

COURSE OF STUDY **TWO-YEAR MASTER OF SCIENCE PROGRAMME
IN MATHEMATICS**

ACADEMIC YEAR **2023-2024**

ACADEMIC SUBJECT **MATHEMATICAL METHODS OF PHYSICS**

General information	
Programme year	First
Term	First semester (September 25, 2023 – December 22, 2023)
European Credit Transfer and Accumulation System credits (ECTS)	7
SSD	MAT/07 – Mathematical Physics
Language	Italian
Mode of attendance	Not mandatory

Lecturer	
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Department and office	Department of Mathematics, room 13 second floor
Virtual meeting room	
Web page	https://www.dm.uniba.it/it/members/ligabo
Office hours	by appointment via email

Work schedule				
	Total	Lectures	Hands-on learning (recitations)	Self-study
Hours	175	40	30	105
ECTS credits	7	5	2	

Learning objectives

	Acquiring the basic techniques for the study of the classical equations of mathematical physics, with a particular focus on the mathematical formulation of the corresponding physical model.
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Course prerequisites	
	Mathematical and physical knowledge acquired during the first two years of a degree of L-35 class. Especially: Mathematical Analysis of functions one and several variables, Linear Algebra, Electromagnetism and Hamiltonian Mechanics.

Syllabus	
Course contents	<p>Transport equation with constant coefficients. General solution and characteristic lines. Initial value problem. Weak solutions. Nonhomogeneous problem. Method of characteristics.</p> <p>Laplace's equation: Outline of electrostatic and physical interpretation. Harmonic functions. Fundamental solution in \mathbb{R}^n. Representation formula for Poisson's equation. Mean-value theorem. Maximum principle. Uniqueness. Smoothness. Estimates on derivatives. Liouville's theorem. Analyticity. Harnack's inequality. Green's functions and representation formula for Poisson's equation with boundary conditions. Symmetry of Green's function and reciprocity principle. Green's function for a half-space and a ball. Poisson's formulae. Energy methods, uniqueness, and Dirichlet's minimum principle.</p> <p>Heat equation: Thermal conduction equation. Physical interpretation. Fundamental solution in \mathbb{R}^n. Cauchy problem and representation formula. Nonhomogeneous problem and Duhamel's principle. Parabolic cylinder and heat ball. Mean-value theorem. Maximum principle. Uniqueness in bounded domains. Maximum principle in \mathbb{R}^n and uniqueness of the Cauchy problem. Regularity. Estimates on derivatives. Energy methods, Forward and backward uniqueness.</p> <p>Wave equation: Heuristic derivation and physical interpretation. Solution in 1D. D'Alembert's formula. Wave equation on the half-line. Reflection method. Spherical means and Euler-Poisson-Darboux equation. Cauchy problem in 3D. Kirchhoff's formula. Wave equation in 2D. Method of descent and Poisson's formula. Representation formulae in arbitrary even and odd dimensions. Regularity. Domain of dependence and cone of influence. Huygens's principle. Nonhomogeneous problem and retarded potentials. Energy methods. Uniqueness. Finite propagation speed.</p>
Reference books	<p>L. C. Evans, Partial Differential Equations, Graduate studies in Mathematics, vol 19, Amer. Math. Soc., Providence, 1998.</p> <p>A.N. Tikhonov and A.A. Samarskii, Equations of Mathematical Physics, Dover Publications, 1990.</p>

	F. John, Partial Differential Equations, Springer Verlag, 1982.
Additional course materials	
Repository	Microsoft Teams

Expected learning outcomes	
Knowledge and understanding	Acquisition of fundamental concepts and strategies for the study of a differential equation. Acquisition of the relative demonstration techniques.
Applying knowledge and understanding	The theoretical knowledge acquired is used in a large part of the differential equations of physics.
Soft skills	<p><i>DD3 Making judgments:</i></p> <ul style="list-style-type: none"> - Ability to evaluate the consistency of logical reasoning used in a proof. - Ability to identify the right mathematical tools and the right techniques to tackle complex problems.
	<p><i>DD4 Communication skills:</i></p> <ul style="list-style-type: none"> - Acquisition of advanced physical/mathematical language and formalism, necessary for consulting and understanding texts. - Exposure of the knowledge acquired through the description, analysis and resolution of problems.
	<p><i>DD5 Learning skills:</i></p> <p>Acquisition of an adequate study method, supported by the consultation of texts and by the resolution of exercises and questions proposed periodically during the course.</p>

Teaching methods	
	Frontal lessons

Assessment	
Assessment methods	Oral exam
Evaluation criteria	<p><i>Knowledge and understanding:</i> acquisition and mastery of the definitions and theoretical results covered by the course.</p> <p><i>Applied knowledge and understanding:</i> ability to apply the acquired theoretical knowledge to the study of partial differential equations;</p> <p><i>Making judgments:</i> critical approach to concepts, ability to choose solution methods and ability to provide examples and counterexamples.</p> <p><i>Communication skills:</i> mastery of language and quality of presentation.</p> <p>Ability to learn: ability to organize knowledge, critical reasoning and possible independent study.</p>

Grading policy

The final mark is expressed out of thirty. The exam is passed if the final mark is greater than 17.

Further information

Attendance at lectures and tutorials is strongly recommended.