further study of the subject at Phd level.

Course prerequisites	
	Basic knowledge of smooth manifolds and Lie groups. Elementary notions about Riemannian metrics: Levi-Civita connection, geodesics, curvature.

Syllabus	
Course contents	**Homogeneous spaces.** Smooth actions of Lie groups on manifolds. Fundamental vector fields. Quotient of a manifold by a regular equivalence relation: theorem of Godement. The exponential map of a Lie group. The closed subgroup theorem. Quotient of a Lie group by a closed subgroup. Examples. **Riemannian homogeneous spaces.** Homogeneous Riemannian manifolds. Examples. Isotropy representation. Criterion for the existence of invariant

	metrics on a homogeneous space. Existence of bi-invariant metrics on compact Lie groups. Homogeneous spaces with irreducible isotropy representation. The Levi-Civita connection of an invariant metric in the reductive case. Killing fields and their characterization. Nomizu's formula for the curvature tensor of a reductive homogeneous Riemannian manifold. Naturally reductive homogeneous spaces. Normal metrics, Samelson's theorem. Invariant metrics on Lie groups. **Isometries of compact Riemannian manifolds.** Outline of the Myers-Steenrod theorem about the ̀isometry group of a Riemannian manifold. Maximum dimension of the isometry group. Divergence theorem. Killing vector fields on compact manifolds: Bochner's theorem. Every Riemannian homogeneous space with negative semi-definite definite Ricci tensor is a flat torus. Killing vector fields on compact manifolds with positive curvature: Berger's theorem. **Exponential map and Jacobi fields.** Exponential map of a Riemannian manifold and its naturality. Normal neighborhoods and geodesic balls. Jacobi vector fields. Every Killing vector field restricts to a Jacobi field along a geodesic. Maximal dimension of the Lie algebra of Killing vector fields. Geodetic completeness of Riemannian homogeneous spaces. **Riemannian symmetric spaces.** Geodesic reflections. Riemannian symmetric spaces. Examples. Canonical representation of a symmetric space Riemannian as a homogeneous reductive space; Cartan decomposition. Curvature and Ricci tensor of a symmetric space. Compact and non-compact type spaces and sign of the sectional curvatures. Every Riemannian homogeneous space with nonpositive sectional curvature and negative definite Ricci tensor is simply connected (theorem of Kobayashi).
Reference books	-F. Warner: Foundations of differentiable manifolds and Lie groups. Graduate Texts in Mathematics, 94. Springer-Verlag, Berlin 1983. -J.M. Lee: Riemannian manifolds. Graduate Texts in Mathematics 176, Springer-Verlag, New York, 1997. -B. O'Neill: Semi-Riemannian geometry. Academic Press, San Diego, 1983. -M. Postnikov: Geometry VI. Riemannian geometry. Encyclopaedia of Mathematical Sciences 91, Springer-Verlag, Berlin, 2001.
Additional course materials	
Repository	

Expected learning outcomes	
Knowledge and understanding	Acquiring some fundamental concepts and proof techniques in modern differential geometry.
Applying knowledge and understanding	The acquired theoretical knowledge provides the necessary appropriate language to study many topics of theoretical physics.
Soft skills	*Making judgements*: Ability to comprehend and rework the proofs of meaningful mathematical results. Ability to test some general facts on specific examples.
	Communication skills: Students should acquire the mathematical language and formalism necessary to read and comprehend advanced textbooks and specialized literature on the subject and to explain the acquired knowledge.

	Learning skills: Acquiring suitable learning methods, supported by text consultation and by elaborating on questions periodically suggested during the course.

Teaching methods	
	Classroom lectures

Assessment	
Assessment methods	Oral exam
Evaluation criteria	• *Knowledge and understanding*: Acquiring some fundamental concepts and proof techniques in modern differential geometry. • *Applying knowledge and understanding*: Acquiring a modern scientific language used in a great part of pure mathematics and of theoretical physics. • *Making judgement*: Ability to comprehend and rework the proofs of meaningful mathematical results. Ability to test some general facts on specific examples. • *Communication skills*: Students should acquire the mathematical language and formalism necessary to read and comprehend advanced textbooks and specialized literature on the subject and to explain the acquired knowledge. • *Learning skills*: Acquiring suitable learning methods, supported by text consultation and by elaborating on questions periodically suggested during the course.
Grading policy	The final grade, on a 18-30 scale, is assigned based on the quality of the presentation by the candidate of some key concepts and results, also taking into account the ability to establish connections between the various parts of the course and the knowledge of significant examples.

Further information	