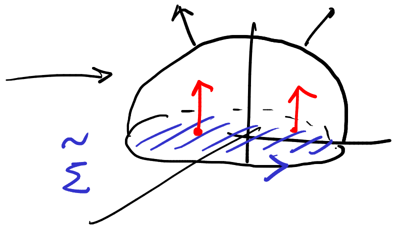
3 giugno 2025

Osservazioni su campi vettoriali di "tipo rotore"

Σ :



$F(x,y,z) = (xy,\ x^2,\ yz)$

$\vdots$

$\text{rot}\, F(x,y,z) = (z, 0, x)$

$$\Phi_\Sigma(\text{rot}\, F) = \Phi_{\tilde\Sigma}(\text{rot}\, F) = (*)$$

$\tilde\Sigma$: $\quad \sigma(u,v) = (u,v,0) \qquad (u,v) \in \bar{B}_1(0,0)$

$$N_\sigma(u,v) = (0,0,\underbrace{1}_{>0}) \quad \checkmark$$

$$(*) = \iint_{\bar{B}_1(0,0)} \text{rot}\, F(\sigma(u,v)) \cdot N_\sigma(u,v)\, du\, dv$$

$$= \iint_{\bar{B}_1(0,0)} (0,0,u) \cdot (0,0,1)\, du\, dv$$

$$= \iint_{\bar{B}_1(0,0)} u\, du\, dv = \iint_{[0,1]\times[0,2\pi]} \rho\cos\theta\, \rho\, d\rho\, d\theta$$

$$= \int_0^1 \rho^2\, d\rho \underbrace{\int_0^{2\pi} \cos\theta\, d\theta}_{=0} = 0 \qquad \square$$

Esempi (calcolo di divergenza)

- $F(x,y,z) = (xy,\ x^2,\ yz) \qquad F \in C^1(\mathbb{R}^3, \mathbb{R}^3)$

  $\text{div}\, F(x,y,z) = y + 0 + y = 2y$

- $F(x,y,z) = (x, y, z) \qquad F \in C^1(\mathbb{R}^3, \mathbb{R}^3)$

  $\text{div}\, F(x,y,z) = 1 + 1 + 1 = 3$

- $F(x,y,z) = (y, -x, 0) \qquad F \in C^1(\mathbb{R}^3, \mathbb{R}^3)$

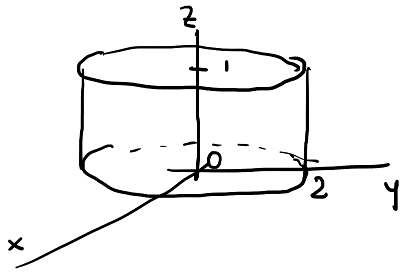
  $\text{div}\, F(x,y,z) = 0 + 0 + 0 = 0$

Esempio

Verificare la validità del teor. della divergenza

per $F(x,y,z) = (xy^2,\ x^2y,\ (x^2+y^2)z^2)$ $\qquad F \in C^1(\mathbb{R}^3,\mathbb{R}^3)$

e T :



① $\operatorname{div} F(x,y,z) = y^2 + x^2 + (x^2+y^2)\,2z = (x^2+y^2)(2z+1)$

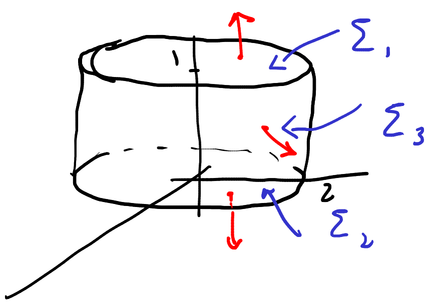
$$\iiint_T \operatorname{div} F(x,y,z)\,dx\,dy\,dz = \int_0^1 \Big(\iint_{T_z} (x^2+y^2)(2z+1)\,dx\,dy \Big) dz$$

$T_z = \bar{B}_2(0,0) \quad \forall z \in [0,1]$

$$= \int_0^1 (2z+1)\,dz \iint_{\bar{B}_2(0,0)} (x^2+y^2)\,dx\,dy$$

$$= [z^2+z]_0^1 \iint_{[0,2]\times[0,2\pi]} \rho^2 \rho \, d\rho\, d\theta = 2\int_0^2 \rho^3 d\rho \;\; 2\pi$$

$$= 4\pi \left[\frac{\rho^4}{4}\right]_0^2 = 4\pi \cdot 4 = 16\pi$$

② 

$\partial T = \Sigma_1 \cup \Sigma_2 \cup \Sigma_3$

Σ_1 : $\quad \sigma(u,v) = (\overset{x}{u}, \overset{y}{v}, \overset{z}{1}) \qquad (u,v) \in \bar{B}_2 \;\; (= \bar{B}_2(0,0))$

