UNIVERSITÀ DEGLI STUDI DI BARI ALDO MORO

CONSIGLIO INTERCLASSE IN MATEMATICA

COURSE OF STUDY — TWO-YEAR MASTER OF SCIENCE PROGRAMME IN MATHEMATICS

ACADEMIC YEAR — 2023-2024

ACADEMIC SUBJECT — ELEMENTS OF ADVANCED MATHEMATICAL PHYSICS

General information	
Term	First semester (September 25, 2023 – December 22, 2023)
European Credit Transfer and Accumulation System credits (ECTS)	7
SSD	MAT/07 – Mathematical Physics
Language	Italian
Mode of attendance	Not mandatory

Lecturers		
Name and surname	Marilena Ligabò (instructor of record)	Arcangelo Labianca
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Telephone	+39 080 544 2695	+39 080 544 2656
Department and office	Department of Mathematics room 13 second floor	Department of Mathematics room 7 second floor
Virtual meeting room		
Web page	https://www.dm.uniba.it/it/members/ligabo	https://www.dm.uniba.it/it/members/labianca
Office hours	by appointment via email	by appointment via email

Work schedule				
	Total	Lectures	Hands-on learning	Self-study
Hours	175	56		119
ECTS credits	7	7		

Learning objectives	
	The course aims to introduce the advanced mathematical methodologies of modern physics.

Course prerequisites	
	Real and Complex Analysis, Linear Algebra

Syllabus	
Course contents	*Review*: metric spaces, Banach spaces, Hilbert spaces and measure theory. *Linear operators on Hilbert spaces:* $C*$-algebra of bounded operators. Normal, self-adjoint, unitary and projection operators. Some element of quantum mechanics. Unbounded operators. Adjoint. Symmetric and self-adjoint operators. Examples: multiplication and derivation operators. Essentially self-adjoint operators. Fundamental criteria of self-adjointness and essentially self-adjointness. Graph, closure and inverse of an operator. Self-adjoint extensions of positive operators. Example: kinetic energy in a segment. Self-adjointness of observables, Kato-Rellich theorem. *Spectrum and dynamics:* Resolvent operator, resolvent set and spectrum. Examples: position and momentum operators. First resolvent formula and analytic properties. Neumann series. Spectrum and Weyl sequences. Spectrum of self-adjoint, unitary and projection operators. Projection-valued measures and resolution of the identity. Integration on PVM of bounded functions. Expectation value of the resolvent. Spectral family of a self-adjoint operator and spectral theorem. Functional calculus. Spectral projections and spectral types. Quantum dynamics and unitary evolution groups. Energy conservation. Stone's theorem. Return and transition probability. Spectral types and return probability. Pure point spectrum and quasi periodic orbits. RAGE theorem.
Reference books	M. Reed, B. Simon, Methods of Modern Mathematical Physics, Vol. 1, Academic Press, New York, 1980 G. Teschl, Mathematical Methods in Quantum Mechanics, American Mathematical Society, Providence, 2009
Additional course materials	
Repository	Microsoft Teams

Expected learning outcomes	
Knowledge and understanding	Understanding of advanced mathematical tools used in fundamental and applied physics research. Knowledge of the modern mathematical structures of functional analysis with particular reference to the theory of operators in Hilbert spaces, necessary to address more advanced problems of Modern Physics
Applying knowledge and understanding	The theoretical knowledge acquired is used in a large part of the differential equations of physics.

Soft skills	*DD3 Making judgments:* - Ability to evaluate the consistency of logical reasoning used in a proof. - Ability to identify the right mathematical tools and the right techniques to tackle complex problems.
	DD4 Communication skills: - Acquisition of advanced physical/mathematical language and formalism, necessary for consulting and understanding texts. - Exposure of the knowledge acquired through the description, analysis and resolution of problems.
	DD5 Learning skills: Acquisition of an adequate study method, supported by the consultation of texts and by the resolution of exercises and questions proposed periodically during the course.

Teaching methods	
	Frontal lessons

Assessment	
Assessment methods	Oral exam
Evaluation criteria	*Knowledge and understanding:* acquisition and mastery of the definitions and theoretical results covered by the course. *Applied knowledge and understanding:* ability to apply the acquired theoretical knowledge to the study of partial differential equations; *Making judgments:* critical approach to concepts, ability to choose solution methods and ability to provide examples and counterexamples. *Communication skills:* mastery of language and quality of presentation. Ability to learn: ability to organize knowledge, critical reasoning and possible independent study.
Grading policy	The final mark is expressed out of thirty. The exam is passed if the final mark is greater than 17.

Further information	
	Attendance at lectures and tutorials is strongly recommended.