29 aprile 2025

Esempi (curve reg., reg. a tratti)

- $r(t) = (\underbrace{t(t-1)}_{=:x(t)}, \underbrace{t(t-1)(2t-1)}_{=:y(t)})$ $\quad t \in \mathbb{R}$

x, y di classe C^1 in $\mathbb{R}$ ✓

Riscrivo $\quad x(t) = t^2 - t \quad , \quad y(t) = (t^2-t)(2t-1)$

$$= 2t^3 - 3t^2 + t$$

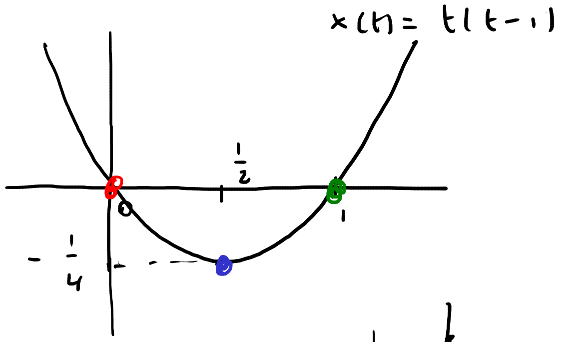
$\forall t \in \mathbb{R}: \quad x'(t) = 2t - 1, \quad y'(t) = 6t^2 - 6t + 1$

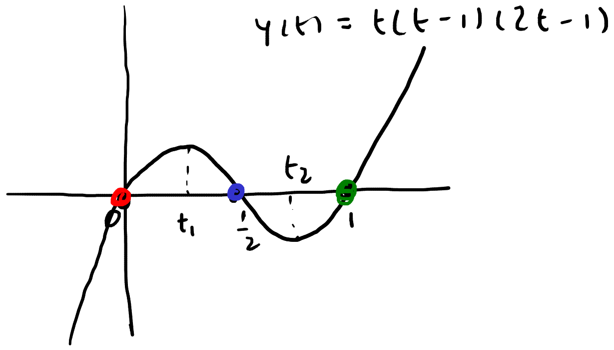
$r'(t) = (0,0) \iff x'(t) = 0$ e $y'(t) = 0$

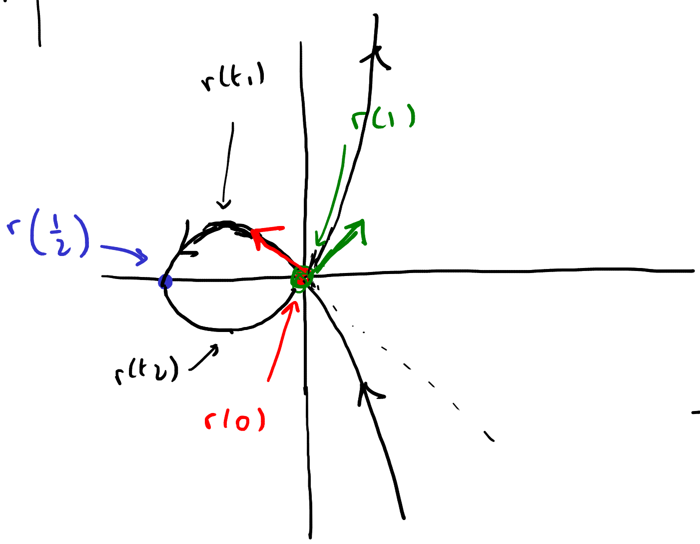
$$\iff \begin{cases} 2t - 1 = 0 \\ 6t^2 - 6t + 1 = 0 \end{cases} \iff \begin{array}{l} t = 1/2 \\ 6 \cdot \frac{1}{4} - 6 \cdot \frac{1}{2} + 1 = -\frac{1}{2} \neq 0 \end{array}$$

$\Rightarrow \quad r'(t) \neq (0,0) \quad \forall t \in \mathbb{R}$

Quindi: la curva è regolare







La curva NON è semplice

$r(0) = r(1)$

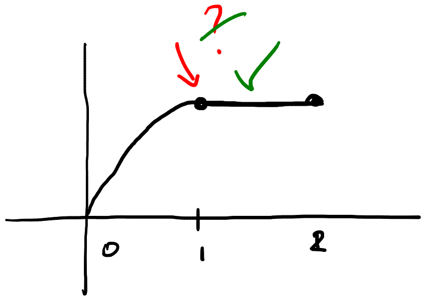
$$T(0) = \frac{r'(0)}{\|r'(0)\|} = \frac{(-1,1)}{\sqrt{2}}$$

$$\neq$$

$$T(1) = \frac{r'(1)}{\|r'(1)\|} = \frac{(1,1)}{\sqrt{2}}$$

- $r(t) = \begin{cases} (2t - t^2,\ 0) & t \in [0,1) \\ (1,\ (t-1)^2) & t \in [1,2) \end{cases}$