$N_\sigma(u,v) = (0,0,\underbrace{1}_{>0})$ $\qquad$ punta verso l'alto : ✓

$$\Phi_{\Sigma_1^+}(F) = \iint_{\bar{B}_2} (uv^2,\ u^2v,\ (u^2+v^2)\cdot 1) \cdot (0,0,1)\,du\,dv$$

$$= \iint_{\bar{B}_2} (u^2+v^2)\,du\,dv \overset{\text{vedi sopra}}{=} \dots = 8\pi$$

$\Sigma_2 : \quad \sigma(u,v) = (u,v,0) \qquad (u,v) \in \bar{B}_2$

$N_\sigma(u,v) = (0,0,\underbrace{1}_{>0})$ punta verso l'alto !!!

$$\Phi_{\Sigma_2^+}(F) = - \iint_{\bar{B}_2} (uv^2, u^2v, 0) \cdot (0,0,1) \, du\,dv$$

$$= - \iint_{\bar{B}_2} 0 \, du\,dv = 0$$

$\Sigma_3 : \quad \sigma(\theta,z) = (2\cos\theta, 2\sin\theta, z) \qquad (\theta,z) \in \overbrace{[0,2\pi]\times[0,1]}^{=:K}$

$N_\sigma(\theta,z) = (2\cos\theta, 2\sin\theta, 0)$ punta verso l'esterno di T ✓

$$\Phi_{\Sigma_3^+}(F) = \iint_K (2\cos\theta \cdot 4\sin^2\theta, 4\cos^2\theta \cdot 2\sin\theta, 4z^2) \cdot (2\cos\theta, 2\sin\theta, 0) \, d\theta\,dz$$

$$= \iint_K (16\cos^2\theta\sin^2\theta + 16\cos^2\theta\sin^2\theta + 0) \, d\theta\,dz$$

$$= \iint_K 32\cos^2\theta\sin^2\theta \, d\theta\,dz$$

$$= \int_0^{2\pi} 32\cos^2\theta\sin^2\theta \, d\theta \cdot \int_0^1 dz$$

$$= 8 \cdot \int_0^{2\pi} 4\cos^2\theta\sin^2\theta \, d\theta$$

$$= 8 \cdot \int_0^{2\pi} (\sin 2\theta)^2 \, d\theta = 8 \cdot \int_0^{2\pi} \frac{1 - \overbrace{\cos 4\theta}^{\text{contributo nullo}}}{2} \, d\theta$$

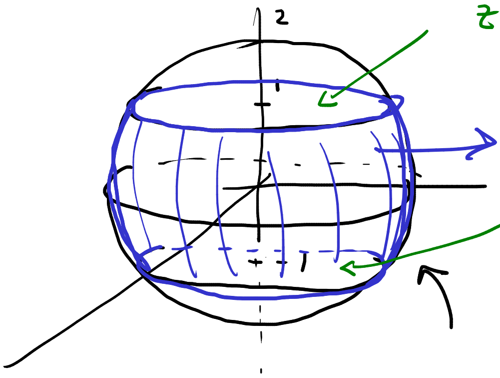
$$= 8\pi$$

Quindi:

$$\int_{\partial T^+} F \cdot n \, dS = \Phi_{\Sigma_1^+}(F) + \Phi_{\Sigma_2^+}(F) + \Phi_{\Sigma_3^+}(F)$$

$$= 8\pi + 0 + 8\pi = 16\pi \qquad \square$$

Esercizio



$z = 1 \iff 2\cos\varphi = 1$

$\varphi = \frac{\pi}{3}$

$z = -1 \iff 2\cos\varphi = -1$

$\varphi = \frac{2}{3}\pi$

$$\sigma(\varphi,\theta) = (2\sin\varphi\cos\theta,\ 2\sin\varphi\sin\theta,\ 2\cos\varphi)$$

$$(\varphi,\theta) \in \left[\frac{\pi}{3}, \frac{2}{3}\pi\right] \times [0, 2\pi]$$

