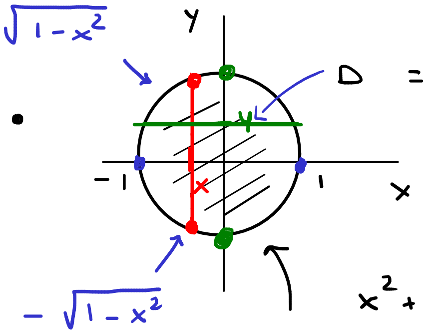
6 maggio 2025

Esempi (insiemi normali in $\mathbb{R}^2$)



- $$D = \{(x,y) \mid \underbrace{-1}_{a} \le x \le \underbrace{1}_{b},\ \underbrace{-\sqrt{1-x^2}}_{\alpha(x)} \le y \le \underbrace{\sqrt{1-x^2}}_{\beta(x)}\}$$

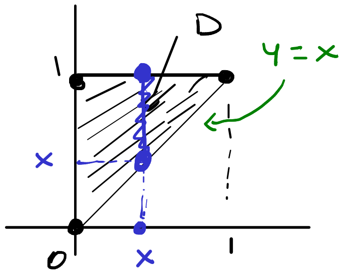
normale rispetto all'asse x

$x^2 + y^2 = 1$

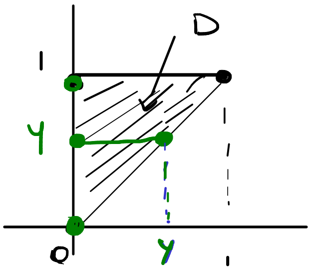
$y^2 = 1 - x^2 \qquad y = \pm\sqrt{1-x^2}$

Rispetto all'asse y:

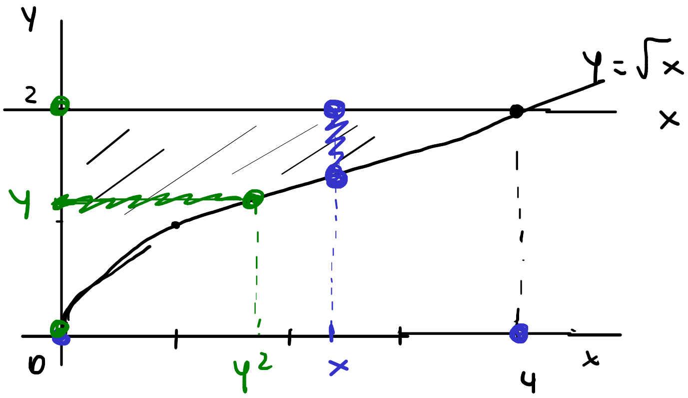
$$D = \{(x,y) \mid -1 \le y \le 1,\ -\sqrt{1-y^2} \le x \le \sqrt{1-y^2}\}$$



- $$D = \{(x,y) \mid 0 \le x \le 1,\ \underbrace{x}_{\alpha(x)} \le y \le \underbrace{1}_{\beta(x)}\}$$



