UNIVERSITÀ DEGLI STUDI DI BARI ALDO MORO

CONSIGLIO INTERCLASSE IN MATEMATICA

COURSE OF STUDY TWO-YEAR MASTER OF SCIENCE PROGRAMME IN MATHEMATICS

ACADEMIC YEAR 2023-2024

ACADEMIC SUBJECT NUMERICAL METHODS AND MODELLING

General information	
Programme year	First
Term	First semester (September 25, 2023 – December 22, 2023)
European Credit Transfer and Accumulation System credits (ECTS)	7
SSD	MAT/08 – Numerical Analysis
Language	Italian
Mode of attendance	Not mandatory

Lecturers		
Name and surname	Luciano Lopez (instructor of record)	Cinzia Elia
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Department and office	Department of Mathematics room 15 second floor	Department of Mathematics room 7 third floor
Virtual meeting room	Microsoft Teams - code: zl4c0un	Microsoft Teams - code: zl4c0un
Web page	https://www.dm.uniba.it/it/members/lopez	https://www.dm.uniba.it/it/members/elia
Office hours	Monday and Wednesday 11:00-13:00 and by appointment via email	Thursday 14:00-16:00 and by appointment via email

Work schedule				
	Total	Lectures	Hands-on learning (recitations)	Self-study
Hours	175	40	30	105
ECTS credits	7	5	2	

Learning objectives	
	Acquiring methods and techniques of applied mathematics for the qualitative analysis and simulation of continuous and discrete models of dynamical systems. Mathematical modeling skills of simple phenomena

Course prerequisites	
	Basic knowledge of Mathematical Analisys , systems of ODEs, Linear Algebra and Numerical Analysis.Bases of a programming language. All requirements are achieved in the Bachelor class L-35.

Syllabus	
Course contents	**1. DISCRETE DYNAMICAL SYSTEMS** First order difference equations and their solutions. Theory of linear difference equations od order k. Homogeneous difference equations. Computation of solutions. Characteristic polynomial: single roots and multiple roots. Operator p(E) and its properties. Computation of particular

	solutions. Equilibria of difference equations and their stability. Formal series. Variation of constants formula. Linear systems of difference equations. Stability of solutions of difference equations. Matrix functions and their properties. Asymptotic study of A^n and exp(tA). Discrete models: Cobweb simple and complex; Lesley model of population dynamic; Indian Natchez. **2. CONTINUOUS DYNAMICAL SYSTEMS** Autonomous linear systems of ODES: Principal matrix solution and its properties. Matrix exponential. Continuity of solutions with respect to the initial condition and concept of stability. Invariant subspaces: stable, unstable and central space. Examples: planar linear systems; node, saddle point, focus and center. Autonomous nonlinear systems. Properties of the flow. Equilibria and concepts of asymptotic stability, stability and instability. Equilibria and linearization. Lyapunov functions. Periodic orbits and limit cycles. Long time behavior of solutions. Continuous models: harmonic oscillator with damping and forcing. Duffing oscillator with damping and forcing. Nonlinear pendulum. Van der Pol. Malthus and Verhulst model. Lotka-Volterra, competing species, Darwin. **Dynamical systems on networks:** Elements of networks. Adjacency and Laplacian matrices; properties. Epidemic Diffusion model on networks: SI andSIR models and stability conditions. General form of a dynamical system on networks. Symmetric equilibrium points. Different cases of coupling and conditions for the asymptotic stability. Gossip model. Examples of synchronous solutions: coupled oscillators. **Neural network and dynamical systemss:** the biological and artificial neuron. Activation functions. The discrete and continuous Perceptron. Various types of neural networks. The training set. Supervised and unsupervised training. Construction of the error function. The backpropagation algorithm. Optional argument: The recurring and dynamic networks. Hopfield's discrete and continuous networks. The energy function. **3. RUNGE KUTTA METHODS** Forward Euler (FWE): local and global error. Linear stability. FWE applied to the harmonic oscillator. Runge Kutta method of order 2. Local and global error. Linear stability and behavior of solutions of the harmonic oscillator. Spurious fixed points. Runge Kutta methods, general theory: consistency, convergence, local and global error, upper bounds. Verification of order of convergence in function of the discretization stepsize. Linear stability: stability function and region of absolute stability. **4. PROGRAMMING** Creation of Matlab functions for explicit Runge Kutta methods. Geometrical and quantitive verification of order of convergence. Creation of Matlab function to trace the absolute stability regione ofthe method. Simulation of models of ODEs with particular emphasis on the qualitative behaviour of the numerical solutions.
Reference books	V. Lakshmikantham, D. Trigiante, Theory of difference equations: numerical methods and applications, Academic Press Inc, 1988. D.G. Luemberger, Introduction to dynamic systems, J. Weley and Sons, 1979.

	M. Braun, Differential Equations and Their Applications: An Introduction to Applied Mathematics: An Introduction to Applied Mathematics. Springer, 1983. L. Perko, Differential Equations and Dynamicla Systems, Springer, 1991. J.D. Lambert, Numerical Methods for Ordinary Differential Systems: The Initial Value Problem, Wiley and Sons
Notes on reference books	The student may find in these books the models presented during the lessons
Additional course materials	Class Webpage https://sites.google.com/site/eliametnum/

Expected learning outcomes	
DD1 Knowledge and understanding	Qualitative behaviour of continuous and discrete dynamical systems.
DD2 Applying knowledge and understanding	Numerical simulation of continuous and discrete dynamical systems Qualitative behaviour of numerical solutions;
DD3-5 Soft skills	*DD3 Making judgements*: predict the behaviour of a model described during the lectures and labs via qualitative and numerical analysis.
	DD4 Communication skills: use formal mathematical language to describe the qualitative and numerical analyses
	DD5 Learning skills: predict the behaviour of a real world model via qualitative and numerical analysis.

Teaching methods	

Assessment	
Assessment methods	The student will have to present a theoretical and numerical qualitative study of a model. The model could either be one already introduced or suggested in class, or a new model. About a week after the presentation, the student will have to take on oral exam.
Evaluation criteria	• *Knowledge and understanding*: The student should be able to perform the qualitative analysis of a dynamical system as the ones presented in class. • *Applying knowledge and understanding*: The student should be able to study numerically a model as the ones presented in class. • *Making judgement*: The student should be able to predict the long term behaviour of a given model via qualitative and numerical analysis. • *Communication skills*: quality in presentation and ability to communicate the notions acquired. • *Learning skills*: autonomous and group organization skills in the study.
Grading policy	The final grade is given out of 30 with a minimum passing grade of 18. The practical exam is a week before the scheduled oral exam and it consists in the theoretical and numerical study of a given model with particular attention to the qualitative behavior of solutions. Upon passing the practical exam, the student will be admitted to the oral exam.

Further information	
	attendance of lessons and exercises in the classroom and in the laboratory is optional but strongly recommended