⋮

In alternativa:

$$\iiint_T \operatorname{div} F \, dx\,dy\,dz = \int_{\partial T^+} F \cdot n \, dS$$

$$= \Phi_{\Sigma^+}(F) + \Phi_{\Sigma^+_{TOP}}(F) + \Phi_{\Sigma^+_{BOTTOM}}(F)$$

$$\Rightarrow \Phi_{\Sigma^+}(F) = 3 \iiint_T 1 \, dx\,dy\,dz - \Phi_{\Sigma^+_{TOP}}(F) - \Phi_{\Sigma^+_{BOTTOM}}(F)$$

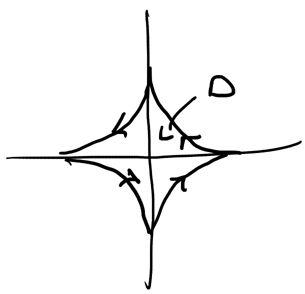
per strati facile facile

. . .

Esempio (applicazione del teor. di Gauss-Green)

Calcolare l'area racchiusa dall'asteroide.

$r(t) = (\cos^3 t, \sin^3 t) \qquad t \in [0, 2\pi]$



Per Gauss-Green:

$$m_2(D) = \int_{\partial D^+} F(P) \cdot dP = \circledast$$

con $F(x,y) = \left(-\frac{y}{2}, \frac{x}{2}\right)$

$$(*) = \int_0^{2\pi} \left(-\frac{\sin^3 t}{2}, \frac{\cos^3 t}{2}\right)\cdot\left(-3\cos^2 t \sin t, 3\sin^2 t \cos t\right) dt$$

$$= \int_0^{2\pi} \left(\frac{3}{2}\cos^2 t \sin^4 t + \frac{3}{2}\cos^4 t \sin^2 t\right) dt$$

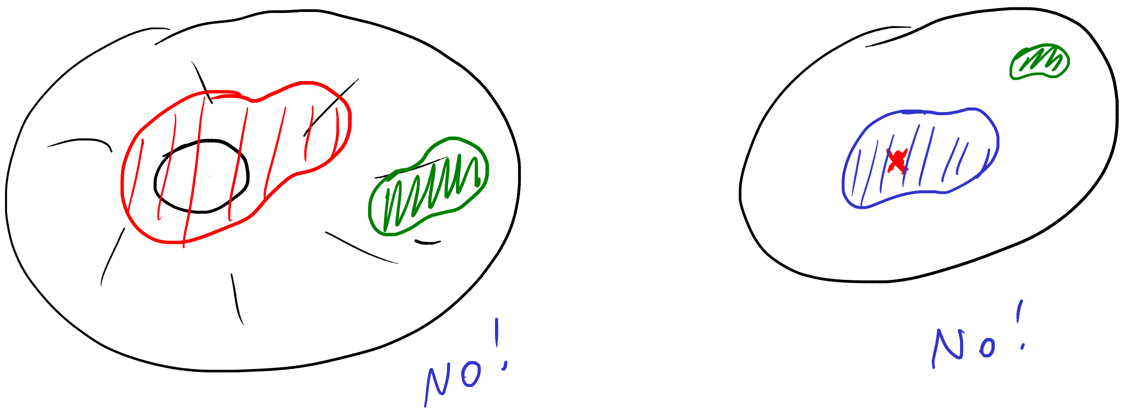
$$= \int_0^{2\pi} \frac{3}{2}\cos^2 t \sin^2 t \,(\sin^2 t + \cos^2 t)\, dt$$

$$= \int_0^{2\pi} \frac{3}{2}\cos^2 t \sin^2 t \, dt = \frac{3}{8}\int_0^{2\pi} (\sin 2t)^2 dt$$

$$= \frac{3}{8}\int_0^{2\pi} \frac{1-\cos 4t}{2} dt = \frac{3}{8}\pi$$

$\dots = 0$

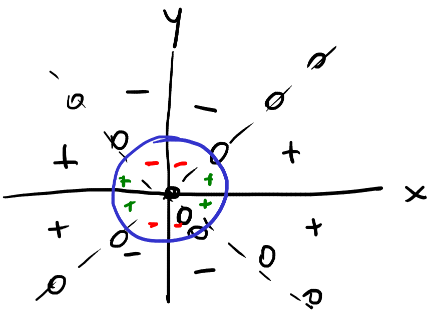
Es. (insiemi non semplicemente stellati)



Esempi (punti di estremo)