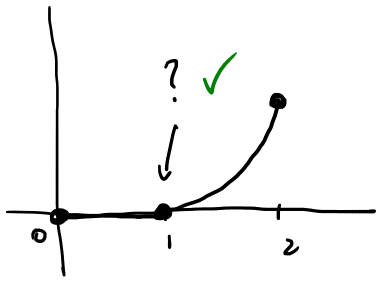
$$x(t) = \begin{cases} 2t - t^2 & t \in (0,1) \\ 1 & t \in [1,2] \end{cases}$$



$$x'(t) = \begin{cases} 2 - 2t & t \in [0,1) \\ 0 & t \in \boxed{[}1,2] \end{cases}$$

conseguenza di Lagrange (AM 1)

$$y(t) = \begin{cases} 0 & t \in [0,1) \\ (t-1)^2 & t \in [1,2] \end{cases}$$



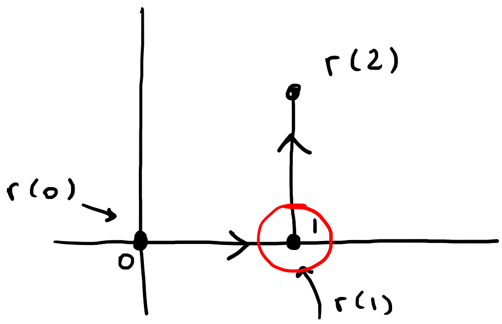
$$y'(t) = \begin{cases} 0 & t \in [0,1\boxed{]} \\ 2(t-1) & t \in (1,2] \end{cases}$$

$x, y \in C^1$; $\quad (x'(t), y'(t)) = (0,0) \iff t = 1$

Quindi: la curva non è regolare ;

è regolare a tratti.

Sostegno ?

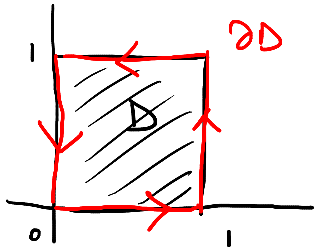


$s \longmapsto r(t_0) + s\, r'(t_0) = \begin{pmatrix} t_0 \\ f(t_0) \end{pmatrix} + s \begin{pmatrix} 1 \\ f'(t_0) \end{pmatrix}$

$\begin{cases} x = t_0 + s \\ y = f(t_0) + s\, f'(t_0) \end{cases}$ $\qquad s = x - t_0$

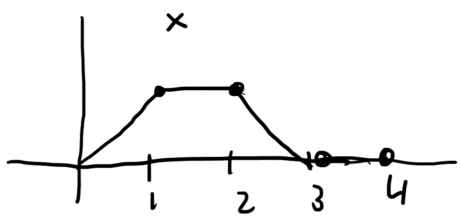
$\qquad y = f(t_0) + f'(t_0)(x - t_0)$

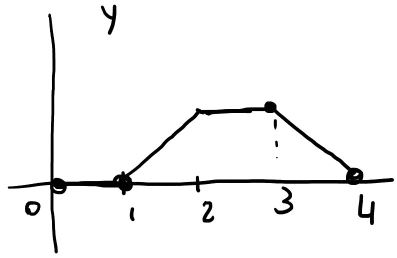
Es. (domini regolari)

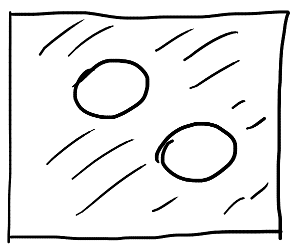


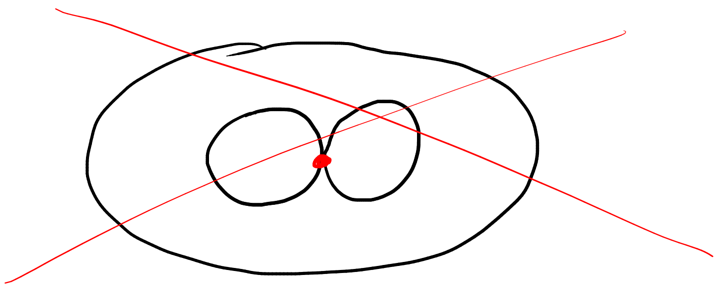
$$r(t) = \begin{cases} (t, 0) & t \in [0,1] \\ (1, t-1) & t \in (1,2] \\ (3-t, 1) & t \in (2,3] \\ (0, 4-t) & t \in (3,4] \end{cases}$$

↑ continua









Es.