$$D = \{(x,y) \mid 0 \le y \le 1,\ 0 \le x \le y\}$$

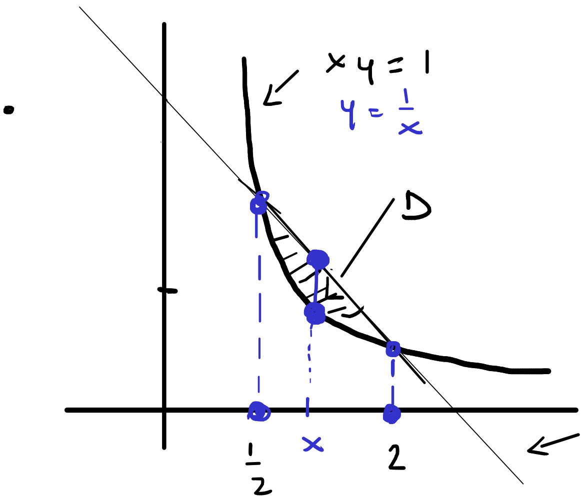


$x = y^2$

$$\begin{cases} y = \sqrt{x} \\ y = 2 \end{cases}$$

$$D = \{(x,y) \mid 0 \le x \le 4,\ \sqrt{x} \le y \le 2\}$$

$$D = \{(x,y) \mid 0 \le y \le 2,\ 0 \le x \le y^2\}$$



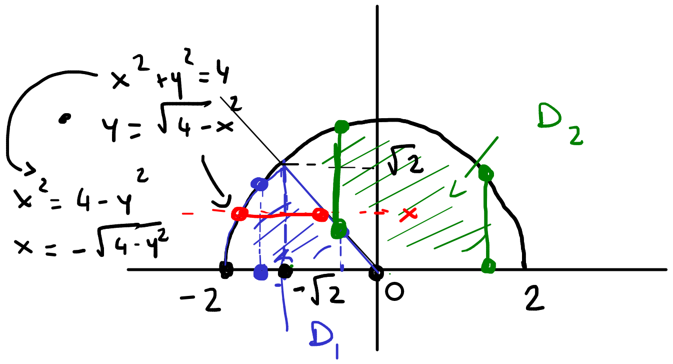
$$\begin{cases} xy = 1 \\ 2x + 2y = 5 \end{cases} \qquad x + y = \frac{5}{2}$$

$$x = \frac{1}{2} \quad \text{opp.} \quad x = 2$$

$$2x + 2y = 5$$

$$y = \frac{5}{2} - x$$

$$D = \{(x,y) \mid \underbrace{\tfrac{1}{2}}_{a} \le x \le \underbrace{2}_{b}, \ \underbrace{\tfrac{1}{x}}_{\alpha(x)} \le y \le \underbrace{\tfrac{5}{2} - x}_{\beta(x)} \}$$



$$\begin{cases} x^2 + y^2 = 4 \\ x + y = 0 \end{cases} \qquad 2x^2 = 4$$

$$D_1 = \{(x,y) \mid -2 \le x \le 0, \ 0 \le y \le \beta(x)\}$$

$$\text{con} \quad \beta(x) = \begin{cases} \sqrt{4 - x^2} & -2 \le x \le -\sqrt{2} \\ -x & -\sqrt{2} < x \le 0 \end{cases}$$

$\uparrow$ continua

$$= \{(x,y) \mid -2 \le x \le -\sqrt{2}, \ 0 \le y \le \sqrt{4 - x^2}\} \cup$$

$$\cup \{(x,y) \mid -\sqrt{2} \le x \le 0, \ 0 \le y \le -x\}$$

In alternativa:

$$D_1 = \{(x,y) \mid 0 \le y \le \sqrt{2}, \ -\sqrt{4 - y^2} \le x \le -y\}$$

$$D_2 = \{(x,y) \mid -\sqrt{2} \le x \le 0, \ -x \le y \le \sqrt{4 - x^2}\} \cup$$

$$\cup \{0 \le x \le 2, \ 0 \le y \le \sqrt{4 - x^2}\}$$

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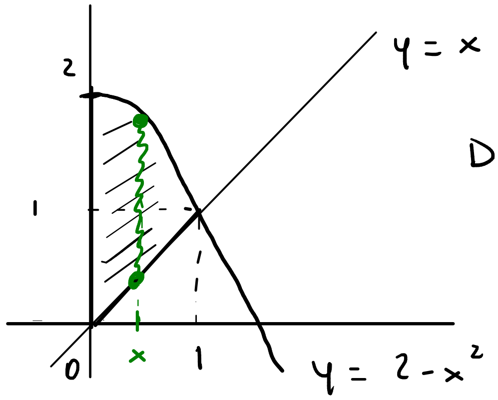

$$D = \{ (x,y) \mid 0 \le x \le 1, \quad x \le y \le 2 - x^2 \}$$

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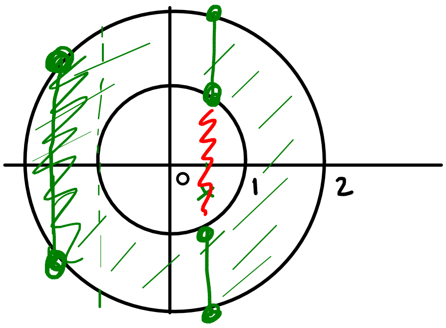
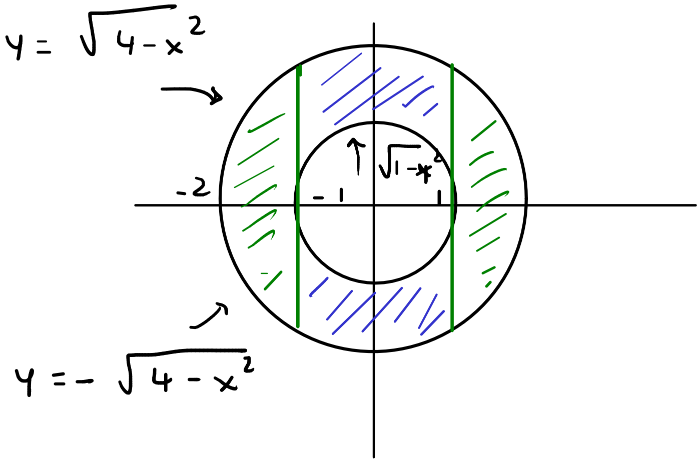