- $f(x,y) = x^2 - y^2$

$f(0,0) = 0$

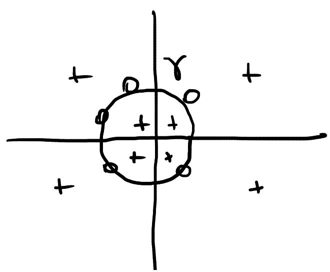


$x^2 > y^2$

$|x| > |y|$

- $f(x,y) = (x^2+y^2-1)^2$

Tutti i punti di γ sono di minimo (globale)



LEZIONE POMERIDIANA (RECUPERO DI UN "PONTE")

Riprendo gli esempi:

- $f(x,y) = x^2 + y^2$

Cerco i punti stazionari:

$$\begin{cases} \dfrac{\partial f}{\partial x}(x,y) = 2x = 0 \\ \dfrac{\partial f}{\partial y}(x,y) = 2y = 0 \end{cases}$$

Unica sol. $(0,0)$

Già notato: $\left.\begin{array}{l} f(0,0) = 0 \\ f(x,y) > 0 \quad \forall (x,y) \in \mathbb{R}^2 \setminus \{(0,0)\} \end{array}\right| \Rightarrow$

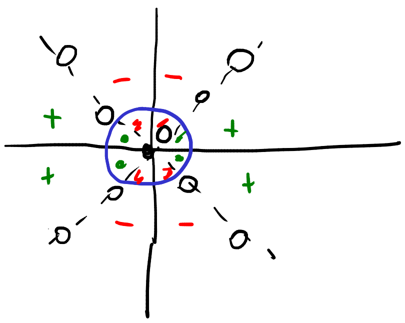
$(0,0)$ punto di minimo (globale)

- $f(x,y) = x^2 - y^2$

per brevità

$$\begin{cases} f_x(x,y) = 2x = 0 \\ f_y(x,y) = -2y = 0 \end{cases} \iff (0,0)$$

Già notato: $(0,0)$ né punto di max, né di min



$(0,0)$ punto di sella

- $f(x,y) = (x^2+y^2-1)^2$

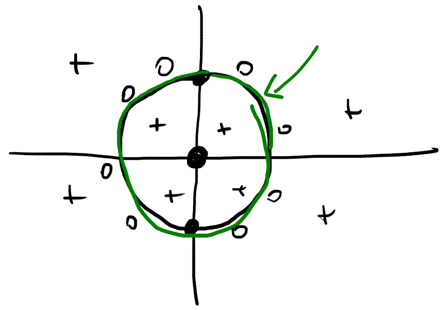
$f_x(x,y) = 2(x^2+y^2-1)\,2x\,, \qquad f_y(x,y) = 2(x^2+y^2-1)\,2y$

$$\begin{cases} f_x(x,y) = 0 \\ f_y(x,y) = 0 \end{cases} \iff \begin{cases} \cancel{4}x\,(x^2+y^2-1) = 0 \\ \cancel{4}y\,(x^2+y^2-1) = 0 \end{cases}$$

1° caso: $\begin{cases} x = 0 \\ y(y^2-1) = 0 \end{cases}$ $\Big<$ (0,0) (0,1) (0,−1)

2° caso: $\begin{cases} x^2+y^2-1 = 0 \\ y \cdot 0 = 0 \end{cases}$ $\forall y$

tutti i punti della circonf. unitaria sono stazionari



I punti della circonferenza sono tutti di minimo (globale)

E (0,0) ?

Considerazioni su teor. di Weierstrass + teor. di Fermat mi portano a concludere che (0,0) sia punto di massimo globale per $f|_{\bar{B}_1(0,0)}$, quindi locale per f

Oss: $\underset{\text{asse } x}{f(x,y)} = f(x,0) = (x^2-1)^2 \xrightarrow[x \to \pm\infty]{} +\infty$

$\Rightarrow$ f è illimitata superiormente

$\Rightarrow$ f non ha massimo globale

In alternativa a "Weierstrass + Fermat", osservo che

$f(0,0) = 1$

$f(x,y) \overset{?}{\le} 1 \iff (x^2+y^2-1)^2 \le 1$

$\iff -1 \le x^2+y^2-1 \le 1$

$\iff 0 \le x^2+y^2 \le 2$

$\uparrow$ vera sempre $\qquad$ $\uparrow$ vera se $(x,y) \in \bar{B}_{\sqrt{2}}(0,0)$

Ritrovo: (0,0) punto di max loc.