$r(t) = (\cos t, \sin t)$ $\quad t \in [0, 2\pi]$

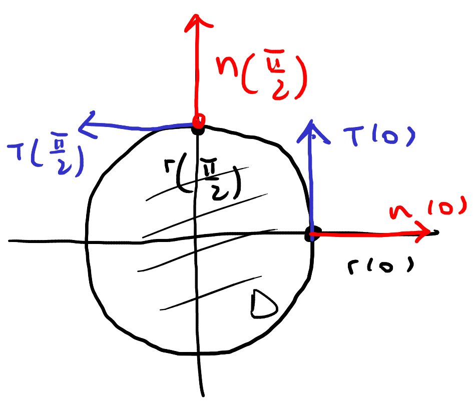
$r'(t) = (-\sin t, \cos t)$; $\|r'(t)\| = \sqrt{\sin^2 t + \cos^2 t} = 1$

$T(t) = (-\sin t, \cos t)$, $n(t) = (\cos t, \sin t)$

$r(0) = (1, 0)$

$T(0) = (0, 1)$

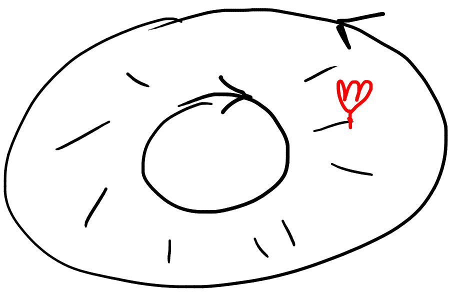
$n(0) = (1, 0)$

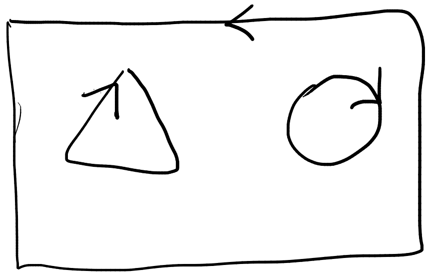


$r\left(\frac{\pi}{2}\right) = (0, 1)$

$T\left(\frac{\pi}{2}\right) = (-1, 0)$

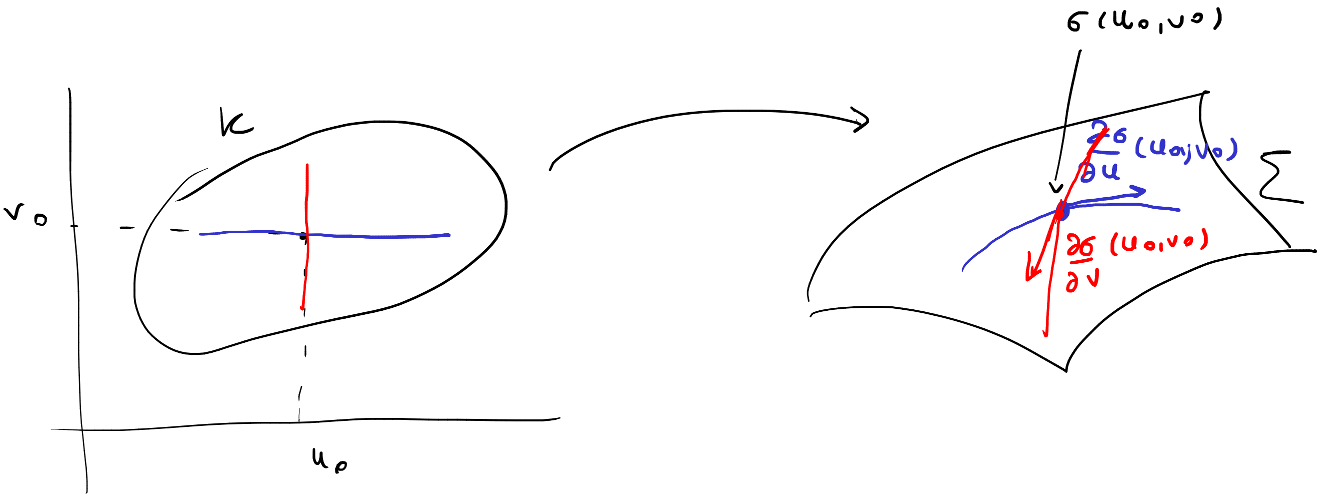
$n\left(\frac{\pi}{2}\right) = (0, 1)$





Es: $(2,-4,1) \times (-3,1,0)$

$$\begin{vmatrix} e_1 & e_2 & e_3 \\ 2 & -4 & 1 \\ -3 & 1 & 0 \end{vmatrix} = (-1, -3, -10)$$



$r(t) = \sigma(t, v_0)$ $\qquad$ $r'(t) = \frac{\partial \sigma}{\partial u}(t, v_0)$

Es. (sup. cilindrica)

