COURSE OF STUDY THREE-YEAR BACHELOR PROGRAMME IN MATHEMATICS

ACADEMIC YEAR 2023-2024

ACADEMIC SUBJECT ELEMENTS OF ADVANCED ANALYSIS 1

General information	
Programme year	Third
Term	First semester (September 25, 2023 – December 22, 2023)
European Credit Transfer and Accumulation System credits (ECTS)	7
SSD	MAT/05 – Mathematical Analysis
Language	Italian
Mode of attendance	Not mandatory

Lecturers		
Name and surname	Monica Lazzo (instructor of record)	Anna Valeria Germinario
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Telephone	+39 080 544 2503	
Department and office	Department of Mathematics room 6 fourth floor	Department of Mathematics room 17 second floor
Virtual meeting room	Microsoft Teams, code hr9trx4	Microsoft Teams, code hr9trx4
Web page	https://www.dm.uniba.it/it/members/lazzo	https://www.dm.uniba.it/it/members/germinario
Office hours	By appointment, to be scheduled by e-mail	By appointment, to be scheduled by e-mail

Work schedule				
	Total	Lectures	Hands-on learning (recitations)	Self-study
Hours	175	40	30	105
ECTS credits	7	5	2	

Learning objectives	
	Acquiring language and techniques of modern analysis, specifically: measure theory, L^p spaces and Hilbert spaces elementary theory, basic complex variable analysis.

Course prerequisites	
	Mathematical knowledge usually acquired during the first two years of a bachelor programme in Mathematics, specifically classical analysis in one and several variables, general topology, linear algebra.

Syllabus	
Course contents	**Real Analysis** **1. Measure and abstract integration theory** σ–algebras and measurable sets; measurable functions; measures and their elementary properties; integration of positive functions and complex–valued functions; sequences of integrals: monotone convergence theorem, Fatou lemma, dominated convergence theorem; series of integrals; completion of a

	measure; Severini-Egoroff theorem; Vitali convergence theorem. **2. Positive Borel measures** Lebesgue measure in $\mathbf{R}^N$; criteria for Lebesgue measurability; existence of non measurable sets; regular measures; translation–invariant positive Borel measures; Lebesgue measure and linear transformations. **3. L^p spaces** Jensen, Hölder, and Minkowsky inequalities; norms in L^p spaces; completeness of L^p spaces; continuity properties of measurable functions in $\mathbf{R}^N$; density of di $C_c(\mathbf{R}^N)$ in $L^p(\mathbf{R}^N)$; density of $C_c(\mathbf{R}^N)$ in $C_0(\mathbf{R}^N)$; separability of $L^p(\mathbf{R}^N)$. **4. Elementary theory in Hilbert spaces** Schwarz inequality, triangle inequality; existence of the smallest norm element for closed convex sets; orthogonal projections; Riesz representation theorem in Hilbert spaces; the best approximation problem; maximal orthonormal sets; Bessel and Parseval identities; $L^2(T)$ space and Fourier series; $H^s(T)$ space, embedding into C(T). **Complex Analysis** **5. Introduction to holomorphic function theory** Complex valued functions, limits and continuity; holomorphic functions, Cauchy–Riemann equations; constant holomorphic functions, Liouville theorem; primitives of complex functions; geometric meaning of complex differentiability, conformal maps; multivalued functions and selections; exponential and logarithm functions, trigonometric and hyperbolic functions; curves and their length; path integral, path integral and differential forms; characterization of the existence of primitives of complex functions; complex power series, convergence radius, holomorphy; analytic functions. **6. Cauchy Theorem and analiticity of holomorphic functions** Analiticity of the Cauchy integral; winding number theorem, Goursat theorem, existence of local primitives, Cauchy formula, analyticity of holomorphic functions, Morera theorem, Cauchy formula and Cauchy estimates for derivatives, fundamental theorem of algebra, Liouville theorem for bounded holomorphic functions, Morera–Weierstrass theorem; Cauchy theorem (general case) and its applications. **7. Zeros of holomorphic functions and properties of harmonic functions** Theorem about the zeros of holomorphic functions and corollaries; holomorphic and harmonic functions; mean value property; maximum principle for harmonic functions; maximum modulus principle, minimum modulus principle. **8. Residue Theorem and applications** Isolated singularities; Laurent series; classification of isolated singularities and characterizations; residues theorem and its applications; logarithmic index theorem; Rouché theorem; open mapping theorem; inverse function theorem. A more detailed description of the course contents will be posted at the end of the semester on the course homepage (https://elearning-mat.hosting.uniba.it).
Reference books	W. Rudin, Real and Complex Analysis, McGraw–Hill Book Company S. Lang, Complex Analysis, Springer–Verlag G. Gilardi, Analisi 3, Mc Graw–Hill G.B. Folland, Real Analysis, Wiley-Interscience For details on the construction of Lebesgue mesure:

	N. Fusco, P. Marcellini, C. Sbordone, Analisi Matematica due, Zanichelli (Liguori for older editions)
Additional course materials	
Repository	Slides, lecture notes, etc are posted on the course homepage (https://elearning-mat.hosting.uniba.it)

Expected learning outcomes	
Knowledge and understanding	Knowledge of principles of modern real and complex analysis and of theorem proving techniques.
Applying knowledge and understanding	Ability to solve problems by utilizing theoretical knowledge and selecting adequate strategies.
Soft skills	Making judgements: ability to assess the soundness of the logical reasoning used in a proof; ability to select the appropriate mathematical tools and techniques to deal with complex mathematical problems.
	Communication skills: mastery of the mathematical language and syntax necessary to communicate the acquired knowledge and to describe, analyze and solve problems.
	Learning skills: ability to study independently and to consult and make us of relevant literature.

Teaching methods	
	Lectures and recitations are held in a classroom, using slides partly prepared in advance, partly generated in class. After each session these slides are made available on the course homepage (https://elearning-mat.hosting.uniba.it).

Assessment	
Assessment methods	Oral exam, consisting of the discussion of definitions, theoretical results (with proof), examples, counterexamples and short problems.
Evaluation criteria	*Knowledge and understanding*: the student must be able to explain definitions and to state and prove theoretical results, providing also examples and counterexamples. *Applying knowledge and understanding*: the student must be able to independently solve simple practical or theoretical problems. *Making judgements*: the student must be able to select the theoretical and practical tools most appropriate for a given problem. *Communication skills*: the student must be able to explain theoretical results and solutions to given problems clearly and completely, using precise mathematical language and syntax. *Learning skills*: the student must know the specific terminology of the course material and must be able to identify the context of each concept.
Grading policy	The final grade is based on 30 points; the minimum passing grade is 18.

Further information	