Esempi (autovalore nullo)

- $f(x,y) = x^2 + y^4$ $\quad f \in C^2(\mathbb{R}^2, \mathbb{R})$

$f_x(x,y) = 2x$

$f_y(x,y) = 4y^3$

$$\begin{cases} f_x = 0 \\ f_y = 0 \end{cases} \iff \begin{cases} x = 0 \\ y = 0 \end{cases}$$

$(0,0)$ unico punto staz.

Oss: $(0,0)$ punto di min. (glob.)

$$H_f(x,y) = \begin{pmatrix} 2 & 0 \\ 0 & 12y^2 \end{pmatrix} \Rightarrow H_f(0,0) = \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}$$

Oss: per una matrice diagonale, gli autovalori si "leggono" sulla diagonale principale e corrispondenti autovettori sono gli elementi della base canonica

Per $H_f(0,0)$: autovalore $\lambda = 2$ con $v = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

autovalore $\lambda = 0$ con $v = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Restringo f alla retta passante per $(0,0)$ individuata da $v = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, cioè: $\begin{pmatrix} 0 \\ 0 \end{pmatrix} + t\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ t \end{pmatrix}$

$g(t) := f(0,t) = t^4$ $\quad t = 0$ è punto di <u>min.</u>

(non mi sorprende!)

- $f(x,y) = x^2 - y^4$ $\quad f \in C^2(\mathbb{R}^2, \mathbb{R})$

$$\begin{cases} f_x(x,y) = 2x \\ f_y(x,y) = -4y^3 \end{cases}$$

Unico punto staz. $(0,0)$

$f_{xx}(x,y) = 2$, $\quad f_{xy}(x,y) = 0$, $\quad f_{yy}(x,y) = -12y^2$

$$H_f(0,0) = \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}$$ come nell'es. precedente!

$\lambda = 0$ con autovett. $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Restringo f a $\begin{pmatrix} 0 \\ 0 \end{pmatrix} + t\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ t \end{pmatrix}$

$g(t) = f(0,t) = -t^4$ ha in $t=0$ un punto di massimo

• $f(x,y) = x^2 + y^3$ $\qquad f \in C^2(\mathbb{R}^2, \mathbb{R})$

$f_x(x,y) = 2x$
$f_y(x,y) = 3y^2$ $\qquad$ p. staz. $(0,0)$

$f_{xx}(x,y) = 2$, $\quad f_{xy}(x,y) = 0$, $\quad f_{yy}(x,y) = 6y$

$$H_f(0,0) = \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}$$

Qui: $f(0,t) = t^3$ ha in $(0,0)$ un punto di flesso

Es. (classificazione di punti stazionari)

• $f(x,y) = x^3 - 3xy + y^2$ $\qquad f \in C^2(\mathbb{R}^2, \mathbb{R})$

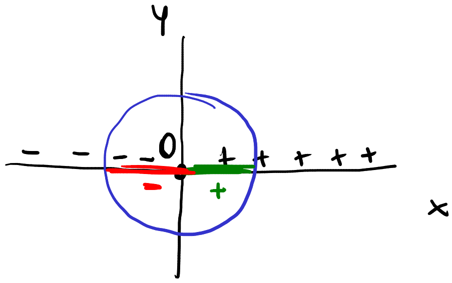
$\forall (x,y) \in \mathbb{R}^2$: $\quad f_x(x,y) = 3x^2 - 3y$
$\qquad f_y(x,y) = -3x + 2y$

Punti stazionari:

$$\begin{cases} f_x = 0 \\ f_y = 0 \end{cases} \iff \begin{cases} 3x^2 - 3y = 0 \\ -3x + 2y = 0 \end{cases} \quad \begin{cases} x^2 - \frac{3}{2}x = 0 \\ y = \frac{3}{2}x \end{cases}$$

$$\begin{cases} x = 0 \\ y = 0 \end{cases} \quad \text{oppure} \quad \begin{cases} x = \frac{3}{2} \\ y = \frac{9}{4} \end{cases}$$

$$(0,0) \qquad \left(\frac{3}{2}, \frac{9}{4}\right)$$



Oss: $f(0,0) = 0$

$f(x,0) = x^3$

$\Rightarrow (0,0)$ punto di sella

Classifico $\left(\frac{3}{2}, \frac{9}{4}\right)$ con la matrice hessiana.