$r > 0$

$\sigma(\theta, z) = (r\cos\theta,\ r\sin\theta,\ z)$ $\qquad$ $(\theta, z) \in \underbrace{[0, 2\pi] \times \mathbb{R}}_{K}$

$\in C^1$? sì! In $\mathbb{R} \times \mathbb{R}$, quindi:

anche in $(0, 2\pi) \times \mathbb{R}$

$$\begin{array}{cccc} & e_1 & e_2 & e_3 \\ \dfrac{\partial\sigma}{\partial\theta}(\theta,z) = & (-r\sin\theta, & r\cos\theta, & 0) \\ \dfrac{\partial\sigma}{\partial z}(\theta,z) = & (0, & 0, & 1) \end{array}$$

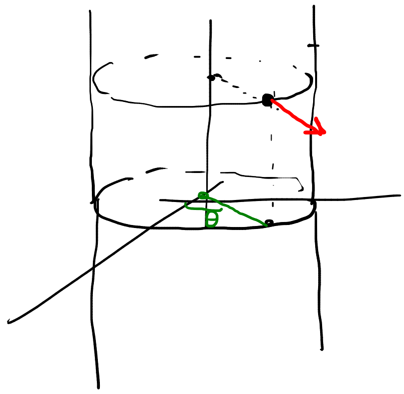
$$\frac{\partial\sigma}{\partial\theta}(\theta,z) \times \frac{\partial\sigma}{\partial z}(\theta,z) = (r\cos\theta,\ r\sin\theta,\ 0)$$

$$\left\| \frac{\partial\sigma}{\partial\theta}(\theta,z) \times \frac{\partial\sigma}{\partial z}(\theta,z) \right\| = \sqrt{r^2\cos^2\theta + r^2\sin^2\theta + 0^2} = r > 0$$

$\Rightarrow$ la sup. è regolare; $\forall \theta \in (0,2\pi) \times \mathbb{R}$:

$$N_\sigma(\theta,z) = (r\cos\theta,\ r\sin\theta,\ 0)$$

$$n_\sigma(\theta,z) = (\cos\theta,\ \sin\theta,\ 0)$$

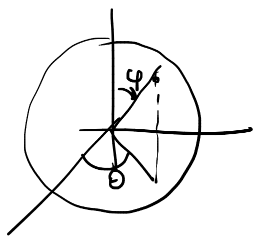


Es. (sup. sferica)

$$\sigma(\varphi,\theta) = (r\underbrace{\sin\varphi\cos\theta}, \ r\underbrace{\sin\varphi}\sin\theta,\ r\underbrace{\cos\varphi})$$

C^1 in $\mathbb{R}\times\mathbb{R}$

$\Rightarrow$ C^1 in $(0,\pi)\times(0,2\pi)$ ✓



$$\begin{array}{cccc} & e_1 & e_2 & e_3 \\ \dfrac{\partial\sigma}{\partial\varphi}(\varphi,\theta) = & (r\cos\varphi\cos\theta, & r\cos\varphi\sin\theta, & -r\sin\varphi) \\ \dfrac{\partial\sigma}{\partial\theta}(\varphi,\theta) = & (-r\sin\varphi\sin\theta, & r\sin\varphi\cos\theta, & 0) \end{array}$$

$$N_\sigma(\varphi,\theta) = (r^2\sin^2\varphi\cos\theta,\ r^2\sin^2\varphi\sin\theta,\ r^2\sin\varphi\cos\varphi)$$

$$= r\sin\varphi\ (r\sin\varphi\cos\theta,\ r\sin\varphi\sin\theta,\ r\cos\varphi)$$

$$= r\sin\varphi\ \sigma(\varphi,\theta)$$

$$\Rightarrow \quad \| N_\sigma(\varphi,\theta) \| = |r\sin\varphi|\ \|\sigma(\varphi,\theta)\|$$

$$= r\sin\varphi\ r = r^2\sin\varphi$$

$$N_\sigma(\varphi,\theta) = 0 \iff \sin\varphi = 0 \iff \varphi = 0,\ \varphi = \pi$$

$$\Rightarrow \quad \forall (\varphi,\theta) \in (0,\pi)\times(0,2\pi):\quad N_\sigma(\varphi,\theta) \neq 0 \quad \checkmark$$

Quindi: è una sup. regolare e

$\forall (\varphi,\theta) \in (0,\pi)\times(0,2\pi)$:

$$n_\sigma(\varphi,\theta) = \frac{N_\sigma(\varphi,\theta)}{\| N_\sigma(\varphi,\theta)\|} = \frac{r\sin\varphi\ \sigma(\varphi,\theta)}{r^2\sin\varphi}$$

$$= \frac{\sigma(\varphi,\theta)}{r} = (\sin\varphi\cos\theta,\ \sin\varphi\sin\theta,\ \cos\varphi)$$