General information		Academic year 2022-2023
Academic subject	**Advanced Mechanics**	
Degree programme	L-35 - Mathematics	
Programme year	Third	
Term	Second semester (February 20, 2023 – May 26, 2023)	
European Credit Transfer and Accumulation System credits (ECTS)		6
Language	Italian	
Attendance	Strongly encouraged	

Lecturer	
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Syllabus	
Learning objectives	Advanced knowledge of results and tools of fluiddynamics and magnetofluiddynamics to face up to research arguments.
Course prerequisites	Linear and affine algebra. General topology. Calculus of functions of many variables. Mechanics of point mass, of system of particles and of rigid continuous bodies. Elements of thermodynamics and electrodynamics.
Course contents	**Preliminaries.** Limits, continuity and differentiability for functions between euclidean vector and affine spaces. Scalar and vector functions. Limits and derivability of vector functions. Point functions: limits and derivability. Recall of tensor algebra. Scalar and vector fields: convergence and differentiability. Directional derivative of scalar and vector fields. Vector and tensor fields: convergence and differentiability. Curves in affine spaces. Regular surfaces. Curvilinear coordinates and natural reference frame. Differential operators. Gradient of scalar, vector and tensor fields. Divergence of vector and tensor fields. Laplacian of scalar and vector fields. Curl of vector fields. Differential identities. Circulation on a curve of a vector field. Field lines. Flux through a surface of a vector field. Field tubes. Gauss' lemma. Gradient's, divergence's and curl's theorem. Stokes' theorem. Linear differential forms. Exact and closed forms. Irrotational and solenoidal fields. Harmonic functions. Poincaré inequality in the cube for solenoidal fields. **Deformable continuous systems.** Continuous systems. Regular motions for continuous systems. Eulerian and lagrangian representations. Eulerian, lagrangian and material derivatives of a quantity. Flux lines and stream lines. Stationary motions. Material lines, surfaces and volumes. Reynolds transport theorem. **Kinematic of deformable continua.** Fundamental relation of the kinematic of continuous systems. Vortex vector. Strain rate and vortex tensors. Linear expansion coefficient. Angular deformation. Volume expansion coefficient. Incompressible systems.

	Dynamics of deformable systems. Postulate of density. Axiom of mass balance. Equation of continuity of mass. Axiom of balance of linear and angular momentum. Internal stresses. Postulate of Euler-Cauchy on stresses. Tension and pressure. Cauchy's stress theorem. Stress tensor. Boltzmann postulate and symmetry of stress tensor. Indefinite equation of continuum mechanics. Kinetic energy of a continuous system. Theorem of kinetic energy for deformable bodies. **Thermodynamics of deformable systems.** Energy balance axiom. Heat flux and heat current vector. Absolute temperature. Entropy axiom. Inequalities of Clausius-Duhem and Clausius-Planck. Fourier law. Reversible and irreversible transformations. Free energy. Dissipation inequality. **Constitutive equations.** Perfect fluids. Cauchy's remark. Objective quantities. Objectivity of the strain rate tensor. Principles of inheritance, of equipresence, of material objectivity and of dissipation. Homogeneous memoryless systems. Barotropic fluids. Viscous fluids. **Stokesian and newtonian fluids.** Stokes' fluid. Relation among eigenvalues of **D** and **T**. Quadratic expression of the stress tensor in a stokesian fluid. Cauchy-Poisson's laws and newtonian fluids. Volume and shear viscosity coefficients. Stokes' relation. Navier-Stokes equation for compressible and incompressible fluids. General equation for heat transport in newtonian fluids. Heat equation for incompressible newtonian fluids at rest. Beltrami's diffusion equation. Helmholtz vorticity equation. Cauchy's integral. Helmholtz' theorem. Cauchy-Lagrange's theorem. **Elements of partial differential equations.** Linear and non linear equations. Quasi linear and semilinear equations. Examples of I and II order equations. Classification of II order equations in two variables. Examples of non linear equations. Cauchy's, Dirichelet's, Neumann's and Robin's conditions. Initial and boundary value problems. Regular and lipshitz domains. Well posedness. **Problems in fluid dynamics.** Boundary conditions for velocity, vorticity and temperature. Compressible newtonian fluid. Barotropic viscous or perfect newtonian fluid. Incompressible newtonian fluid and incompressible perfect fluid. Dimensionless equations. Adimensionalization and Reynolds number. IBVP for Navier-Stokes equations. Kinematic admissibility condition. Plane layer. Plane motions. Laminar motions. Laminar solutions of Navier-Stokes equations: Couette's flow, Poiseuille's flow and Couette-Poiseuille's flow. Rayleigh-Stokes oscillatory motion. Hydrostatics. Stevin's law. Archimede's principle and buoyant force. Bernoulli's equation for barotropic perfect fluids. Torricelli's theorem. Irrotational motions. **Stability.** Basic solution, perturbed solution and perturbation. Liapunov stability and asymptotic stability. Linear and non linear stability. Eigenvalues and linear stability. Criticality and principles of exchange of stabilities. Linearized problems and linear instability. Liapunov's functionals. Liapunov's theorem. **Uniqueness.** Time derivative of the kinetic energy for incompressible newtonian fluids. Reynolds-Orr equation for perturbation energy. Uniqueness theorem for Navier-Stokes IBVP. Energy stability of hydrodynamic equilibrium. **Bénard problem.** Quasi-incompressibility approximation of Oberbeck-Boussinesq. Thermodiffusive equilibrium in a horizontal plane layer. Rayleigh-Bénard problem. Adimensionalization, Rayleigh number and