$f_{xx}(x,y) = 6x, \quad f_{xy}(x,y) = -3, \quad f_{yy}(x,y) = 2$

$$\Rightarrow H_f\left(\frac{3}{2}, \frac{9}{4}\right) = \begin{pmatrix} 9 & -3 \\ -3 & 2 \end{pmatrix}$$

Cerco gli autovalori:

$$\det\begin{pmatrix} 9-\lambda & -3 \\ -3 & 2-\lambda \end{pmatrix} = (9-\lambda)(2-\lambda) - 9$$
$$= 18 - 11\lambda + \lambda^2 - 9$$
$$= \underbrace{\lambda^2 - 11\lambda}_{\text{var.}} \underbrace{+ 9}_{\text{var.}} \quad \Rightarrow \quad \lambda_1, \lambda_2 > 0$$

$\overset{\text{cor.}}{\Rightarrow} \left(\frac{3}{2}, \frac{9}{4}\right)$ punto di <u>min. loc.</u>

(<u>non</u> globale)

- $f(x,y,z) = x^3y - y + x^2z^2 \qquad f \in C^2(\mathbb{R}^3, \mathbb{R})$

$\forall (x,y,z)$:

$f_x(x,y,z) = 3x^2y + 2xz^2$

$f_y(x,y,z) = x^3 - 1 \qquad f_z(x,y,z) = 2x^2z$

Punti stazionari:

$$\begin{cases} f_x = 0 \\ f_y = 0 \\ f_z = 0 \end{cases} \qquad \begin{cases} 3x^2y + 2xz^2 = 0 \\ x^3 - 1 = 0 \\ 2x^2 z = 0 \end{cases} \qquad \begin{cases} y = 0 \\ x = 1 \\ z = 0 \end{cases}$$

$(1,0,0)$

Determino $H_f(1,0,0)$:

$f_{xx}(x,y,z) = 6xy + 2z^2, \quad f_{xy}(x,y,z) = 3x^2, \quad f_{xz}(x,y,z) = 4xz$

$f_{yy}(x,y,z) = 0, \quad f_{yz}(x,y,z) = 0, \quad f_{zz}(x,y,z) = 2x^2$

$$\Rightarrow H_f(1,0,0) = \begin{pmatrix} 0 & 3 & 0 \\ 3 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

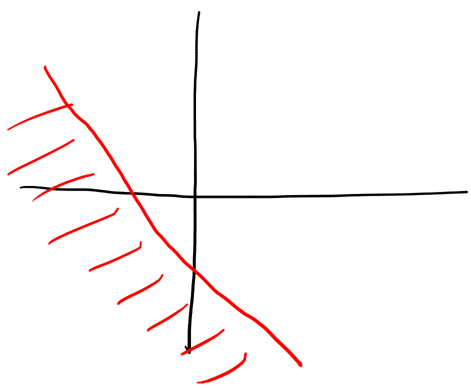
Cerco gli autovalori:

$$\det \begin{pmatrix} -\lambda & 3 & 0 \\ 3 & -\lambda & 0 \\ 0 & 0 & 2-\lambda \end{pmatrix} = (2-\lambda)(\lambda^2 - 9)$$

$\lambda = 2 \qquad \underbrace{\lambda = 3 \qquad \lambda = -3}_{\text{discordi}}$

$\Rightarrow (1,0,0)$ punto di sella.

- $f(x,y) = \ln(1 + x + y) - x - y^2$



$\mathrm{dom}(f) = \{(x,y) \mid 1 + x + y > 0\}$

$\uparrow$

aperto, $f \in C^2$

$\forall (x,y) \in \mathrm{dom}(f)$:

$$f_x(x,y) = \frac{1}{1+x+y} - 1 \qquad f_y(x,y) = \frac{1}{1+x+y} - 2y$$

Punti stazionari:

$$\begin{cases} f_x = 0 \\ f_y = 0 \end{cases} \qquad \begin{cases} \dfrac{1}{1+x+y} - 1 = 0 \\ \dfrac{1}{1+x+y} - 2y = 0 \end{cases} \qquad \begin{cases} \cancel{1}+x+y = \cancel{1} \\ 2y = 1 \end{cases} \qquad \begin{cases} x = -\frac{1}{2} \\ y = \frac{1}{2} \end{cases}$$

$$\left(-\frac{1}{2}, \frac{1}{2}\right)$$

Calcolo:

$$f_{xx}(x,y) = -\frac{1}{(1+x+y)^2} \qquad f_{xy}(x,y) = -\frac{1}{(1+x+y)^2}$$

$$f_{yy}(x,y) = -\frac{1}{(1+x+y)^2} - 2$$

$$\Rightarrow H_f\left(-\frac{1}{2}, \frac{1}{2}\right) = \begin{pmatrix} -1 & -1 \\ -1 & -3 \end{pmatrix}$$