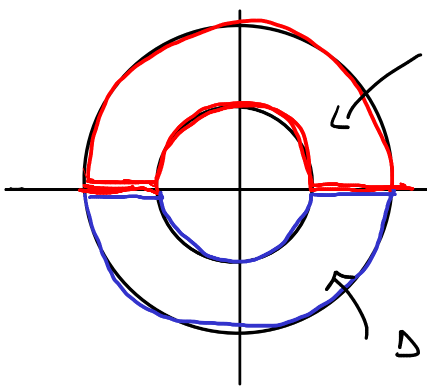
$$D = \{ (x,y) \mid -2 \le x \le -1, \quad -\sqrt{4-x^2} \le y \le \sqrt{4-x^2} \} \cup$$

$$\cup \{ (x,y) \mid -1 \le x \le 1, \quad \sqrt{1-x^2} \le y \le \sqrt{4-x^2} \} \cup$$

$$\cup \{ (x,y) \mid -1 \le x \le 1, \quad -\sqrt{4-x^2} \le y \le -\sqrt{1-x^2} \} \cup$$

$$\cup \{ (x,y) \mid 1 \le x \le 2, \quad -\sqrt{4-x^2} \le y \le \sqrt{4-y^2} \}$$

In alternativa:

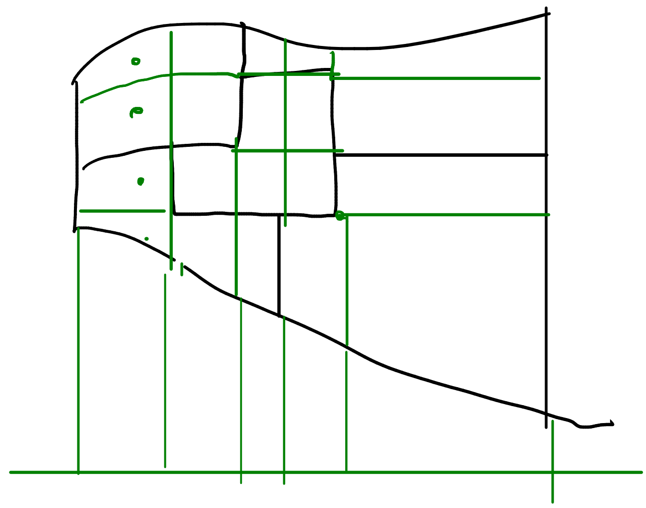


$$D_1 = \{ (x,y) \mid -2 \le x \le 2, \ \alpha(x) \le y \le \sqrt{4-x^2} \}$$

$$\text{con} \quad \alpha(x) = \begin{cases} 0 & -2 \le x \le -1 \\ \sqrt{1-x^2} & -1 < x < 1 \\ 0 & 1 \le x \le 2 \end{cases}$$

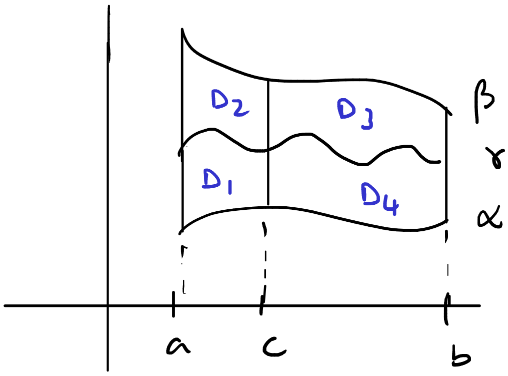
$$D_2 = \{ (x,y) \mid -2 \le x \le 2, \ -\sqrt{4-x^2} \le y \le \beta(x) \}$$

$$\text{con} \quad \beta(x) = \begin{cases} 0 & -2 \le x \le -1 \\ -\sqrt{1-x^2} & -1 < x < 1 \\ 0 & 1 \le x \le 2 \end{cases}$$