	Prandtl number. Perturbations problem. Linear stability of thermodiffusive equilibrium. Boundary conditions for rigid or free planes. Analysis in terms of “normal modes” in a periodicity cell. Criticality analysis for free planes. Eigenvalues problem and linear stability condition for the Rayleigh number. **Elements of electromagnetism.** Postulate of the electric charge density. Balance of charge density axiom. Electric and magnetic fields. Electric displacement and magnetic flux density fields. Electric current density. Conduction, convection and displacement currents. Maxwell’s equations. Ampére’s law. Neumann-Lenz’s theorem. Gauss and magnetic Gauss’ theorems. Ohm’s law. **Magnetofluiddynamics.** Magnetic field equation. Dimensionless equations and magnetic Reynolds number. Electroconducting fluids. Lorentz’s law. Energy equation and Joule effect. Galileo-invariant equations: non relativistic magnetofluiddynamics axioms. Maxwellian tensor. Perfectly conducting fluids. Poynting vector. Magnetic energy evolution. Magnetic field decay in a fluid at rest. Alfven’s theorem. Faraday induction law for moving surfaces. Boundary conditions for electric and magnetic field. Conditions for insulated or conducting walls. Dielectric polarization. Magnetization: diamagnetic, paramagnetic and ferromagnetic media. **Magnetofluiddynamics solutions.** Electromagnetic plane waves. Progressive and regressive waves and propagation speed. Alfven magnetohydrodynamics waves for fluids at rest. Infinitesimal perturbations for incompressible newtonian, perfect and perfectly conducting fluids. Magnetosonic waves for barotropic, perfect and perfectly conducting fluids. Force free magnetic fields in magnetofluidstatics. Force-free field decay. Magnetofluiddynamic laminar solutions in a plane layer. Hartmann number. Hartmann flow and magnetohydrodynamic Couette flow.
Reference books	J. Serrin, *Mathematical Principles of Classical Fluid Mechanics*, Handbuch der Physik, Band VIII/1 S. Rionero, *Appunti di Magnetofluidodinamica*
Additional course materials	Teacher-provided course notes.

Work schedule				
	Total	Lectures	Hands-on learning (recitations/laboratories /seminars/other)	Self-study
Hours	150	40	15	95
ECTS credits	6	5	1	

Teaching methods	
	Lectures and exercise sessions.

Expected learning outcomes	
Knowledge and understanding	Acquiring the language, the mathematical formalism and tools and the knowledge of the main results that make it possible the reading and understanding of fluid dynamics textbooks and articles.
Applying knowledge and understanding	Acquiring the language, the mathematical formalism and tools and the knowledge of the main results that let to describe, analyze and solve fluid dynamics problems.

Making judgements	Reaching the capability of selecting techniques and theoretical tools needed to face up to fluid dynamics problems.
Communication skills	Mastering the introduction and presentation of objects and results already studied with mathematical accuracy and physical sense.
Learning skills	Reaching the skill in self-studying and self-learning new knowledge about fluid dynamics from books and articles.

Assessment and feedback	
Assessment methods	Oral final exam.
Evaluation criteria	• *Knowledge and understanding*: Correctness and completeness in the exposition of fluid dynamics concepts and results. • *Applying knowledge and understanding*: Correctness and precision in problem solving techniques. • *Making judgements*: Appropriateness in usage of techniques and results in theorical and applicative questions. • *Communication skills*: Strictness and correctness in the usage of mathematical and physical reasoning in fluid dynamics arguments. • *Learning skills*: Resolution of problems similar to that presented in the lessons.
Grading policy	Final evaluation expressed in 30th and exam passed with at least 18 over 30.

Additional information	