Autovalori:

$$\det \begin{pmatrix} -1-\lambda & -1 \\ -1 & -3-\lambda \end{pmatrix} = (1+\lambda)(3+\lambda) - 1$$

$$= \lambda^2 \underbrace{+}_{\text{perm.}} 4\lambda \underbrace{+}_{\text{perm.}} 2 \Rightarrow \lambda_1, \lambda_2 < 0$$

$$\Rightarrow \left(-\frac{1}{2}, \frac{1}{2}\right) \text{ punto di max loc.}$$

- $f(x,y,z) = x^2 + y^3 + z^2 - xy - xz \qquad f \in C^2(\mathbb{R}^3, \mathbb{R})$

$$\begin{cases} f_x(x,y,z) = 2x - y - z = 0 \\ f_y(x,y,z) = 3y^2 - x = 0 \\ f_z(x,y,z) = 2z - x = 0 \end{cases} \qquad \begin{cases} 6y^2 - y - \frac{3}{2}y^2 = 0 \\ x = 3y^2 \\ z = \frac{x}{2} = \frac{3}{2}y^2 \end{cases}$$

$$\frac{9}{2}y^2 - y = 0 \qquad y\left(\frac{9}{2}y - 1\right) = 0$$

$$\begin{cases} y = 0 \\ x = 0 \\ z = 0 \end{cases} \qquad y = \frac{2}{9}, \quad x = \frac{4}{27}, \quad z = \frac{2}{27}$$

Punti stazionari: $(0,0,0)$ $\left(\frac{4}{27}, \frac{2}{9}, \frac{2}{27}\right)$

$$H_f(x,y,z) = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 6y & 0 \\ -1 & 0 & 2 \end{pmatrix}$$

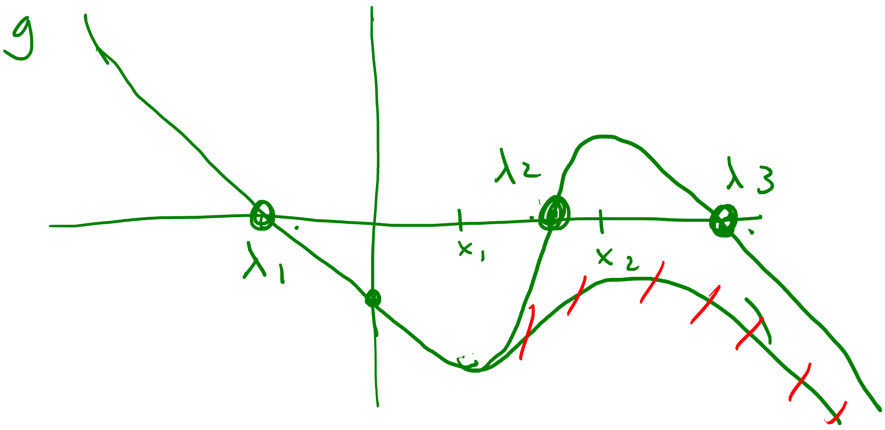
$$\Rightarrow \quad H_f(0,0,0) = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 0 & 0 \\ -1 & 0 & 2 \end{pmatrix}$$

Autovalori:

$$\det \begin{pmatrix} 2-\lambda & -1 & -1 \\ -1 & -\lambda & 0 \\ -1 & 0 & 2-\lambda \end{pmatrix} = -(-\lambda) + (2-\lambda)(-2\lambda + \lambda^2 - 1)$$

$$= \underline{\lambda} - \underline{4\lambda} + \underline{2\lambda^2} - 2 + \underline{2\lambda^2} - \underline{\lambda^3} + \underline{\lambda}$$

$$= -\lambda^3 + 4\lambda^2 - 2\lambda - 2 \quad =: g(\lambda)$$



$\lambda_1 < 0$

$\lambda_2, \lambda_3 > 0$

$\Rightarrow$ $(0,0,0)$ punto di sella

$$g'(\lambda) = -\underbrace{3\lambda^2}_{\text{var.}} + \underbrace{8\lambda - 2}_{\text{var}}$$

$g'(\lambda)$ si annulla in due punti a dx di 0

	x_1		x_2	
g'	$-$	$+$	$-$	
g	$\searrow$	$\nearrow$	$\searrow$	

Classifica l'altro punto staz.