Motivazione per

$$m_2(D) = m_2(D_1) + \dots + m_2(D_k)$$



$$m_2(D) \overset{\text{def}}{=} \int_a^b (\beta(x) - \alpha(x))\,dx$$

additività

$$= \int_a^c (\beta(x) - \alpha(x))\,dx + \int_c^b (\beta(x) - \alpha(x))\,dx$$

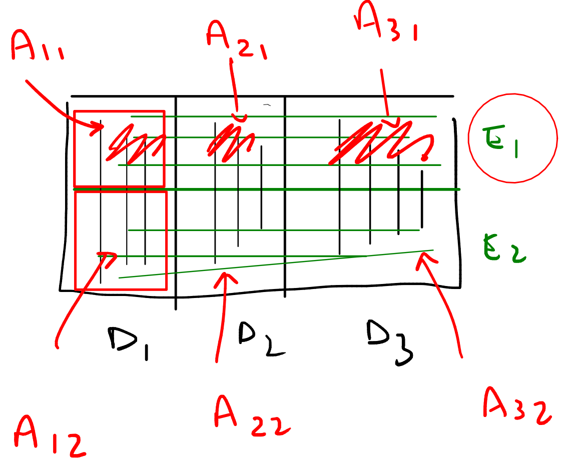
$$= \int_a^c (\beta(x) - \gamma(x) + \gamma(x) - \alpha(x))\,dx + \int_c^b (\beta(x) - \gamma(x) + \gamma(x) - \alpha(x))\,dx$$

linearità

$$= \underbrace{\int_a^c (\beta(x) - \gamma(x))\,dx}_{= m_2(D_2)} + \int_a^c \underbrace{(\gamma(x) - \alpha(x))}_{= m_2(D_1)}\,dx +$$

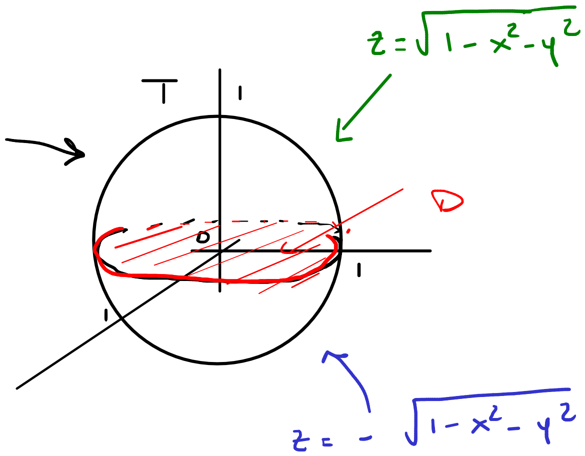
$$+ \int_c^b \underbrace{(\beta(x) - \gamma(x))}_{= m_2(D_3)}\,dx + \int_c^b \underbrace{(\gamma(x) - \alpha(x))}_{= m_2(D_4)}\,dx$$

$\square$



Esempi (ins. normali in $\mathbb{R}^3$)

- $x^2+y^2+z^2=1$

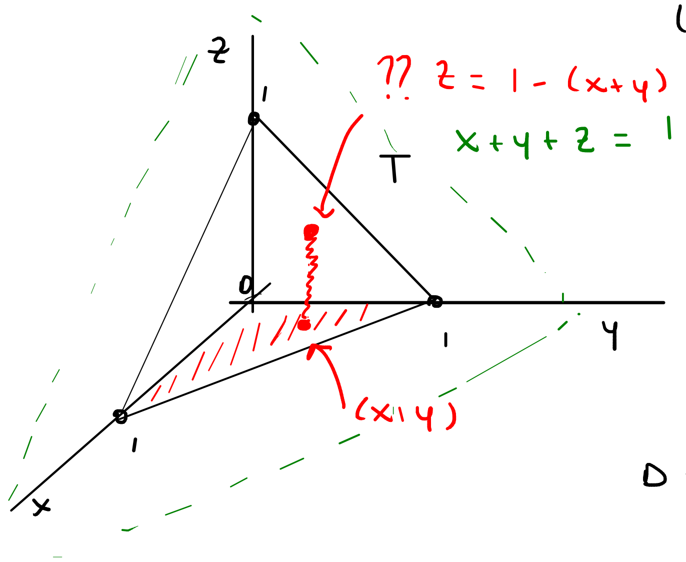


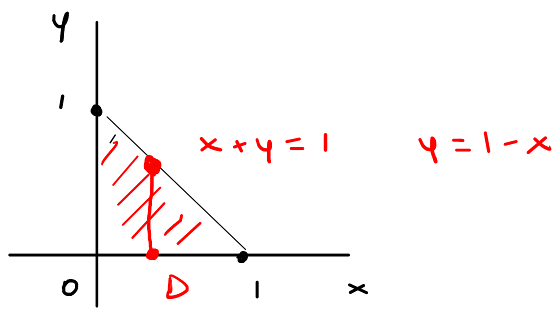
$$T=\{(x,y,z)\mid (x,y)\in D,\ -\sqrt{1-x^2-y^2}\le z\le\sqrt{1-x^2-y^2}\}$$

con D : palla chiusa di centro $(0,0)$ e raggio 1 nel piano xy

$$=\{(x,y)\mid -1\le x\le 1,\ -\sqrt{1-x^2}\le y\le\sqrt{1-x^2}\}$$

- Tetraedro di vertici $(0,0,0)$, $(0,0,1)$, $(0,1,0)$, $(1,0,0)$

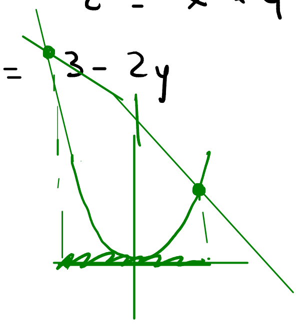


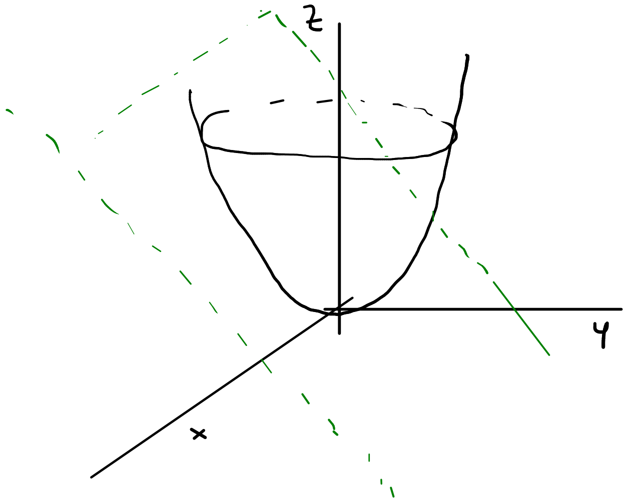


$$D=\{(x,y)\mid 0\le x\le 1,\ 0\le y\le 1-x\}$$

$$T=\{(x,y,z)\mid (x,y)\in D,\ \underbrace{0}_{\gamma(x,y)}\le z\le \underbrace{1-(x+y)}_{\delta(x,y)}\}$$

- T delimitato da
  - paraboloide $z = x^2 + y^2$
  - piano $z = 3 - 2y$





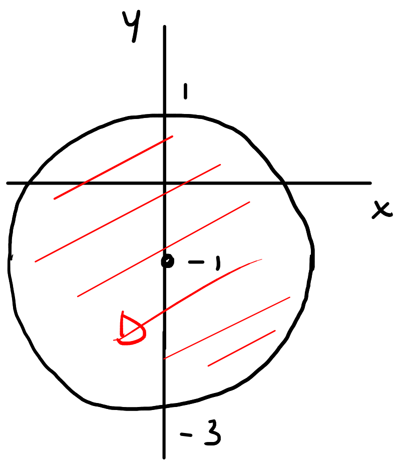
$$T = \{ (x,y,z) \mid (x,y) \in D \ , \ \underbrace{x^2+y^2}_{\gamma(x,y)} \le z \le \underbrace{3-2y}_{\delta(x,y)} \}$$

$\uparrow$?

Determino D!

$$\begin{cases} z = x^2 + y^2 \\ z = 3 - 2y \end{cases}$$

$$x^2 + y^2 = 3 - 2y$$
$$x^2 + y^2 + 2y = 3$$
$$x^2 + y^2 + 2y + 1 = 4$$
$$x^2 + (y+1)^2 = 2^2$$



Circonferenza di centro $(0,-1)$ e raggio 2