$$Hf\left(\frac{4}{27}, \frac{2}{9}, \frac{2}{27}\right) = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 4/3 & 0 \\ -1 & 0 & 2-\lambda \end{pmatrix}$$

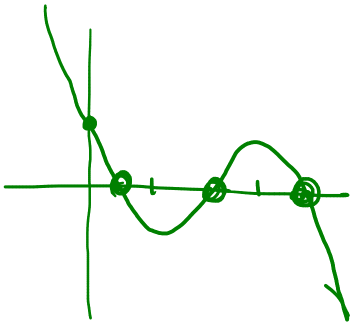
$$\det \begin{pmatrix} 2-\lambda & -1 & -1 \\ -1 & 4/3-\lambda & 0 \\ -1 & 0 & 2-\lambda \end{pmatrix} =$$

$$= -1\left(\frac{4}{3} - \lambda\right) + (2-\lambda)\left(\frac{8}{3} - 2\lambda - \frac{4}{3}\lambda + \lambda^2 - 1\right)$$

$$= -\frac{4}{3} + \lambda + \frac{16}{3} - \frac{20}{3}\lambda + 2\lambda^2 - 2 - \frac{8}{3}\lambda + \frac{10}{3}\lambda^2 - \lambda^3 + \lambda$$

$$= -\lambda^3 + \frac{16}{3}\lambda^2 + \left(2 - \frac{28}{3}\right)\lambda + 2$$

$$= -\lambda^3 + \frac{16}{3}\lambda^2 - \frac{22}{3}\lambda + 2 =: g(\lambda)$$



$$g'(\lambda) = -3\lambda^2 + \frac{32}{3}\lambda - \frac{22}{3}$$

si annulla in $x_1, x_2 > 0$

Gli autovalori sono tutti $> 0 \Rightarrow$ il punto è di minimo.

• $f(x,y,z) = x^2 y + y^2 z + z^2 - z$ $f \in C^2(\mathbb{R}^3, \mathbb{R})$

$$\begin{cases} f_x(x,y,z) = 2xy = 0 \\ f_y(x,y,z) = x^2 + 2yz = 0 \\ f_z(x,y,z) = y^2 + 2z = 0 \end{cases}$$

$1°$: $\begin{cases} x = 0 \\ yz = 0 \\ y^2 + 2z = 0 \end{cases}$ $\begin{matrix} x=0 \\ y=0 \\ z=0 \end{matrix}$; $\begin{matrix} x=0 \\ z=0 \\ y^2=0 \end{matrix}$ $\quad (0,0,0)$

$$2^\circ: \begin{cases} y = 0 \\ x^2 = 0 \\ 2z = 0 \end{cases} \quad — \quad (0,0,0)$$

$$H_f(x,y,z) = \begin{pmatrix} 2y & 2x & 0 \\ 2x & 2z & 2y \\ 0 & 2y & 2 \end{pmatrix}$$

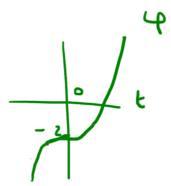
$$H_f(0,0,0) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

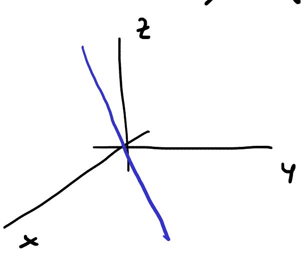
All' autovalore $\lambda = 2$ corrisponde autovalore $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$;
la restrizione di f alla retta di equazione
$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + t \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ t \end{pmatrix}$ ha in $(0,0,0)$ un minimo locale
(per il teorema)

All'autovalore $\lambda = 0$ corrispondono autovalori $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ e $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$.
La restrizione di f alla retta di equazione
$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + t \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} t \\ 0 \\ 0 \end{pmatrix}$ è $g(t) := f(t,0,0) \equiv -2$;
la restrizione di f alla retta di equazione
$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + t \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ t \\ 0 \end{pmatrix}$ è $h(t) = f(0,t,0) \equiv -2$

In base a questo non posso né escludere né confermare che $(0,0,0)$ sia punto di minimo locale.

Restringo f alla retta di equazione $\begin{pmatrix} t \\ t \\ 0 \end{pmatrix}$:





$$\varphi(t) = f(t,t,0) = t^3 - 2.$$

φ ha in $(0,0,0)$ un punto di sella
$\Rightarrow (0,0,0)$ punto di sella per f.